A linear programming based heuristic algorithm for charge and discharge scheduling of electric vehicles in a building energy management system

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A B S T R A C T

Electric vehicles (EVs) are becoming an attractive alternative to gasoline vehicles owing to the increase of greenhouse gas emissions and gasoline prices. EVs are also expected to function as battery storages for stabilizing large fluctuations in the power grid through the vehicle-to-grid power system, which requires smart charge and discharge scheduling algorithms. In this paper, we develop a linear programming based heuristic algorithm on a time-space network model for charge and discharge scheduling of EVs. We also develop an improved two-stage heuristic algorithm to cope with uncertain demands and departure times of EVs, and evaluate the effect of the smart charge and discharge scheduling of EVs on a peak load reduction in a building energy management system.

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1. Introduction

With rising greenhouse gas emission and gasoline prices, electric vehicles (EVs) are becoming an attractive alternative to gasoline vehicles. However, the rapid growth in electricity demand may cause large and undesirable peak loads in the power grid. Fortunately, EVs can flexibly coordinate charge schedules, and most owners will not be inconvenienced, providing that the EV batteries are full before departure. A wide variety of models and algorithms has been proposed for charge scheduling of EVs. Clement et al. [2] proposed a quadratic programming model to minimize power losses and voltage deviations. Deilami et al. [4] reported a fast heuristic algorithm, called the maximum sensitivity selection algorithm, to minimize the total cost involved with the additional electricity demands of EVs and power losses. Sotomme et al. [22] described and compared three optimization models; minimizing power losses, minimizing load variance, and maximizing load factor. Soares et al. [21] proposed a linear programming (LP) model that minimizes deviations between expected and actual demands, which was suitable for quasi-real time applications because of its low computational cost. Hernández-Arauzo et al. [11] formulated the scheduling problem as a sequence of constraint satisfaction problems (CSPs) over time called the dynamic CSP, and decomposed each CSP into three instances of a one-machine scheduling problem. Kim et al. [17] analyzed performance measures of two typical charge scheduling methods: the first-in-first-out and the processor sharing under a realistic stochastic model for EV battery charging stations. EVs can offer further benefits to the power grid by discharging electricity from their batteries, which is called vehicle-to-grid power [15,16]. The rapid growth of intermittent renewable energy sources, such as photovoltaic and wind power generation, requires huge number of battery storages for stabilizing the large fluctuations in the power grid. EV batteries are expected to provide an alternative to expensive stationary battery storages and to play an important role in the emerging power grid that has a large number of renewable energy sources.

Several optimization models and algorithms have been proposed for charge and discharge scheduling of EVs. Han et al. [8] reported a dynamic programming algorithm to optimize frequency regulation, in which charge and discharge scheduling of individual EVs was considered rather than that of multiple EVs. Clement et al. [3] proposed an LP model and He et al. [10] proposed a quadratic programming model to minimize the total charge cost. Zakariazadeh et al. [26] formulated a multi-objective model to minimize operational costs and emissions as a mixed integer nonlinear programming (MINLP) model and it solved with Bender’s decomposition technique. Kawashima et al. [14] reported a mixed integer linear programming (MILP) model to minimize a total
charge cost, and García-Villalobos et al. [7] presented a comprehensive review of models and algorithms for charge and discharge scheduling of EVs.

We have considered an optimal electric power management in a home energy management system (HEMS) using photovoltaics (PVs) and stationary battery storages, and developed an MILP formulation to minimize electricity costs on a time–space network model [5]. In this paper, we investigate peak load reduction in a building energy management system (BEMS), and develop an LP based heuristic algorithm on a time–space network model for charge and discharge scheduling of EVs. Japanese electric utilities use the net feed-in tariff system; they are obliged to purchase surplus electricity generated by renewable energy sources immediately, and are not allowed to purchase electricity from battery storages. Therefore, the electricity in EV batteries is available for only onsite demand. Conventional studies of BEMS have mainly focused on managing appliance demand, such as air conditioning, lighting, and elevators [23,12,6]. Therefore, peak load reduction in a BEMS may encourage individual enterprises to introduce EVs under the current Japanese electricity tariff system.

The proposed algorithm is aimed at working as a sub-routine of various types of BEMS, in which demands and departure times of EVs are given by some prediction algorithms [13] and updated frequently. This approach has been often referred as the rolling horizon in logistics and production planning [24,20]. Exact optimization algorithms, which are time-consuming, are not suitable for charge and discharge scheduling of EVs, because the schedules must be updated whenever demands and departure times of EVs are updated. Thus, we develop a fast LP based heuristic algorithm for computational efficiency and a two-stage heuristic algorithm to cope with uncertain demands and departure times of EVs.

The rest of the paper is organized as follows. In Section 2, we illustrate the considerations for a BEMS with EVs and a time–space network model to describe the charge and discharge schedules of EVs. In Section 3, we formulate an optimization problem for charge and discharge scheduling of EVs in the time–space network model. We present an LP based heuristic algorithm in Section 4 and a two-stage heuristic algorithm to cope with uncertain demands and departure times of EVs in Section 5. We report the computational results in Section 6 and make concluding remarks in Section 7.

2. Building energy management system with electric vehicles

We focus on a local electric power network of a BEMS that includes a number of EVs, in which every EV is used not only as a means of transportation but also as a battery storage. EVs repeatedly charge and discharge their batteries with supplementary electricity to satisfy the demand for appliances in the building. Fig. 1 illustrates the local electric power network of the BEMS. The local electric power network has an alternating current (AC) and a direct current (DC) electric transmission system. Several electric devices are connected with each other through converters and inverters resulting in electric decay.

The main functions of EVs are travel and charging and discharging their batteries. We consider the charge and discharge scheduling of EVs supposing that the travel schedules of all EVs are given. EVs are usually disconnected from the local electric power network while they are away from their parking lots. We accordingly use a time–space network model to describe the dynamic changes in the local electric power network over the time horizon. The time–space network model is an expansion of the standard network model, which illustrates a dynamic network varying over time, and the model has a wide variety of applications, such as airline scheduling [9], forest management [1], vehicle scheduling [19], and evacuation routing [25]. A time–space network contains a copy of the node set of the underlying network for every time period, in which each pair of nodes in consecutive time periods is connected by a forward directed arc.

Fig. 2 shows a time–space network model for the local electric power network, in which copies of a node of the underlying network are arranged in a row. The possible charge and discharge operations of EVs are represented by forward directed arcs. We introduce internal nodes, called AC and DC systems, to aggregate the electricity supply through AC and DC converters, respectively. The time–space network model makes it easy to describe the structural changes in the local electric power network; we can describe the absence of EVs from their parking lots by removing the corresponding nodes from the time–space network.

3. Mixed integer linear programming formulation

We consider an optimization problem to achieve peak load reduction in the BEMS. We formulate the problem as an MILP model on the time–space network model. The switching operations between charging and discharging EV batteries are described with binary variables, in order to control the frequency of switching operations appropriately while satisfying AC and DC loads that change significantly in a short time. The inflow and outflow at each node represents the total electricity supply and demand, respectively. We set the lower and upper bounds of the amount of inflow and outflow at each node, which represents the demands and limits of the electric equipment in the local electric power network, e.g., the electricity demands of AC and DC loads and EVs, the maximum electricity supply from the power grid, and the maximum charge and discharge of EV batteries per unit time period. We also set the departure and arrival times of EVs supposing that all EVs depart their parking lots once or twice in the scheduling period. The sets, parameters and variables in the MILP formulation are defined as follows.

**Sets**

- \( N \) set of EVs.
- \( J_i \) set of trips of EV \( i \).
- \( T \) set of time periods.
- \( T_i \) set of time periods when EV \( i \) stays in the parking lot.
- \( T_{peak} \) set of time periods on peak hours.

**Parameters**

- \( c_i \) battery capacity of EV \( i \).
- \( f_i \) maximum electricity charge per unit time of EV \( i \).
- \( g_i \) maximum electricity discharge per unit time of EV \( i \).
- \( a_{ij} \) departure time of the \( j \)th trip of EV \( i \).
- \( b_{ij} \) arrival time of the \( j \)th trip of EV \( i \).
The electric utilities should immediately generate the sufficient quantity of the electric power to meet almost all demands at that moment, because the total capacity of battery storages is much larger than the actual electricity consumption $e_{ij}$, so that EV $i$ is never fully discharged. The average electricity supply from the power grid during peak hours $T_{\text{peak}}$ is defined as follows:

$$z = \frac{1}{T_{\text{peak}}} \sum_{t=1}^{T_{\text{peak}}} (x_t^A + x_t^D). \tag{1}$$

The electric utilities should immediately generate the sufficient quantity of the electric power to meet almost all demands at that moment, because the total capacity of battery storages is much smaller than the total amount of electricity demands in the global electric power network. However, most power generators are not capable to meet the rapid change of electricity demands. Leveling the variation of electricity demands is also important as well as minimizing the total amount of electricity demands during peak hours. We accordingly introduce the optimization model to minimize the average and total deviation of the electricity supply from the power grid simultaneously. The MILP formulation for charge and discharge scheduling of EVs is described as follows:

$$\text{minimize} \quad z + \frac{\alpha}{T_{\text{peak}}} \sum_{t=T_{\text{peak}}}^{T_{\text{peak}}} |z_t^A + z_t^D - z| \tag{2}$$

subject to \quad \begin{align*}
  f_i & x_{ti} + u_{ti} \leq c_i, \quad t \in T_i, \quad i \in N, \\
  z_t^A + z_t^D & \leq L, \quad t \in T, \\
  x_{ti} + y_{ti}^A + y_{ti}^D & \leq 1, \quad t \in T_i, \quad i \in N, \\
  x_{ti}^A + x_{ti}^D + \sum_{i \in N} g_{ti} y_{ti}^A = E_t^A, \quad t \in T, \\
  x_{ti}^A + x_{ti}^D + \sum_{i \in N} g_{ti} y_{ti}^D = E_t^D, \quad t \in T, \\
  z_t^A + \sum_{i \in N} g_{ti} y_{ti}^D = E_t^D + \sum_{i \in N} f_i x_{ti}, \quad t \in T, \\
  f_i x_{ti} + u_{ti} \geq d_{ti}, \quad j \in J, \quad i \in N, \\
  u_{ti-1} + f_i x_{ti-1} = u_{ti} + g_{ti} (y_{ti}^A + y_{ti}^D), \quad t \in T_i, \quad i \in N, \\
  u_{ti} - u_{bi} = e_{ij}, \quad j \in J, \quad i \in N, \\
  x_{ti} \in \{0, 1\}, \quad y_{ti}^A \in \{0, 1\}, \quad y_{ti}^D \in \{0, 1\}, \quad t \in T_i, \quad i \in N, \\
  0 \leq u_{ti} \leq c_i, \quad t \in T_i, \quad i \in N, \\
  x_t^A + x_t^D \geq 0, \quad t \in T. \tag{3}
\end{align*}$$

The objective function (2) minimizes both the average and the total deviation of the electricity supply from the power grid during peak hours, where $\alpha$ is a parameter controlling their trade-off. Eq. (3) shows the battery capacity constraints of EVs. Eq. (4) describes the electricity supply constraints from the power grid. Eq. (5) shows the exclusive constraints of charge and discharge operations of EVs. Eqs. (6)–(8) show the demand constraints of AC and DC loads and EVs, where we assume $x_{ti} = y_{ti}^A = y_{ti}^D = 0$ for all $t \in T \setminus T_i, i \in N$. Eq. (9) shows the flow conservation constraints of EV batteries. Eq. (10) shows the actual electricity consumption for EV trips. In addition, we also introduce a set of dummy nodes to avoid any infeasible instances caused by short parking times for EVs satisfying their electricity demands.

### 4. Linear programming based heuristic algorithm

The optimization problem often becomes hard to solve exactly for a long scheduling period, because the size of the MILP formulation is in proportion to the length of the scheduling period. In addition, the EV charge and discharge schedules must be updated immediately when the demands and departure times of EVs are updated. We accordingly attain a feasible solution of the MILP formulation by rounding an optimal solution of its LP relaxation problem rather than applying time-consuming exact algorithms.
The LP based heuristic algorithm consists of the following two steps: (i) solving the LP relaxation problem of the MILP formulation and (ii) rounding an optimal (fractional) solution of the LP relaxation problem to attain a feasible (integer) solution of the MILP formulation. Here, the LP relaxation problem is defined by replacing the binary constraints (11) with 0 \leq y^A_i \leq 1, 0 \leq y^D_i \leq 1 for all \ t_A \in T_i, i \in N. If the values of y^A_i; t and y^D_i; t are fractional, then it rounds down them to zero and increases the values of z^A_i; t, z^D_i; t, and u_i; t to satisfy Eqs. (6), (7), and (9), respectively. If Eq. (3) (the battery capacity constraint) is violated by increasing the value of u_i; t, then it rounds down the value of the last electricity charge x(s) > 0 (s < t) and increases the value of z^D_i; t to satisfy Eq. (7) (the demand constraint) at time s. If the values of x_i; t are fractional, then it rounds them up to one and increases the values of z^D_i; t to satisfy Eq. (7). Fig. 3 illustrates the rounding procedures.

5. Two-stage heuristic algorithm

We need to handle the uncertainties and fluctuations of the input data for real applications; the demands and departure times of EVs often change because of heavy traffic and changes to routes. Otherwise, EVs will not hold a sufficient charge for their trips or supply AC and DC loads during peak hours. Next, we consider two uncertainties for the input data illustrated in Fig. 4: (a) departure times are made earlier than the original schedule, and (b) electricity demands become larger than the original demands. In Fig. 4, schedule B can meet the electricity demand more than schedule A in both cases (a) and (b). Thus, we can make room to change the travel schedules of EVs by charging EV batteries with a certain amount of supplementary electricity as soon as possible. However, the standard algorithm for the LP relaxation problem, called the simplex algorithm, can hardly afford supplementary electricity to cope with the uncertainties because it basically converges to an extreme point of the feasible region that often has no margins for most of the constraints.

We develop a two-stage heuristic algorithm to maximize the amount of supplementary charged electricity while minimizing the average and total deviation of the electricity from the power grid during peak hours: (i) the algorithm first obtains an objective value \( \zeta^* \) of a feasible solution by applying the LP based heuristic algorithm to the MILP formulation in Section 3, and then (ii) it obtains another feasible solution of the same objective value \( \zeta^* \) obtained in stage (i) by solving the following MILP formulation.

Parameters
\[ \zeta^* \] objective value of a feasible solution obtained in stage (i).

Variables
\[ \beta_{ij} \] amount of supplementary electricity charged for the jth trip of EV i.

Problem
\[
\begin{align*}
& \text{maximize} & \sum_{i \in N} \sum_{j \in J_i} \beta_{ij} \\
& \text{subject to} & f_i x_{ij} + u_i x_{ij} &= d_{ij} + \beta_{ij}, & j \in J_i, & i \in N, \\
& & & & (15)
\end{align*}
\]
We replace Eqs. (2) and (8) with Eqs. (14) and (15), respectively, to maximize the amount of supplementary charged electricity. We also add Eq. (16) to achieve the same objective value $\zeta^*$ obtained in stage (i). If it is hard to charge any supplementary electricity in the MILP formulation, then it is an alternative to replace the right-hand side of Eq. (16) with $\zeta^* + \epsilon$, where $\epsilon > 0$ is a parameter.

6. Computational results

We conduct computational experiments to evaluate the effect of the smart charge and discharge schedules of EVs obtained by the two-stage heuristic algorithm in Section 5 and the computational efficiency of the LP based heuristic algorithm in Section 4.

The performance of the EV batteries is estimated according to [18], where the discharge electric current is 50 A and the voltage is 250 V, and the normal charge electric current is 15 A and the voltage is 200 V. The capacity of the batteries is set to 21 kWh. We obtain the total AC and DC load measured every 10 min during a day from a real eight-story office building (the total floor area is 11,822 m$^2$), as shown in Fig. 5. The scheduling period is set to a day and half, where the peak hours are set to 8:30–18:00. We generate five traveling schedules artificially and assign them to all EVs in turn. Here, we assume that all EVs are used for only commercial services (not for commuting employees) and all EVs stay in the parking lots of the office building during outside of working hours. The number of EVs is set to 10, and all EVs depart their parking lots once or twice during the scheduling period. Their departure time is from 7:00 to 19:00 and their travel duration is from two to seven hours. The initial charge of all EVs is from 30% to 55% of the battery capacity, while the actual consumption $e_{ij}$ of all EVs is from 20% to 80% of the battery capacity, where recall that $d_{ij} \geq e_{ij}$ holds for all $i \in N$, $j \in J_i$ so that EV $i$ is never fully discharged. The unit time period of the time–space network is set to 10 min. All input data can be obtained from the electronic supplementary material. Under these conditions, we generate instances of the MILP formulation, which have approximately 8000 constraints and 7700 variables. Here, the parameter $\alpha$ is set to 12.70 for leveling the variation of electricity demands during peak hours; this value is adjusted so as to minimize the top 5% of the electricity demands during peak hours on preliminary computational experiments. We use a latest solver called CPLEX12.6 for solving the LP relaxation problem. For each evaluation, we run the algorithms for five different data sets of traveling schedules of all EVs.

Initially, we evaluate the effect of the smart charge and discharge scheduling of EVs on a peak load reduction. Fig. 6 shows the electricity supply from the power grid for every 10 min, which is illustrated by a simple moving average of 30 min. The light gray line labeled “No EVs” shows the electricity supply without EVs, which is equal to the total AC and DC load. The dark gray line labeled “Only charge” shows the results for the smart charge schedules of EVs obtained with the two-stage heuristic algorithm with no EV discharge. The black line labeled “Charge and discharge” shows the results for the smart charge and discharge schedules of EVs obtained with the two-stage heuristic algorithm.

Table 1 shows the average, maximum, and standard deviation of electricity supply from the power grid during peak hours, and we observe that the smart charge and discharge scheduling of EVs reduces the electricity supply required during peak hours. Fig. 7 shows the state of charge for each EV with their corresponding schedules, where the gray zone illustrates the absence of EVs from their parking lots. To the extent of our computational experiments, we succeed to increase the amount of supplementary charge at stage (ii) while keeping the same objective value $\zeta^*$ obtained in stage (i). We evaluate the robustness of the schedule by the margins for charge time up to departure and the amount of charge for each EV trip. Stage (ii) increases the margins for charge time up to departure by 72 min and the amount of charge for each EV trip by 0.90 kWh on average. That makes room to charge the travel schedules of EVs such as departing earlier time and traveling longer distances.

Next, we evaluate the effect of the smart charge and discharge scheduling with different numbers of EVs. Fig. 8 shows the electricity supply from the power grid for every 10 min with different numbers of EVs. Table 2 shows the average, maximum, and standard deviation of electricity supply from the power grid during peak hours. Although the total amount of electricity supply over
the scheduling period increases in proportion to the number of EVs, it is possible to reduce the average and standard deviation of electricity supply during peak hours more effectively with a larger number of EVs. We note that the maximum value of the electricity supply does not decrease monotonically because the optimization model is not to minimize the maximum value during peak hours. Finally, we evaluate the computational efficiency of the LP based heuristic algorithm for solving the MILP formulation in

![Graphs showing state of charge for EVs](http://dx.doi.org/10.1016/j.omega.2016.04.005i)

Fig. 7. State of charge for EV1–10 obtained with the smart charge and discharge schedules.
Table 2
Statistics for electricity supply from the power grid during peak hours with different numbers of EVs (kWh).

<table>
<thead>
<tr>
<th>Statistics</th>
<th>No EVs</th>
<th>5 EVs</th>
<th>10 EVs</th>
<th>20 EVs</th>
<th>40 EVs</th>
<th>80 EVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg.</td>
<td>457.22</td>
<td>450.07</td>
<td>447.68</td>
<td>442.98</td>
<td>441.93</td>
<td>427.49</td>
</tr>
<tr>
<td>Max.</td>
<td>565.55</td>
<td>553.38</td>
<td>557.62</td>
<td>540.41</td>
<td>560.27</td>
<td>557.55</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>69.60</td>
<td>62.21</td>
<td>57.69</td>
<td>50.48</td>
<td>44.06</td>
<td>40.70</td>
</tr>
</tbody>
</table>

Table 3
Computational results of the LP based heuristic algorithm and the MILP solver for solving the MILP formulation in Section 3.

<table>
<thead>
<tr>
<th>#EV</th>
<th>LP based heuristic</th>
<th>MILP solver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obj.</td>
<td>Gap</td>
</tr>
<tr>
<td>5</td>
<td>154.51</td>
<td>6.14%</td>
</tr>
<tr>
<td>10</td>
<td>139.52</td>
<td>8.55%</td>
</tr>
<tr>
<td>20</td>
<td>110.05</td>
<td>18.33%</td>
</tr>
<tr>
<td>40</td>
<td>80.74</td>
<td>35.57%</td>
</tr>
<tr>
<td>80</td>
<td>70.57</td>
<td>41.41%</td>
</tr>
</tbody>
</table>

Fig. 8. Electricity supply from the power grid for every 10 min with different numbers of EVs (kWh).

Fig. 9. Time courses of the objective values for the LP based heuristic algorithm and the MILP solver with 80 EVs.

Section 3, by comparing it with a latest MILP solver called CPLEX12.6. Table 3 shows the computational results for the algorithms on a MacBook Pro laptop computer with a 2.7 GHz Intel Core i7 processor. The second column shows the optimal values of the LP relaxation problems. The third and sixth columns show the objective values (not the electricity supply from the power grid) of the LP based heuristic algorithm and the MILP solver, respectively. The fourth and seventh columns show the relative gap (%) $100 \times (\zeta - \zeta_{LP})/\zeta$ of the objective value $\zeta$. The fifth, eighth and ninth columns show the computation time (s) of the algorithms. The computation time to solve the LP relaxation problems is almost same as that of the LP based heuristic algorithm, because we need little computational effort for rounding an optimal (fractional) solution of the LP relaxation problem. The eighth column shows the computation time (s) of the MILP solver to the best upper bounds (not the optimal values) with time limits of 3600 s, abbreviated to TTB. The ninth column shows the computation time (s) of the MILP solver to improve the upper bounds obtained by the LP based heuristic algorithm, abbreviated to TTI. We first observe that the MILP solver attains no optimal solutions for any cases within the time limits. Although the MILP solver achieves better upper bounds than the LP based heuristic algorithm, it consumes much more computation time to achieve them. Fig. 9 shows the time courses of the objective values for the LP based heuristic algorithm and the MILP solver with 80 EVs. The dotted horizontal line shows the optimal value $\zeta_{LP}$ of the LP relaxation problem. The gray and black lines show the upper bounds obtained by the LP based heuristic algorithm and the MILP solver, respectively. We observe that the MILP solver takes 465.00 s to improve the upper bound obtained by the LP based heuristic algorithm. The family of simplex algorithms performs efficiently for solving a series of LP instances with small modifications, known as the warm start technique, which uses the optimal solution of the solved LP instance as a starting point and solves the modified LP instance faster than it can be solved from scratch. Based on these observations, the two-stage heuristic algorithm is suitable for coordinating charge and discharge schedules of EVs with uncertain demands and departure times.

7. Conclusion

We examined reducing the peak load in a BEMS by coordinating the charge and discharge schedules of EVs. We presented an LP based heuristic algorithm in a time–space network model to create a smart charge and discharge schedule of EVs within a limited computation time. We also developed an improved two-stage heuristic algorithm to cope with uncertain demands and departure times of EVs. According to computational experiments for a BEMS, the two-stage heuristic algorithm achieves a peak load reduction and handles the uncertain demands and departure times of EVs within a limited computation time. Our computational results provide an incentive for individual enterprises to introduce EVs despite the limitation of the net feed-in tariff system.

Appendix A. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.omega.2016.04.005.

References

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