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Collaborative optimization for train scheduling and train stop planning on high-speed railways☆

Lixing Yang*, Jianguo Qi, Shukai Li, Yuan Gao

State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

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ABSTRACT

Focusing on providing a modelling framework for train operation problems, this paper proposes a new collaborative optimization method for both train stop planning and train scheduling problems on the tactic level. Specifically, through embedding the train stop planning constraints into train scheduling process, we particularly consider the minimization of the total dwelling time and total delay between the real and expected departure times from origin station for all trains on a single-track high-speed railway corridor. Using the stop planning indicators as important decision variables, this problem is formally formulated as a multi-objective mixed integer linear programming model, and effectively handled through linear weighted methods. The theoretical analyses indicate that the formulated model is in essence a large-scale optimization model for the real-life applications. The optimization software GAMS with CPLEX solver is used to code the proposed model and then generate approximate optimal solutions. Two sets of numerical examples are implemented to show the performance of the proposed approaches. The experimental results show that, even for the large-scale Beijing–Shanghai high-speed railway, the CPLEX solver can efficiently produce the approximate optimal collaborative operation strategies within the given gaps in acceptable computational times, demonstrating the effectiveness and efficiency of the proposed approaches.

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1. Introduction

With the development of social economy, a large scale of high-speed railways have been put into operation or been being under construction in some countries to meet large passenger flow demands. Thus, effectively managing and operating the high-speed railways then becomes an important issue for the different railway companies. On the high-speed railway corridors, train stop planning and train scheduling, which are regarded as the most important parts of train operations and managements, have often been studied separately up to now due to the complexity of each involved problem. In practice, as a sub-problem of the train operating managements, the train stop plan is usually made on the basis of predictions of the potential passenger flow for different origin–destination pairs, and the generated stop plan needs to be adjusted repeatedly in order to satisfy the realistically changing requirements over the service time horizon. With the specified stop plan design, the scheduling process then aims to determine the arrival and departure times at each predetermined service station such that no operational conflicts occur between different trains, and expectedly, the resource utilization of the railway traffic system can be maximized. Typically, in comparison to the train stop plan, a train schedule can provide more detailed operational instructions for all the involved trains on the tactic level-based decision strategies.

In general, train stop planning and train scheduling are usually included in different pre-trip planning stages. For clarity, Lusby et al. [36] gave a detailed flowchart to show the railway planning process, as shown in Fig. 1. In this process, once a pre-specified train stop plan is changed according to the realistic requirements (for instance, busy transport during the Spring Festival in China), a new train schedule on the railway needs to be regenerated to satisfy the varied stop plan constraints. Obviously, this process essentially increases the complexity of railway operations. Aiming to produce a comprehensive operational plan on the tactic levels, we are particularly interested in how to design effective methods to collaboratively optimize these two problems, and then generate a system-optimization based planning strategy. Since this topic still has not attracted sufficient attention in the literature, we hereinafter shall address this issue formally.

1.1. Literature review

Practically, the line planning and train scheduling are two significant parts for determining a detailed train operation plan. In the stage of line planning, one needs to specify the number of trains, type of trains, the stop plan for each train, etc., which is

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* Corresponding author.
E-mail address: lxyang@bjtu.edu.cn (L. Yang).

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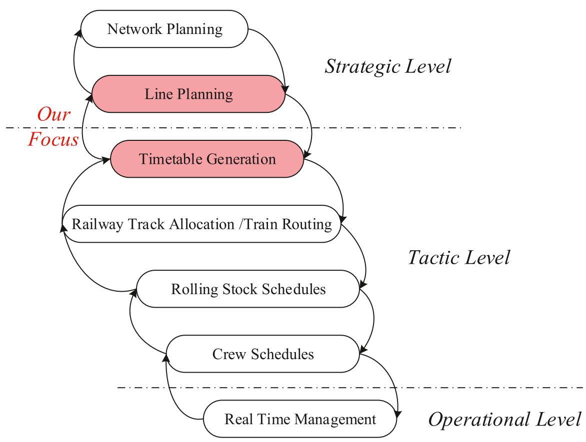

Fig. 1. The railway planning process.

usually included in the strategic level-based decision making. In this process, the train stop planning is of particular importance for the real-world operations once the number and type of trains are provided. For this problem, the main task is to determine the stopping stations for each train on railway lines to satisfy the passenger demands with the minimum cost. In the literature, five kinds of train stop plans are often be considered in reality, including the all-stop operation, skip-stop operation, zonal operation, express/local operation and combined stop operation. Each type of these stop plans has its own advantages, disadvantages and applicable conditions. Typically, although the all-stop operation is obviously the simplest stop planning for satisfying all passenger demands, it might probably enhance the total travel time of long-distance passengers. With this concern, Vuchic [45] further considered the skip-stop operation, zonal operation and express/local operation, and gave evaluation methods of using these stop strategies by summarizing, analyzing and comparing each of them. Zolfaghari et al. [56] pointed out that although stop skipping can effectively reduce the waiting times for passengers on boarding a vehicle and those at downstream stops, it still might increase the waiting time for passengers at skipped stops and those who are requested by the driver to alight at a given stop to wait for the next vehicle in service. According to the operating experience of Japanese Shinkansen, Lan [30] proposed that in designing operation plans of Beijing–Shanghai high-speed railway, staggered stop plan, direct and other forms of stops programs should be taken into account. In addition, Cheng and Peng [6] considered the combined stop plan with elastic demands. The computational results showed that the combined stop plan is more suitable for some special passenger flow.

In the literature, the majority of existing researches focus on investigating different train stop strategies through a variety of optimization methods. For instance, the zonal operation was studied by Salzbom [41] and Ghoneim and Wirasinghe [16]; skip-stop operation by Suh et al. [43], Zheng et al. [55], Wang et al. [46], and Cao et al. [3]; and express/local operation by Nemhauser [38], Song et al. [42], and Xiong [47]. In addition, Guo [21] explicitly studied the characteristics and applicable conditions of these three stop planning types, and formulated the corresponding optimization models for these three stop strategies. By considering different realistic conditions, Lee et al. [31,32] proposed a mathematical model to optimize the skip-stop operation strategy in urban rail transit systems, and also designed an efficient genetic algorithm to search for a near-optimal solution. Goossens et al. [18,19] and Chang et al. [5] treated train stop planning problem as a subproblem of the entire operation plan. Note that stakeholders of the train operation plans are associated with both passengers and railway companies, the objective function can be considered as (i) minimization of total operation cost and passengers total travel time loss (e.g., Chang et al. [5]), (ii) maximization of the saved total passenger travel time (e.g., Zheng et al. [55]) and (iii) minimization of the generalized travel cost and stop quantity (e.g., Deng et al. [13]).

With the given stop plan for each train, a train timetable can be scheduled to instruct the detailed operations of different trains on the railway, which is included in the framework of the job-shop scheduling problem and its variants (e.g., Pan and Ruiz [39] and Rustogi and Strusevich [40]). An efficient train timetable on a railway line or a railway network is always required to ensure that the resources of railway infrastructure can be utilized optimally. Since Szpigel's [44] work with the objective of minimizing the total travel time, motivated by Greenberg's [20] branch-and-bound approach for the job-shop scheduling problem, the research of the train scheduling problem has attracted tremendous attention from numerous researchers and engineers. In general, there are three classes of techniques proposed to obtain a desirable operation plan, as pointed by Yang et al. [51], namely, (1) optimization methods studied by Szpigel [44], Higgins et al. [23], Barrena et al. [1], Harrod [22], Yang et al. [50,53], Kang et al. [26], Jaehn et al. [25], Li et al. [35] and D'Ariano et al. [11,12]; (2) simulation methods studied by Dorfman and Medanic [14], Li et al. [34] and Xu et al. [48,49]; and (3) expert systems studied by Chang and Thia [4], Iida [24] and Komaya et al. [27,28]. For optimization approaches, Higgins et al. [23] presented a two-objective optimization model on single line rail corridor to minimize the delay time and fuel consumption cost; with the purpose of optimizing the energy consumption and passengers' total travel time, Ghoseiri et al. [17] investigated a multi-objective train scheduling model on a railroad network which includes single and multiple tracks, as well as multiple platforms with different train capacities; Barrena et al. [1] presented three formulations for designing and optimizing train timetable adapted to a dynamic demand environment, with the aim of minimizing passenger average waiting time; D'Ariano et al. [11,12] and Corman et al. [7–10] explored a series of realistic railway traffic management problems (routing, scheduling, dispatching, etc.) and proposed efficient solution methodologies, such as the local search, branch and bound algorithm and tabu search algorithm. Considering the complexity of real-world situations, many researchers focused on using uncertain programming to get a more robust train timetable. Kroon et al. [29] proposed a stochastic optimization model to improve the robustness of a given cyclic railway timetable; Meng and Zhou [37] presented a two-stage stochastic model to find the robust rescheduling strategies when an incident occurs on the railway link under different forecasted operational conditions, in which the purpose is to minimize the expected additional delay; Yang et al. [51] formulated the rescheduling problem as a two-stage expected fuzzy optimization model on a two-way double-track railway line to minimize the expected total delay; by using a space–time network to represent the choice of train trajectories and introducing a fuzzy variable-based recovery time to capture the uncertainty of incident duration, Yang et al. [52] formulated a credibilistic two-stage fuzzy 0–1 integer optimization model to find a reliable rescheduling plan for trains in a double-track railway network when the capacity reduction is caused by a low-probability incident.

1.2. The proposed methods

From the viewpoints of system optimization, both train stop planing and train scheduling problems can only produce the suboptimal solutions for the entire train operation process. Note that, although the train stop planning and train scheduling were often studied independently in the literature due to the complexity of

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each of them (see Fig. 1), the consistency of these two problems still can be well considered in a unified objective function (for instance, minimizing the passengers' total travel time, minimizing the total cost of operations, etc.), because the main stakeholders for both of them are essentially associated with passengers and operation companies. Moreover, another significant issue is that the decision variables of these two problems are essentially related to each other, since the needed decision variables of train scheduling problem are the departure and arrival times at each station which are also dependent on the predetermined stop plan (that is, the dwelling time of a train at a station can be calculated by the difference of its departure time and arrival time). Then, this relationship enlightens us to find an effective and feasible method to combine the train stop planning and train scheduling process together into a collaborative optimization model to generate a system-based optimal planning strategy. This study aims to provide the following contributions to the framework of the train operation optimization methods.

(1) As a novel idea in literature, this paper first integrates the train stop planning and train scheduling problems together into a fundamental collaborative optimization model, in which the objective function is to minimize the total delay at origin station and dwelling time at intermediate stations. Clearly, in comparison to traditional strategies, our proposed method essentially relaxes the potential restrictions of the pre-specified train stop plan to train schedules, leading to collaboratively optimized stopping plan and train schedule on the system optimization level. In the formulation, the systematic constraints include passenger demand constraints, the minimum dwelling time constraints, headway constraints, etc.

(2) In formulating process, we especially introduce a binary variable to indicate the stop choice at each station for each train. Through formulating passenger demand constraints, we can produce a feasible stopping plan successfully for all trains operated on the railway line; moreover, to generate a collaborative solution strategy, we give a connection between the stopping plan and train schedules by the minimal dwelling time constraints. In the model, the headway constraints are used to guarantee the safe operations of trains on the railway line. The proposed formulation turns out to be a mix-integer linear programming model, and some properties are analyzed to illustrate the complexity of the proposed model.

(3) Numerical examples are implemented to demonstrate the effectiveness and efficiency of our proposed methods. Specifically, the CPLEX solver in GAMS optimization software is used to solve the proposed model. Some real-world operation data on Beijing–Shanghai high-speed railway are collected to generate a collaborative solution. In particular, the performance of the XPRESS and CPLEX solvers on solving the proposed model are also compared on the basis of considered railway corridor. By adopting different sets of parameters and dispatching criteria, the computational results show that CPLEX solver can efficiently generate near-optimal solutions for the considered problems within acceptable computational times, demonstrating the effectiveness of the proposed approaches.

The remainder of this paper is organized as follows. Section 2 gives the detailed problem statements and assumptions, including train speed, train loading capacity and interstation travel times. In Section 3, the collaborative mathematical model is formulated to generate the train stop plan and train schedule simultaneously through introducing interconnection constraints. In Section 4, the proposed model is applied to Beijing–Shanghai high-speed railway corridor by adopting the real-world operation data. Finally, some conclusions and further works are presented in Section 5.

2. Problem statements and assumptions

2.1. Problem statements

Train stop planning, which is regarded as a key component in providing high-quality transport services, can greatly affect the quality of train schedules and service levels for passengers. In practice, the train stop plan is often pre-designed on the basis of predicted passengers demands and required operational efficiency levels. Mathematically, we can use a binary decision variable as an indicator to represent whether a train is allowed to stop at a station or not. When a set of feasible decision variables are obtained, the stop plan for each train can be specified easily.

Next, an example is given to illustrate how the train stop plan affects the quality of the generated schedules. We consider a railway corridor with four stations and three links for simplicity. As shown in Fig. 2(a), three trains, whose origin and destination are nodes 1 and 4, respectively, are taken into account to generate the stop plan. In this figure, the solid dot "●" and hollow dot "○" respectively, represent stop and non-stop operations at the current station. Clearly, the specific stop plans for individual trains are different, in which train A is scheduled as a through train with no stops at intermediate stations, while train C is scheduled to stop at each intermediate station. In comparison, train B is only arranged one stop at station 3. In practice, these three trains will jointly provide different services for the various travel demands of passengers (including long-distance and short-distance demands).

In the real-life operation planning, a high-quality train schedule is needed to provide pre-trip guidance operations once the stop plan is determined, which is used to specify the arrival, departure and dwelling times of each train at each station, and also clarifies the basic information for passengers during their journeys. Typically, the train stop plan needs to be guaranteed in the scheduling process, and moreover some additional constraints are also considered to adjust the arrival and departure times for a part or all of

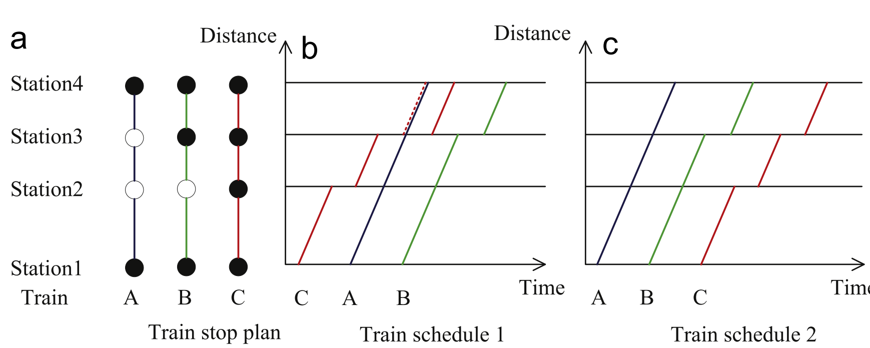

Fig. 2. An illustration for a train stop plan and train schedules.

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trains to keep the safety of train operations in generating the train schedules.

Practically, the generated schedule is closely related to the pre-specified train stop plan and departure sequence. As shown in Fig. 2, a given stop plan may correspond to different schedule patterns, usually depending on different departure times. For instance, if the departure sequence is train C→ train A→ train B such that all the trains are allowed to traverse without obstacles, train A may conflict with train C at station 3. Thus more dwelling time of train C at station 3 should be considered to eliminate this conflict, leading to the enhancement of total operation time (see Fig. 2(b)). However, we can avoid the extra delay if we adjust the above departure sequence as train A→ train B→ train C, shown in Fig. 2(c). On the other hand, when all-stop operations are scheduled for all of these three trains, a more balanced train timetable can be produced accordingly, shown in Fig. 3. Meanwhile, since trains A and B require more stop operations, the total travel time from station 1 to station 4 will be greater than that of train schedule shown in Fig. 2(c). In this illustration, we can easily draw that the final schedule can be impacted by both pre-specified train stop plan and departure sequence. In other words, a desirable train schedule is necessarily dependent on the predetermined stop plan and corresponding departure orders of all the involved trains.

In general, some potential drawbacks may occur in the process of scheduling trains once a stop plan is predetermined. On one hand, although the given train stop plan may be a preferable scheme related to passenger demands, it is possibly not a desirable one from the perspective of designing a train schedule; on the other hand, if a pre-specified train stop plan is changed according to the realistic requirements, an updated train schedule on the railway line needs to be regenerated to satisfy the new stop plan constraints. With this concern, collaboratively considering these two problems to achieve a systematic optimization-based train stop plan and train schedule turns out to be a theoretically challenging issue for the department of railway operations and managements. To the best of our knowledge, few related research has taken both of them into consideration simultaneously for the high-speed railway or regular railway traffic.

In this study, we particularly set up the relationship between arrival/departure times and dwelling time at the station to formulate the minimum dwelling time constraint (see Section 3.3 for more details), which can combine the train stop planning and train scheduling problems together into a collaborative model with the purpose of minimizing the total delay at origin station and dwelling time at intermediate stations for all trains. The following discussion aims to give a general description for operational environments of the involved problem. Assume that there is a single-track high-speed railway line with $|S|$ stations and $|T|$ trains. As shown in Fig. 4, the stations are sequentially numbered as $1, 2, \ldots, |S|$, in which stations 1 and $|S|$, respectively, denote the initial station and terminal station. As we consider the China high-speed railway corridor as a decision-making environment, all the trains in this study are divided into two classes (denoted by G and

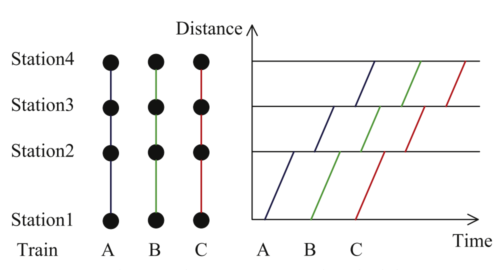

Fig. 3. The all-stop train stop plan and train schedule.

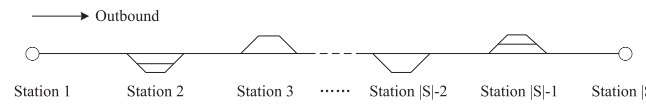

Fig. 4. An illustration for the structure of railway corridor.

D) according to the allowable highest speed of the trains, where G and D represent sets of trains with maximum velocity 300 km/h and 250 km/h, respectively.

2.2. Characteristic analysis and assumptions

Next, some characteristic analyses associated with the stop plan, train timetable and passenger flow will be provided for identifying the detailed features of the high-speed railways considered in this paper, particularly through comparing with those of the urban rail transit traffic.

Stop plans. For the urban rail transit system, some simplified characteristics are usually taken into consideration in scheduling process, such as the same speed of all trains, the prohibition of overtaking operations, and the same dwelling time at each station (see Cao et al. [3], Lee et al. [31,32], Freyss et al. [15] and Zheng et al. [55]). Conceptually, the train stop plan considered in this paper is similar to the train stop-skipping in the urban rail transit. However, compared to the urban rail transit, the operations of high-speed railways are much more complicated. Specifically, (1) in high-speed railways, at least two types of trains with different speeds are operated on the railway corridor, and the high-speed trains are allowable to overtake low-speed trains at each station with enough capacities, which essentially increases the difficulty of searching the optimal scheduling strategies; (2) with the given stop plan, the dwelling time of each train at each station is not a constant, which can be viewed as the returned decision information in this study; (3) the departure sequence from the origin station is also an important factor for the quality of the generated solution; (4) more extra constraints need to be further considered in scheduling trains once a stop plan is pre-specified, for instance, traversing sequence, dwelling time, dwelling capacity, etc.

Train timetable. Train timetables aim to specify the detailed arrival, departure and dwelling times at each station for each train. For the urban rail transit, the passengers are often assumed to arrive at the station with a pre-specified passenger arrival rate (for instance, Cao et al. [3], Freyss et al. [15] and Wang et al. [46]), and the passengers do not need to refer to the timetable for planning their trips because each train can provide the transportation service for any passengers if the capacity of this train is large enough. In this sense, the main role of a train timetable is provided for the dispatchers to guarantee the well-organization of the railway traffic. However, besides of guaranteeing the well-operations of traffic systems, another most important role of a train schedule in the regular/high-speed railway is to release the service information for all the passengers.

Passenger flow. In general, the dynamics of passenger flow on the high-speed railway is less evident than that of the metro system. In the high-speed railways, the majority of passengers make their travel plans according to the timetable proposed by the railway operator. In comparison, the passenger flow in urban rail transit system has a different characteristic, since passengers usually do not care about the train timetables before their trips (see Cao et al. [3], Freyss et al. [15], and Wang et al. [46]), leading to the dynamic (time-dependent) features due to the randomness of demands. Take the regular railway in China as an example. If a passenger plans an intercity trip, he/she is allowed to buy the ticket at most 60 days before his/her trip. This is to say, on the operational levels, the number of passengers is almost

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predetermined by the sold tickets on the basis of the published schedule, since the temporary passengers only occupy a small ratio of the total passengers. With this concern, time-dependency associated with passenger flow is typically less evident. So in this research, we particularly consider the total demands as the parameters in the decision-making process, which are only treated as static constants.

Through the detailed analyses mentioned above, some assumptions will be given in the following discussion for the convenience of formulating the problem, since both train stop planning and train scheduling problems for regular railway operations are hard to solve due to the complexity of the practical requirements.

Assumption 1. On a railway corridor, the entire railway line might be divided into a series of operation zones consisting of subsections of the corridor (see Fig. 5), for meeting various passenger demands (short-distance or long-distance trips). In essence, this realistic situation will enhance the descriptive complexity of determining the departure orders of trains in a certain station. To simplify the problem, all trains in this paper are assumed to start from initial station 1 and end at terminal station $|S|$, which can also be extended to the real-world situations easily. Moreover, a higher speed train is allowed to overtake other trains at any stations if required, and overtaking operations are prohibited on any railway sections.

Assumption 2. In the solution process, one of key constraints in this problem is associated with the passenger demands that need to be satisfied. Since this research aims to generate the train schedule on the tactic levels instead of operational levels, we shall not track the detailed number of passengers getting on/off each train at each station, but consider the estimated loading capacity of each train and the number of passenger demands at each station. It is particularly required that the total capacity of trains stoping at each station is over the total passenger demands at this station.

Assumption 3. If a train is pre-scheduled to stop at a station, deceleration and acceleration operations need to be performed when it enters and leaves this station. These operations will necessarily cause the extra loss of travel time in comparison to the non-stop trains. For simplicity, in this study the time loss caused by deceleration and acceleration at the station will not be taken into account.

Assumption 4. Finally, to guarantee the necessary time for preparing departure operations at origin station, all trains are assumed to leave from the origin station not earlier than their predetermined expected departure times in this paper (see Section 3.3 for the close-to-favorite-departure-time constraints).

3. Mathematical model

This section will provide a rigorous formulation to collaboratively optimize the train stop planning and train scheduling problems. The following discussion mainly focuses on specifying each part of the model, including parameters, decision variables,

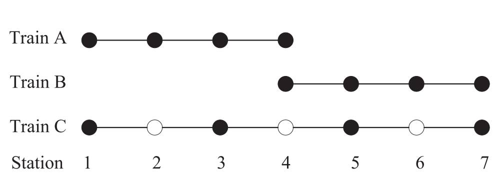

Fig. 5. An illustration of stop plans for operation zones.

systematic constraints, objective functions and complexity of the formulation.

3.1. Notations and parameters

For modelling convenience, Table 1 lists all the relevant subscripts and parameters used in the formulations.

3.2. Decision variables

This paper focuses on generating optimal strategies for the train stop plan and train schedule simultaneously, in which it is necessary to specify the following operation characteristics for each train at every station, including departure time, arrival time, stop plan and departure order, etc. Thus, five types of decision variables will be considered hereinafter, as given below:

1. t_{is}^d : the time that train i departs from station s.
2. t_{is}^a : the time that train i arrives at station s.
3. x_{is} : indicator of stop plan for train i at station s, $=1$, if train i stops at station s; $=0$ otherwise.
4. y_{ijs} : indicator of departure order for trains i and j from station s, $=1$ if train i departs from station s before train j; $=0$ otherwise.
5. z_i^G : type indicator of train i for departure sequence, $=1$ if train i belongs to set G; $=0$, otherwise.

In general, decision variables t_{is}^d and t_{is}^a can be employed to generate a continuous space-time path for each train i; the third binary variable aims to determine whether train i stops at station s, and it also can help to formulate dwelling time constraints for train i at station s, which is the key connection between the stop plan and schedule; the fourth variable specifies the departure order of trains at each intermediate station, which is used to formulate the headway constraints at each station to guarantee the

Table 1
Subscript and parameters used in formulation.

Notations	Definition
S	Set of considered stations.
T	Set of trains, $T=\{1,2,\ldots,|T|\}$.
i,j	Index of trains, $i,j\in T$.
s	Index of stations and also the index of railroad section $[s,s+1]$, $s,s+1\in S$.
$|S|$	The total number of stations.
$|T|$	The total number of trains.
G	Set of high-speed trains.
D	Set of relevant low-speed trains compared with those in G.
$|G|$	The number of trains in set G.
$|D|$	The number of trains in set D, clearly $|D|=|T|-|G|$.
R_s	The required least number of stops at station $s\in S$.
V_G	The speed of trains in set G.
V_D	The speed of trains in set D.
T_i^E	The expected departure time for train i from the initial station.
ΔT	The maximum fluctuation time compared with the expected departure time.
C_i	The maximum capacity of train i.
δ_{is}	The loading capacity coefficient of train i at station s.
Q_s	The total passenger demand at station s.
t_{is}^p	The travel time of train i from station s to station $s+1$.
l_s	The length/distance from station s to station $s+1$.
t_τ	The minimum dwelling time of each train at each station.
t_z	The minimum tracking headway of two consecutive trains.
t_d	The minimum headway of two consecutive trains departing from the same station.
t_a	The minimum headway of two consecutive trains arriving at the same station.
h_a	The maximum of t_a and t_z, i.e., $h_a=\max\{t_a,t_z\}$.
h_d	The maximum of t_d and t_z, i.e., $h_d=\max\{t_d,t_z\}$.
U	A sufficiently large number.

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safe operations; the fifth variable is introduced to denote the type of train i, which can determine the travel time of train i from station s to station $s+1$ and further formulate the travel time constraints as linear constraints (see Eq. (14) for more details).

3.3. Systematic constraints

In this subsection, systematic constraints will be formulated to meet the passenger demands associated with train stop plans, and then guarantee the feasibility and safety of the corresponding train schedule. The involved constraints are formally formulated below.

Close-to-favorite-departure-time constraints: in general, since a variety of trains with different types need to be scheduled on the railway corridor, it is desirable to predetermine the favorite (expected) departure time at origin station for individual trains in order to guarantee the service balance over the considered time horizon, which can also improve attractiveness of some train services to passengers when the departure time is well scheduled. However, if we design the train schedule according to these favorite departure times, some conflicts may probably occur as the stop plan of trains and their velocities are not necessarily consistent. To eliminate these conflicts in scheduling process, and moreover find an optimal solution as soon as possible, we especially relax each favorite departure time to a time interval with the largest allowable derivation ΔT (i.e., $[T_i^E, T_i^E+\Delta T]$). Then, the close-to-favorite-departure time constraints can be formulated as follows:

$$T_i^E \le t_{i1}^d \le T_i^E + \Delta T, \quad \forall i \in T = \{1, 2, \ldots, |T|\} \tag{1}$$

Clearly, these constraints ensure that the scheduled departure time of each train from origin station cannot be earlier than the expected departure time, and also cannot overflow the maximum fluctuation time. In practice, the fluctuation time is impossible to be set as a too large constant with the consideration of the real-world operational conditions.

Headway constraints: to guarantee the safe operations on the railway line, headway constraints need to be formulated to keep a safe distance between the adjacent trains, including the arrival headway at stations, departure headway at stations and tracking headway on railroad segments. Although there are a variety of factors that can influence the minimal headway between adjacent trains (such as braking distance, reaction time, etc.), this study still treats the minimal headway as a constant. As the velocity of the same type of trains is assumed as a constant on each railway section, the distance between adjacent trains can be easily checked by measuring the time difference when they arrive at and depart from each station.

(i) *Arrival and departure headway constraints*: considering the dwelling capacity of each station (usually referring to as the number of available platforms), if a train arrives at/departs from a station, a time period should be left to the preparation for the next train's arrival/departure. In other words, this time period is actually time distance of two trains for their arrival or departure operations. To effectively formulate this constraint, we need to introduce a traversing order indicator for two adjacent trains when they traverse on a common railway link. Here, we use a binary variable y_{ijs} to indicate the departure order of trains i and j at station s. Then, the arrival and departure headway constraints can be formulated as follows:

$$t_{is}^d + t_d \le t_{js}^d + U \cdot (1-y_{ijs}), \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{2}$$

$$t_{is+1}^a + t_a \le t_{js+1}^a + U \cdot (1-y_{ijs}), \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{3}$$

$$y_{ijs} + y_{jis} = 1, \quad \forall i,j \in T, \ i < j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{4}$$

$$y_{ijs} \in \{0, 1\}, \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{5}$$

In this set of constraints, inequality (2), together with equality (4) and the binary condition of variable y_{ijs}, guarantee that the minimum headway of two consecutive trains i and j departing from the same station s is respected, since decision variable y_{ijs} can fully specify the sequence of each train pair (i,j) with the relationship $i \ne j$ on the railway link s. Likewise, the arrival headway can also be guaranteed in a similar way.

(ii) *Tracking headway constraints*: Tracking headway is the minimum time interval to guarantee the safety distance for adjacent tracking trains on each interstation section. It is mainly determined by the braking ability, velocity of adjacent trains and the type of signal interlocking equipments. As we assume that the velocity of the same type of trains is a constant and the overtaking is prohibited on the railway section, the minimum tracking headway on interstation section can be represented by departure headway and arrival headway at corresponding stations for two adjacent trains.

Fig. 6 gives four feasible schedules to show the relationship between different headway constraints explicitly, where h_1 and h_2, respectively, represent the time differences of departure and arrival operations at corresponding stations; h_3 denotes the tracking time difference of two adjacent trains at different locations of the considered railway section. As shown in Fig. 6(a), adjacent trains i and j are the same type of trains (G- or D-trains). In this case, when the required minimum tracking headway t_z is less than or equal to either departure headway h_1 or arrival headway h_2, the tracking headway constraints can be guaranteed on railway section $[s, s+1]$. In Fig. 6(b), if the former train i is a G-train and the follower train j is a D-train, then the minimum tracking headway t_z on section $[s, s+1]$ should not be greater than departure headway at station s. Yet, if train i is a D-train and j is a G-train without overtaking on the interstation section, the minimum tracking headway t_z on section $[s, s+1]$ should not be greater than the arrival headway at station $s+1$. Finally, if G-train j overtakes D-train i at station s, as shown in Fig. 6(d), the minimum tracking headway constraints on section $[s, s+1]$ and section $[s-1, s]$ will become the same situations in Fig. 6(b) and (c), respectively.

With the above analysis, the tracking headway constraints can be formulated as the same forms of departure and arrival headway constraints. If we define h_d as the maximum required headway of t_z and t_d (i.e., $h_d = \max\{t_z, t_d\}$) and h_a as the maximum required headway of t_z and t_a (i.e., $h_a = \max\{t_z, t_a\}$), then the arrival headway constraints, departure headway constraints and tracking headway constraints can be merged into a set of simplified constraints, given below:

$$t_{is}^d + h_d \le t_{js}^d + U \cdot (1-y_{ijs}), \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{6}$$

$$t_{is+1}^a + h_a \le t_{js+1}^a + U \cdot (1-y_{ijs}), \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{7}$$

$$y_{ijs} + y_{jis} = 1, \quad \forall i,j \in T, \ i < j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{8}$$

$$y_{ijs} \in \{0, 1\}, \quad \forall i,j \in T, \ i \ne j, \ s \in \{1, 2, \ldots, |S|-1\} \tag{9}$$

Remark 3.1. As shown in Fig. 7, in scheduling process, a higher speed train (train D) is allowable to overtake a medium speed train (train C) at some station because its travel time on the interstation section is less than that of the medium speed train. Similarly, a train (train A) which stops at more stations might be overtaken by a train (train B) only stop at few stations. In both cases, when train j is scheduled to overtake train i at station s, train i may be required to dwell at the station for more time to wait for train j

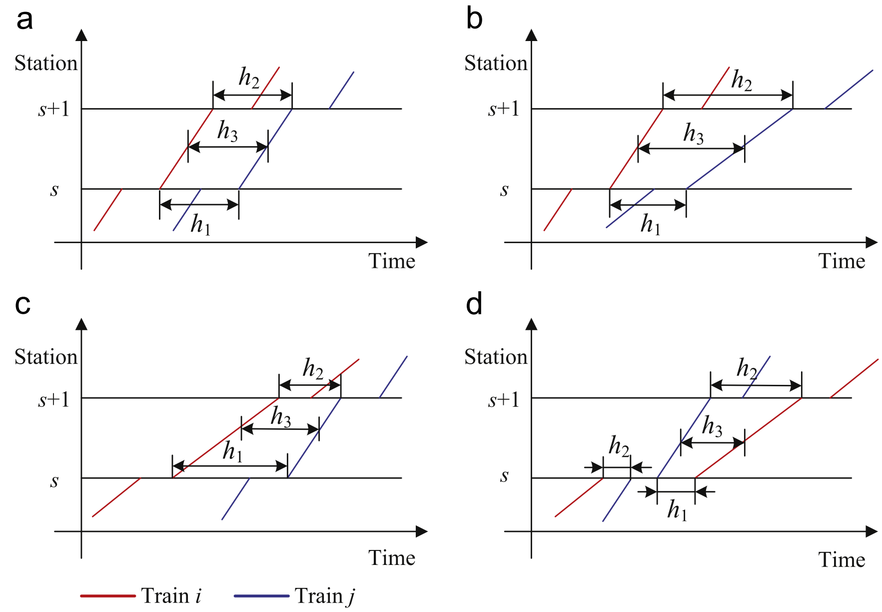

Fig. 6. An illustration for arrival, departure and tracking headway of two adjacent trains.

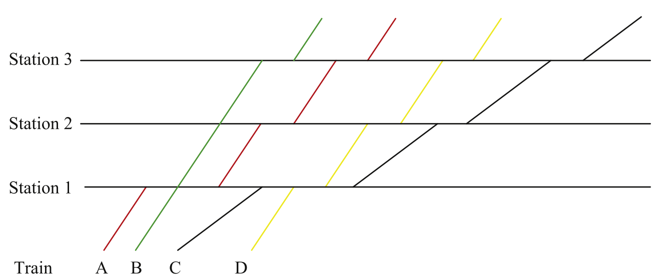

Fig. 7. An illustration for overtaking operations.

passing by. Although these two trains' departure order will be switched on the next section, the overtaking conditions can also be guaranteed by the departure and arrival headway constraints (6)–(9).

Dwelling time constraints: If a train is pre-scheduled to stop at a specific station, some time should be left for this train to conduct necessary operations, such as loading and unloading passengers, changing crews, etc. Then dwelling time constraints, which play key roles to explicitly link the train stop planning problem with train scheduling problem, are needed. To indicate the stop plan for each train, a binary variable x_{is} is used to denote which train i should stop at station s. Obviously, if we know the arrival and departure times of each train i at each station s, the actual dwelling time q_{is} for train i at station s can be calculated by the difference of arrival and departure times t^a_{is} and t^d_{is}, namely $q_{is} = t^d_{is} - t^a_{is}$. Then the dwelling time should be no less than a pre-specified minimum time t_τ for operations. We here give the dwelling time constraints at intermediate stations as follows:

$$t^d_{is} - t^a_{is} \geq x_{is} \cdot t_\tau, \quad \forall i \in T = \{1, 2, \ldots, |T|, \ s \in \{2, 3, \ldots, |S|-1\} \tag{10}$$

In this constraint, if train i is planned to stop at station s, we then have $x_{is} = 1$. Thus, this constraint will guarantee the needed operational time at station s. On the other hand, if no pre-specified stop is imposed for train i at station s, this constraint will vanish automatically due to $x_{is} = 0$.

Passenger demands constraints: As addressed in assumptions, since the train scheduling problem is discussed on the tactic levels, we here only consider the satisfaction of macro demands at each station instead of microcosmically tracking the detailed number of passengers getting on/off each train, where the demands at each station can be approximately obtained through travel demand estimation or historical travel data. To suitably represent the loading capacity of a train at a station, we in particular introduce an estimated loading capacity coefficient $\delta_{is} \in (0, 1)$ associated with each train at each station by synthetically considering the potential reserved empty seats and possible alighting passengers. Then, $\delta_{is} \cdot C_i$ will be regarded as the estimated capacity of train i at station s. Thus, the total loading capacity of involved trains which stop at each station must be large enough to meet the passenger demands at this station.

As for the specific value of this coefficient, it is mainly determined by the expert's experience and the historical statistics. When determining this parameter for a specific train at each station, we need to comprehensively consider different factors, such as the level of trains, level of stations, possible passenger demands, etc. Undoubtedly, much larger parameters can be given at some busy stations for some high level trains. In general, if we set this coefficient as a small value for a specific train, it means that we leave a relatively large number of unsatisfied passengers at this station, probably leading to more stops of other trains at this station to meet the demand constraints. On the contrary, if we set it as a larger value, a relatively small number of passengers will potentially be unsatisfied, which possibly corresponds to few trains that need to stop at this station. In our opinion, all of these criteria can be considered in pre-specifying the relevant parameters. In addition, we need to mention that, practically, a timetable will be adjusted frequently to meet the new situations of the passenger demands according to the real-world applications. Thus, this parameter can also be adjusted to meet the new requirements.

Although the aforementioned second assumption is slightly different from the realistic applications, we can also use it to effectively formulate the passenger demands constraints on the tactic level, listed below:

$$\sum_{i=1}^{|T|} x_{is} \cdot \delta_{is} \cdot C_i \geq Q_s, \quad s \in \{1, 2, \ldots, |S|\} \tag{11}$$

Stop plan constraints: In practice, since the passenger demand is considered over the entire planning horizon at each station, passenger demand constraints might lead to a small number of stops at some stations which have small passenger flow demands. To increase service flexibility for the passengers, we can add an additional constraint to ensure at least a given number of trains to stop at each station. By this method, the convenience and balance characteristics of services can be expectedly improved although it necessarily enhances the total travel time of the long distance travelers. The constraints are formally formulated below:

$$\sum_{i=1}^{|T|} x_{is} \geq R_s, \quad \forall s \in S \tag{12}$$

where notation R_s is the given threshold indicating the required minimum number of stops at station s.

Interstation travel time constraints: In this study, the lost time caused by train accelerations and decelerations is not considered and the velocity of the same type of trains is a constant on each railway section. The travel time of train i from station s to station $s+1$ equals to the value of arrival time of train i at station $s+1$ minus departure time from station s. Namely, the interstation travel time constraints are formulated as below:

$$t_{is}^p = t_{is+1}^a - t_{is}^d, \quad \forall i \in T = \{1, 2, \ldots, |T|\}, \ s \in \{1, 2, \ldots, |S|-1\} \tag{13}$$

In addition, as we consider two classes of trains in our model, the travel times of different types of trains on the same railway section will be distinguished. Through introducing a train type indicator z_i^G, we can formulate the interstation travel time constraints in the following for different trains:

$$\begin{cases} t_{is}^p = z_i^G \cdot l_s / V_G + (1 - z_i^G) \cdot l_s / V_D \\ \sum_{i=1}^{|T|} z_i^G = |G| \\ \forall i \in T = \{1, 2, \ldots, |T|\}, \quad s \in \{1, 2, \ldots, |S|-1\} \end{cases} \tag{14}$$

Clearly, in Eq. (14), the binary variable z_i^G represents the type of trains. When $z_i^G = 1$, train i will be included in set G; otherwise, it should be in set D. Thus the second equality ensures that a total of $|G|$ trains are included in the set of high-speed trains. The first equality implies the interstation travel times of different trains through dividing the distance by the velocity of train i.

Remark 3.2. In this model, the headway constraints and dwelling time constraints can be found in a variety of literatures associated with train scheduling/rescheduling problems, such as Yang et al. [51,52], Meng and Zhou [37], Ghoseiri et al. [17]. However, other constraints are newly proposed constraints that have not attracted enough attention in the existing literature.

3.4. *Objective function*

Practically, it is desirable for railway companies to utilize the minimum cost to evaluate the generated train stop plan and train schedule. With this concern, we can use the actual travel time to represent the considered cost in condition that the passenger demands can be satisfied. Typically, the total travel time of all trains can be formulated as follows:

$$T_{\text{total}} = \sum_{i=1}^{|T|} (t_{i|S|}^a - t_{i1}^d) \tag{15}$$

It is clear that formulation (15) is essentially the sum of each train's actual travel time from its origin to destination. Equivalently, the total travel time can be further decomposed into the following two parts to obtain a more straightforward understanding:

$$T_{\text{total}} = \sum_{i=1}^{|T|} \sum_{s=2}^{|S|-1} (t_{is}^d - t_{is}^a) + \sum_{i=1}^{|T|} \sum_{s=1}^{|S|-1} t_{is}^p \tag{16}$$

in which the first part is associated with the total dwelling time at intermediate stations, and the second part is the sum of link travel times. In this equation, the second part is actually a constant since the link travel time is considered as a constant for each train type in the scheduling process. Then, minimizing the total travel time in this paper is equivalent to minimizing the total dwelling time. Thus, the following discussion will adopt the total dwelling time as one of evaluation indexes, which is re-denoted as follows:

$$T_{\text{dwell}} = \sum_{i=1}^{|T|} \sum_{s=2}^{|S|-1} (t_{is}^d - t_{is}^a) \tag{17}$$

On the other hand, even with the close-to-favorite-departure-time constraints, the departure time of each train from its origin station (i.e., t_{i1}^d) is still an important decision variable in this study, since we relax the favorite departure time as a time interval with an allowable fluctuation time ΔT, especially when we set this fluctuation time as a relative large number. Thus we adopt the total differences (i.e., delay time) between the real and expected departure times of each train from its origin station as the second evaluation index. Since all trains are assumed to depart from origin station not earlier than their predetermined expected departure times, the total delay at origin station can be formulated as below:

$$T_{\text{delay}} = \sum_{i=1}^{|T|} (t_{i1}^d - T_i^E) \tag{18}$$

Typically, minimizing total delay at origin station aims to determine the train timetable according to the expected departure times as close as possible.

In this paper, we consider the total dwelling time at intermediate stations and total delay at origin station as the evaluation indexes, and then formulate the problem of interest as a multi-objective optimization problem. Typically, the linear weighted method can be adopted to handle these two objective functions, leading to the following evaluation index:

$$T_{\text{eval}} = \gamma_1 \cdot \sum_{i=1}^{|T|} (t_{i1}^d - T_i^E) + \gamma_2 \cdot \sum_{i=1}^{|T|} \sum_{s=2}^{|S|-1} (t_{is}^d - t_{is}^a) \tag{19}$$

where γ_1, γ_2 are pre-specified weights of these two evaluation indexes. For this objective function, if one sets γ_1 as a small value and γ_2 as a large value, it means that he/she would like to pay more attention to the total dwelling time at intermediate stations than the total delay at origin station. On the contrary, if the decision maker sets γ_1 as a large value and γ_2 as a small value, the total delay at origin station will be emphasized more. Typically, it is reasonable to determine these two parameters according to the decision maker's preferences and practical operation situations.

Remark 3.3. It is worth mentioning that the total delay at origin station can be regarded as the dwelling time if the expected departure time is treated as the train arrival time at this station. In this case, the problem of interest can be treated as a single-objective programming model. However, note that the two parts in Eq. (19) are not necessarily comonotonic with respect to the solutions, we then handle these indexes separately to satisfy different preferences by pre-setting two weight coefficients in the decision-making process. This method can also balance the preferences between the expected departure time and the minimum dwelling time. Typically, Eq. (19) is the generalization of the aforementioned single objective. Specifically, if $\gamma_1 = 1$ and $\gamma_2 = 0$, the optimal schedule necessarily dispatches all the trains according to their favorite departure times. On the other hand, parameters $\gamma_1 = 0$ and $\gamma_2 = 1$ potentially corresponds to the optimal schedule with the minimum dwelling time. In addition, if $\gamma_1 = 0.5$ and $\gamma_2 = 0.5$, the objective function will equivalently degenerate to the following single objective form:

$$T_{\text{eval}} = \sum_{i=1}^{|T|} \sum_{s=1}^{|S|-1} (t_{is}^{d} - t_{is}^{a})$$

where $t_{i1}^{a} = T_i^{E}$ for any train i. In this sense, we provide a more flexible objective function for the decision makers through presetting their favorite weights γ_1 and γ_2.

3.5. Mathematical model

According to Eqs. (1)–(19), the train stop planning and train scheduling collaborative optimization model can be formulated as follows:

$$\begin{cases} \min\ T_{\text{eval}} = \gamma_1 \cdot \sum_{i=1}^{|T|} (t_{i1}^{d} - T_i^{E}) + \gamma_2 \cdot \sum_{i=1}^{|T|} \sum_{s=2}^{|S|-1} (t_{is}^{d} - t_{is}^{a}) \\ \text{s.t. constraints } (1), (6)-(14) \end{cases} \tag{20}$$

Remark 3.4. The above formulation can be only regarded as a fundamental model for train scheduling and train stop planning collaborative optimization problems. Typically, it has great extensional space for satisfying a variety of particular requirements. (1) This model can determine the departure order of two types of trains from the origin station by using a binary variable z_i^G to specify the travel time for each train at each interstation section. Yet, it also can fix the departure order for a part or all of the trains by providing the departure time directly. (2) In train stop planning problem or train scheduling problem on high-speed railway corridors, some other constraints may also be considered, such as the total number of stops for one train or one grade station, stopping at certain stations for some given trains, etc. For example, in order to determine the train stop plan on a high-speed railway corridor, Li et al. [33] use the node service frequency, inter-station service accessibility and number of one-train stopping as the main constraints to establish a non-linear planning model with the purpose of minimizing the total number of stops for all the trains operated on the line. These constraints can also be added to our model because the decision variables in this research are associated with the stop selection indicator x_{is}, the train type selection indicator z_i^G, the arrival and departure times t_{is}^{a}, t_{is}^{d}. In summary, the constraints that are established by these four variables all can be considered in this model.

3.6. Complexity of formulation

Typically, all the variables in the proposed formulation can be divided into two categories. One is associated with binary decision variables including train stop selection indicators (representing whether the train stops or not at each station), train type indicators (determining a train is a G-train or a D-train) and departure orders of two trains from intermediate stations. The other refers to positive decision variables representing the departure and arrival times of each train at each station. In this sense, this formulation is essentially a large-scale mixed integer linear programming model. In the following, we are particularly interested in analyzing the complexity of the proposed formulation in detail, such as the total number of decision variables and some critical constraints involved in the problem, shown in Table 2.

There is no doubt that the complexity of this model is fully determined by two pre-specified parameters on the considered railway corridor, namely the number of trains (i.e., $|T|$) and the number of stations (i.e., $|S|$). In order to illustrate this problem clearly, an example with 20 stations and 30 trains is given here to state the total number of decision variables. If we predetermine the departure order for two types of trains from the initial station

Table 2
Number of variables and constraints in formulation (20).

Variable or constraints	Total number at most
Variable t_{is}^{d}	$\|T\| \cdot (\|S\|-1)$
Variable t_{is}^{a}	$\|T\| \cdot (\|S\|-1)$
Binary variable x_{is}	$\|T\| \cdot \|S\|$
Binary variable y_{ijs}	$\|T\| \cdot (\|T\|-1) \cdot (\|S\|-1)$
Binary variable z_i^G	$\|T\|$
Close-to-favorite-departure-time constraints (1)	$2 \cdot \|T\|$
Headway constraints ((6)–(8))	$5 \cdot (\|S\|-1) \cdot \|T\| \cdot (\|T\|-1)/2$
Dwelling time constraints (10)	$\|T\| \cdot (\|S\|-2)$
Passenger demand constraints (11)	$\|S\|$
Stop plan constraints (12)	$\|S\|$
Travel time constraints ((13) and (14))	$2 \cdot \|T\| \cdot (\|S\|-1)+1$

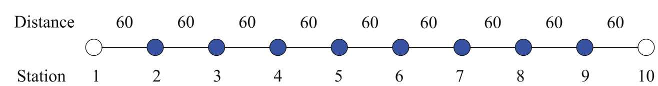

Fig. 8. The structure of single-track railway corridor.

in advance, it will produce 1140 positive variables with respect to t_{is}^{a} and t_{is}^{d} and the total number of binary variables with respect to x_{is}, y_{ijs} will be 17 130, in which 600 binary variables are used to determine whether the train stops or not at different stations and 16530 variables are used to denote the departure order of two trains from intermediate stations. On the other hand, if we use the model to design the departure order for two types of trains from the initial station, a total of 30 extra binary variables are also needed to specify the detailed departure order. Clearly, this problem turns out to be a large-scale mixed linear integer programming model with a total number of 17 160 binary decision variables. In addition, it is easy to see that, in comparison to small-scale cases, the departure order indicators have less effects on the total number of decision variables for the large-scale problems when we consider the departure order as a type of decision variables.

4. Numerical experiments

In this section, we implement several numerical experiments to show the effectiveness and efficiency of our proposed model, in which GAMS optimization software with the CPLEX solver is employed to code the searching process for producing an optimal solution. All the experiments are implemented on a Windows 7 workstation with two Intel Core i3-4130M CPUs and 4G RAM.

4.1. A small-scale example

In this example, we consider an one-way single-track railway line corridor with 10 stations, as shown in Fig. 8, in which stations 1 and 10 are initial and terminal stations respectively, and the remainder stations are referred to as intermediate stations. For simplicity, the distance of any two adjacent stations is assumed to be 60 km. In this set of experiments, a total of 10 trains will be taken into consideration, which can be divided into two sets in accordance to their speeds, i.e., set G in which the train's highest speed is 300 km/h, and set D in which the train's highest speed is 240 km/h. For simplicity, trains in sets G and D, respectively, are referred to as G-trains and D-trains. Moreover, we set $|G| = |D| = 5$. Then, the interstation travel times of trains belonging to G and D, respectively, are 12 min and 15 min on each railway section. Since we pay more attention to the dwelling time at intermediate stations in this set of experiments, the weight coefficients of two parts in the objective function are respectively set as 0.1 and 0.9 (i.e., $\gamma_1 = 0.1, \gamma_2 = 0.9$). In GAMS optimization software, the relative gap (denoted by "OPTCR") between the generated objective and estimated optimal objective can be used as the termination conditions. That is, once a solution that satisfies the termination condition is found, the searching process will be terminated and then output the best solution encountered. In this experiment, we set the parameter OPTCR as 0.05. When a solution within the relative gap 5% is found, it will be outputted as the near-optimal solution to the problem of interest.

For simplicity, we assume that the expected departure time of the first train is at 10 min. Then the following trains are expectedly dispatched in turn with a departure interval 15 min. The detailed expected departure times are given in Table 3 for clarity. In actual scheduling process, we allow 3 min fluctuation around the expected departure time to produce a favorite schedule, i.e., $\Delta T = 3$ min.

In addition, we give the passenger demand at each station in Table 4. If the train's departure order index is an even number, the loading capacity of this train is assumed to be 400 at each station; otherwise, the loading capacity will be set as 300. For operational security, the minimum departure, arrival and tracking headway are all set as 2 min. We fix the minimum dwelling time of each train at its pre-specified stop station as 3 min to guarantee the necessary operational time.

Using the data given above, we design the codes in GAMS software by using CPLEX solver. As a result, we can finally obtain the departure time, arrival time of each train at each station, the stop stations of each train, and also the type of each train. For this experiment, the consumed computation time is only 0.354 s with outputted objective value 144.6 min, in which the total delay and dwelling time of all trains are 6 and 160 min, respectively. The relative gap (given by GAMS software to represent the optimality accuracy of obtained solutions) turns out to be 2.749% in comparison with the possible best solutions, which implies the high-quality of the generated solutions. To understand straightforwardly, the scheduled departure time of each train at origin station and supply capacity at each station are also displayed in Tables 3 and 4, respectively, for a clear comparison with the originally given data. Clearly, in Table 3, since we take the total dwelling time at intermediate stations as a more important evaluation index, only trains T3, T5 and T7 are respectively delayed for 2 min in comparison to their expected departure times so as to reduce the dwelling time at intermediate stations.

In the optimal solution, trains T1–T3, T5, and T10 are high-speed trains in set G and the remainders are medium-speed trains in set D. Then the corresponding train stop plan and train schedule for these 10 trains on the railway corridor are displayed in Figs. 9 and 10, respectively.

Specifically, the solid dot "●" in Fig. 9 represents that this train is required to stop at this station for loading and unloading passengers operations, and the hollow dot "○" means that this train is not forced to stop at this station. It is easy to see from Table 4 and Fig. 9 that, since the passenger demands at the first six stations are much larger than those at other stations, more trains are finally arranged to stop at these stations to provide enough capacity for satisfying passenger demands. In addition, since trains T1, T3, T5, and T7–T9 are only scheduled to stop at a part of stations, the extra dwelling time for long-distance passengers can be expectedly reduced in comparison to the all-stop operation plans. On the other hand, as trains T2, T4, T6, and T10 are required to stop at all stations, this plan can guarantee the necessary service frequency for short-distance passengers. In the train schedule, it is clear and reasonable to see that train T5 overtakes train T4 at station 4 because train T5 is a G-train stopping only at stations 1, 6 and 10, and train T4 is a D-train stopping at all stations.

Table 3
The expected and scheduled departure times of each train from initial station.

Train	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
Expected departure time	10	25	40	55	70	85	100	115	130	145
Scheduled departure time	10	25	**42**	55	**72**	85	**102**	115	130	145

4.2. *Large-scale experiments on Beijing–Shanghai high-speed railway corridor*

To further test the computational performance, the following discussion intends to apply the proposed model to designing the train stop plan and train schedule for Beijing–Shanghai high-speed railway corridor in China. With a total length 1318 km, this high-speed railway consists of 24 stations and 23 railway sections, as shown in Fig. 11. In particular, Tianjin West station will not be taken into account since this station is not on the main line physically. Most of the experimental data used in this set of experiments can be found in Yang [54] and officially released train timetable in 2014. In the following, a total of 62 experiments are implemented based on the given data to test different parameters, which can be expected to get a more comprehensive assessment of the proposed approaches.

4.2.1. *Data preparation*

To perform the experiments, we first pre-generate the basic input data in the solution process, including railway section length, travel time, expected departure time, passenger demands, etc.

Distance and travel time. The distance of each railway section and corresponding travel times of the high- and medium-speed trains are shown in Table 5. Explicitly, the ideal interstation travel time (ITT) is deduced through dividing the distance by the speeds of trains, and all the generated travel times are set to be integers for descriptive convenience. To further coincide with the real-world conditions, we also use the real travel times (RTT) downloaded from the railway official website to solve this problem (according to the train timetable in 2014). Practically, these data can be deduced according to the real-world physical conditions, which are usually influenced by some critical parameters, such as the length of railway sections, speed limits, grade of railways, etc. Obviously, the practical interstation travel times are not less than the ideal travel times due to these physical condition limitations.

Loading capacity and passenger demands. As we know, the rolling stock used on Beijing–Shanghai high-speed railway corridor is mainly the CRH380 train, and there are 8 and 16 marshalling types. For experimental simplicity, we set the loading capacities of trains at each station as follows according to their departure sequences, i.e., 400, 800, 400, 800,⋯(unit: person). That is, trains

Table 4
Passenger demand and supply capacity at each station (unit: person).

Station	1	2	3	4	5	6	7	8	9	10
Passenger demand	3100	2800	2800	2600	2600	3200	2000	2000	1500	3100
Supply capacity	3500	2900	2900	2600	2600	3300	2000	2000	1600	3500

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with 8 and 16 marshalling types will be dispatched in turn during the scheduling process. Since we only provide a theoretical framework of collaborative optimization for train scheduling and train stop planning on high-speed railways, for simplicity the loading capacity coefficient δ_{is} is assumed to be 1.0 for all trains at each station in the implementations. In Table 6, the passenger demand at station i is calculated by OD passenger flow which equals to the total OD passenger flow from station i to other following stations. We only consider the schedule from Monday to Thursday when there are a total of 38 trains operated on the railway line, in which 33 trains belong to set G with highest speed 300 km/h, and 5 trains are in set D with highest speed 250 km/h.

Expected departure time and other parameters. In this experiment, the real departure time and departure order from the initial station, given in the realistic timetable, are all taken as the expected departure time and departure order. Trains D315, D319, D313, D311, and D321 are medium-speed trains and the remainders are high-speed trains. In Table 7, the expected and real departure time of each train from the first station are displayed. For operational security, the minimum dwelling time, departure headway, arrival headway, tracking headway are all taken as 2 min and the maximum fluctuation time is set as 5 min in comparison with the expected departure time. Like example 1, the weight coefficients of total delay at origin station and total dwelling time at intermediate stations are respectively set as $\gamma_1 = 0.1$ and $\gamma_2 = 0.9$. In order to enhance the service quality for some stations with small passenger demands, at least 3 trains will be scheduled to stop at each intermediate station for the passengers getting on/off. Moreover, in GAMS optimization software, the parameter OPTCR is set as 0.01.

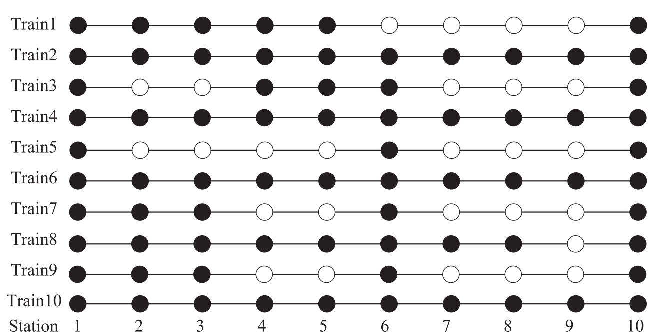

Fig. 9. Train stop plan for 10 trains on the railway corridor.

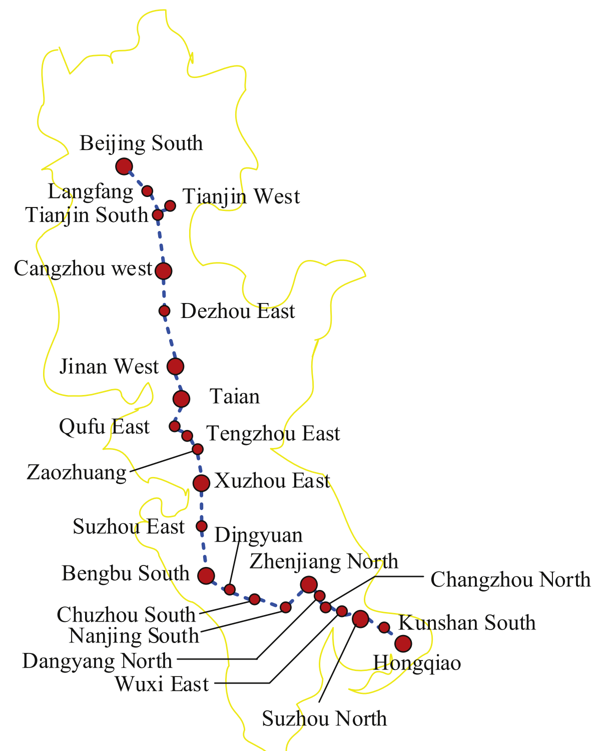

Fig. 11. Map of Beijing–Shanghai high-speed railway corridor.

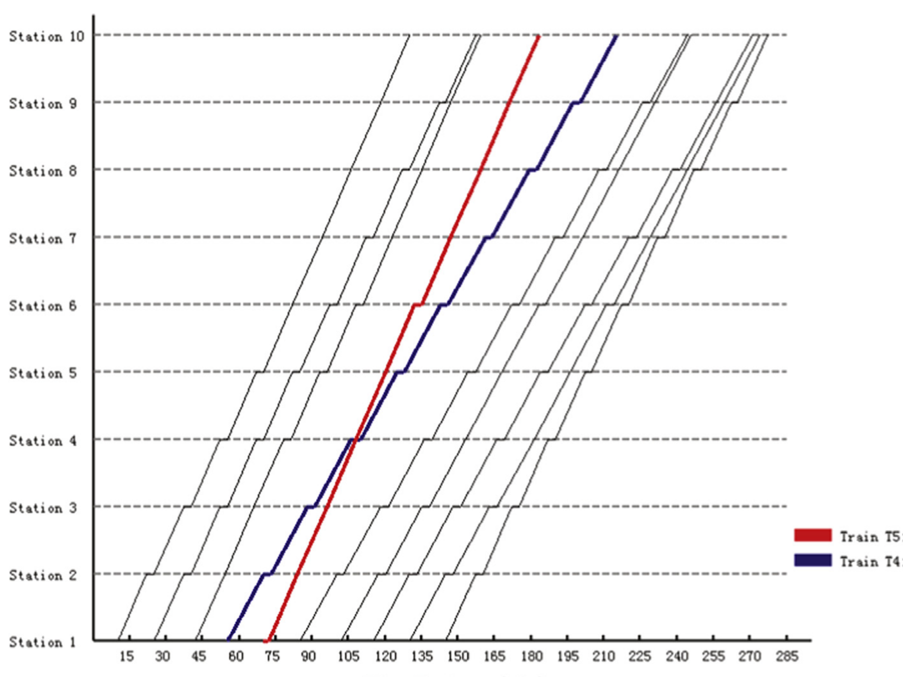

Fig. 10. Train schedule for 10 trains on the railway corridor.

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Table 5
Distance and travel time for each train of each interstation section.

Station section	Distance (km)	Ideal travel time (min)		Real travel time (min)	
		G Trains	*D* Trains	*G* Trains	*D* Trains
Beijing South–Langfang	59	12	14	21	21
Langfang–Tianjin South	72	14	17	18	18
Tianjin South–Cangzhou West	88	18	21	22	23
Cangzhou West–Dezhou East	108	22	26	27	29
Dezhou East–Jinan West	92	18	22	24	26
Qufu East–Tengzhou East	56	11	13	17	19
Tengzhou East–Zaozhuang	36	7	9	13	13
Zaozhuang–Xuzhou East	63	13	15	18	20
Xuzhou East–Suzhou East	79	16	19	19	20
Suzhou East–Bengbu South	77	15	18	23	26
Bengbu South–Dingyuan	53	11	13	16	17
Dingyuan–Chuzhou	62	12	15	19	19
Chuzhou–Nanjing South	59	12	14	18	19
Nanjing South–Zhenjiang South	69	14	17	20	23
Zhenjiang South–Danyang North	25	5	6	11	11
Danyang North–Changzhou North	32	6	8	11	12
Changzhou North–Wuxi East	57	11	14	17	18
Wuxi East–Suzhou North	26	5	6	10	11
Suzhou North–Kunshan South	32	6	8	11	12
Kunshan South–Hongqiao	43	9	10	18	19

Table 6
Passenger demand and supply capacity at each station (unit: person).

Station	Passenger demand	Supply Capacity		Station	Passenger demand	Supply capacity	
		ITT	RTT			ITT	RTT
Beijing South	22 800	22 800	22 800	Bengbu South	9260	9600	9600
Langfang	4000	4000	4000	Dingyuan	7678	8000	8000
Tianjin South	19 370	19 600	19 600	Chuzhou	5096	5200	5200
Cangzhou West	3135	3200	3200	Nanjing South	14 630	14 800	14 800
Dezhou East	4630	4800	4800	Zhenjiang South	5315	5600	5600
Jinan West	13 315	13 600	13 600	Danyang North	5397	5600	5600
Taian	4192	4400	4400	Changzhou North	5479	5600	5600
Qufu East	3479	4400	3600	Wuxi East	8384	8400	8400
Tengzhou East	3392	3600	3600	**Suzhou North**	4986	5200	5600
Zaozhuang	3178	3200	3200	Kunshan South	1370	1600	1600
Xuzhou East	12 000	12 000	12 000	Hongqiao	22 800	22 800	22 800
Suzhou East	5425	5600	5600				

Table 7
The expected and real departure times of each train from the initial station.

Trains	Expected departure time	Real departure time		Trains	Expected departure time	Real departure time	
		ITT	RTT			ITT	RTT
G101	07:00	07:00	07:00	G133	12:48	**12:49**	12:48
G105	07:36	07:36	07:36	G135	13:00	**13:01**	13:00
G11	08:00	08:00	08:00	G137	13:06	13:06	13:06
G107	08:08	08:08	08:08	G139	13:40	13:40	13:40
D315	08:18	08:18	08:18	G3	14:00	14:00	14:00
G111	08:40	08:40	08:40	G43	14:05	14:05	**14:06**
G1	09:00	09:00	09:00	G141	14:16	14:16	14:16
G113	09:05	**09:06**	09:05	G143	14:41	14:41	14:41
G115	09:16	09:16	09:16	G17	15:00	15:00	15:00
G41	09:33	09:33	09:33	G145	15:29	15:29	15:29
G117	09:44	09:44	09:44	G19	16:00	16:00	16:00
G13	10:00	10:00	10:00	G147	16:10	16:10	**16:12**
G119	10:05	**10:06**	10:05	G149	16:15	16:15	16:15
G121	10:45	10:45	10:45	G21	17:00	17:00	17:00
G15	11:00	11:00	**11:01**	G153	17:15	17:15	17:15
G125	11:13	11:13	11:13	G155	17:40	17:40	17:40
G129	11:40	11:40	11:40	D313	19:34	19:34	19:34
D319	12:04	12:04	12:04	D311	21:16	21:16	21:16
G131	12:27	**12:28**	**12:31**	D321	21:23	**21:28**	**21:28**

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4.2.2. Computational results

With the input data prepared above, the GAMS software with CPLEX solver is used to solve this numerical example. To test the steadiness of the optimization software, we first implement the experiments with the ideal link travel times (ITT) for five times. Consequently, with the same near-optimal solution, the difference among the consumed computational times (i.e., $8'44''43, 8'43''417, 8'44''539, 8'43''299$ and $8'43''807$) does not exceed 2 s, demonstrating the stability of CPLEX solver in solving our proposed model. Here, the experiments with both ITT and RTT are implemented to specify the detailed solution characteristics. The optimal objective value, total delay (i.e., the first part) and dwelling time (i.e., the second part) are {353.30, 5.00, 392.00} for ITT and {351.80, 8.00, 390.00} for RTT, respectively. Table 7 is given to elaborate the departure time of each train in the optimal solutions. Clearly, except for trains G113, G119, G15, G131, G133, G135, G43, G147 and D321 (shown in bold) which depart from the initial station later than their expected departure times for avoiding potential conflicts and decreasing extra dwelling time at intermediate stations, the scheduled departure times from initial station for cases of ITT and RTT are all equal to the expected departure times, illustrating the rationalities of pre-specified expected departure times. Moreover, due to the differences of link travel times for ITT and RTT cases, the scheduled departure times from the initial station of these 9 trains are also different. Take train G147 as an example. This train departs from the initial station at its expected departure time for ITT, while 2 min delay occurs in the case of RTT.

The generated train schedules on this railway corridor for ITT and RTT are given in Figs. 12 and 13 respectively. In these two schedules, although the total number of stops at intermediate stations for all the involved trains are respectively 195 and 194 times, the modes of trains stopping at each station are typically different since the actual travel time even for the same train is different, resulting in that the supply capacity at some stations are also distinctive for ITT and RTT, respectively. For instance, the supply capacity for ITT at Qufu East station is 800 more than that for RTT. While the supply capacity for ITT at Suzhou North station is 400 less than that for RTT. Specifically, trains G107, G1, D319, G17, D313, and D321 are scheduled to stop at Qufu East station for ITT in Fig. 12, whereas trains G1, D319, G17 and D313 are replaced by trains G125, G131 and G19 for RTT in Fig. 13. Although the passenger demand at Tianjin South station is 19 370, the supply capacities of different schedules at this station are all 19 600 for ITT and RTT, which can provide more seats for the passengers at this station. In addition, note that the stop plans of the same train for ITT and RTT are also different. For instance, except for the terminal stations (Beijing South and Hongqiao stations), train G105 is also scheduled to stop at twelve intermediate stations (i.e., Langfang, Tianjin South, Jinan West, Xuzhou East, Suzhou East, Bengbu South, Chuzhou, Nanjing South, Danyang North, Changzhou North, Wuxi East and Suzhou North stations) for ITT, while for the case of RTT, the scheduled stops are changed as 11 intermediate stations (i.e., Suzhou East, Bengbu South and Suzhou North stations are replaced by Zhenjiang South and Kunshan South stations) on the railway corridor. Besides, train G15 overtakes train G121 at Xuzhou East station for RTT, but this does not occur for ITT. Additionally, train G131 overtakes train D319 at Qufu East station for ITT, but it is not the case for RTT.

For simplicity, we here only display the train stop plan for ITT in Fig. 14. In this train stop plan and its train schedule, it is easy to see that, in comparison to trains labelled by odd numbers (i.e., indexes of departure sequence), the trains labelled by even numbers are scheduled to stop more times at stations. Since this study only assumes the fixed passenger demands at each station, the produced train stop plan can be treated as an operational strategy on the tactic levels. However, on the operational levels, it is needed to specify the detailed numbers of passengers loading and unloading at each stations to generate more precise stopping strategies plans. This is also a further study of our research. In addition, it is worth mentioning that train G129 is scheduled to stop at Qufu East station only for overtaking operations or security concerns without loading/unloading passengers activities, as denoted by red solid dots in Fig. 14. Essentially, these extra stops can be expectedly restricted within a small range by reasonably setting parameter γ_2 in the objective function.

Here, we need further state that it is practically difficult to obtain the real stop plan used on Beijing–Shanghai high-speed railway corridor, since this railway line might be divided into a series of operation zones consisting of different railway sections, for meeting various passenger demands (short-distance or long-distance trips). For modelling convenience, this study only

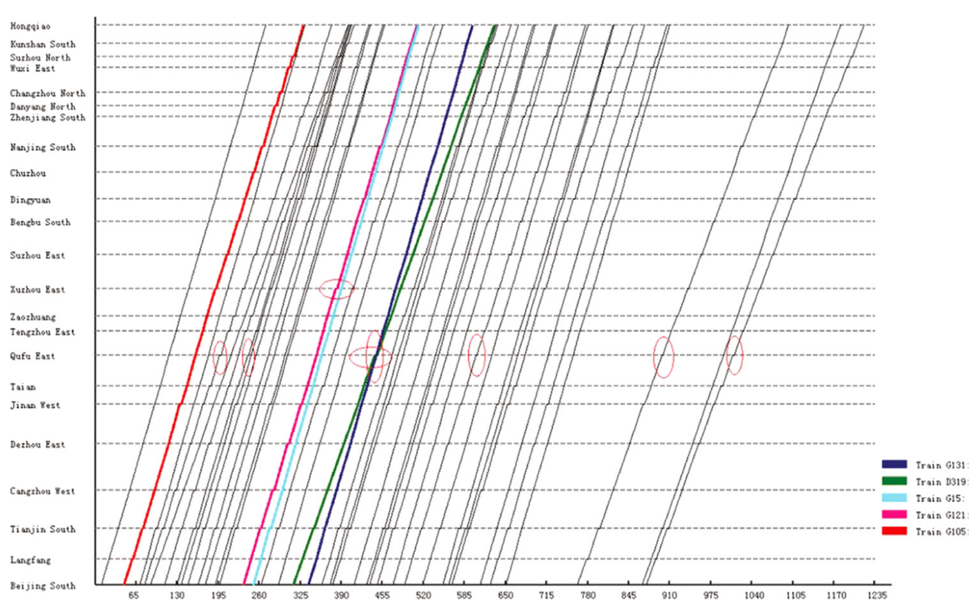

Fig. 12. Train schedule of Beijing–Shanghai high-speed railway corridor for ITT.

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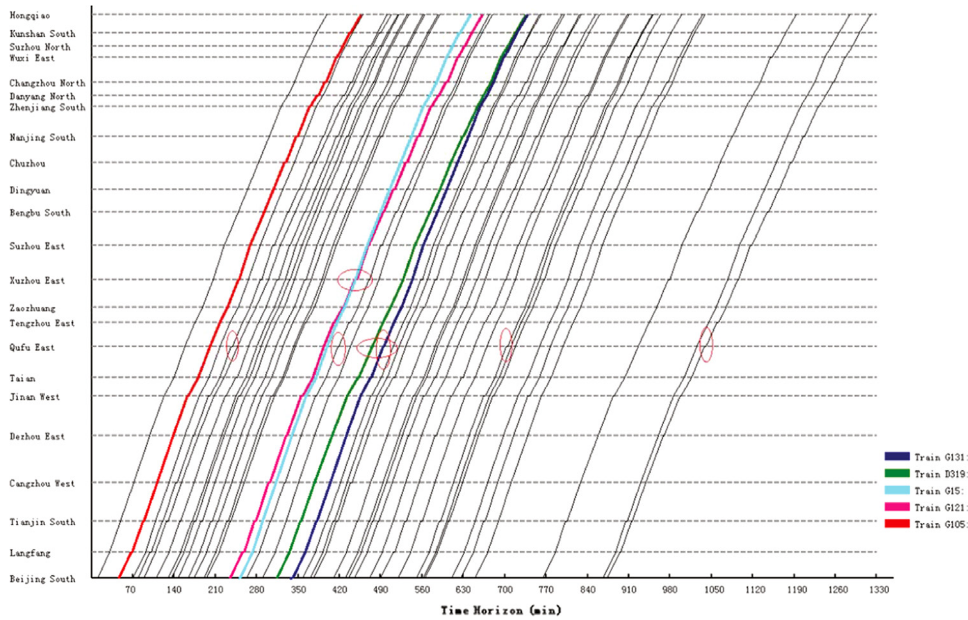

Fig. 13. Train schedule of Beijing–Shanghai high-speed railway corridor for RTT.

assumes that all trains start from initial station 1 and end at terminal station to simplify the problem, which has a little difference with the real-world situations. Actually, this treatment also reduces the possibility of the comparability between the generated stop plan and the realistic stop plan on Beijing–Shanghai high-speed railway. Thus, we omit this work in the following discussion.

4.3. Additional experiments on Beijing–Shanghai high-speed railway corridor

Next, six sets of additional experiments are also implemented by adopting different parameters on the Beijing–Shanghai high-speed railway corridor to further demonstrate the performance of our proposed model and solution quality.

(i) In the first set of experiments, we aim to test the sensitivity of the outputted optimal solution with respect to the parameter OPTCR. Note that the parameter OPTCR, which represents the relative gap between the final objective and the possible best objective, is the key parameter in GAMS software to guarantee the quality of the produced solution. In general, small parameter OPTCR will correspond to more computational time. To explicitly show the relationship between the computational time and this parameter in the experiments, we use different parameters to solve the model for ITT, i.e., OPTCR=0.5, 0.1, 0.05, 0.01, 0.005, 0.001. The computational results are displayed in Table 8.

Clearly, the computational time in the solution process increases drastically with the decrease of OPTCR, since it requires more iterations to obtain a more accuracy solution. When we take small parameters, the returned near-optimal solutions may have little changes to meet the given OPTCR (for instance, when we set OPTCR as 0.01, 0.005 and 0.001 respectively). Practically, setting OPTCR as 0.05, 0.01 and 0.005 can be also acceptable since the final computational time and solution accuracy are all acceptable. Yet, when OPTCR is set as 0.001 in the experiment, the CPLEX solver needs more than 2 h to find a solution that satisfies the high-accuracy requirements. Thus, for the sake of saving the computational time, the OPTCR will be taken as 0.05 in the following experiments if there are no additional statements.

(ii) In the second set of experiments, we intend to test the influence of different weight coefficients in the objective function on the optimized solutions and computational times since different weight coefficients γ_1 and γ_2 might correspond to different optimal departure times of individual trains and total dwelling times. In order to show this influence clearly, we respectively set γ_1, γ_2 as {1.0, 0.0}, {0.9, 0.1}, {0.8, 0.2}, {0.7, 0.3}, {0.6, 0.4}, {0.5, 0.5}, {0.4, 0.6}, {0.3, 0.7}, {0.2, 0.8}, {0.1, 0.9} and {0.0, 1.0} in the experiments for ITT. In addition, the OPTCR is set as 0.01 in this set of experiments to show this influence more accurately, as CPLEX solver possibly returns a near-optimal solution instead of the exact optimal one. The detailed computational results are given in Table 9.

In Table 9, we list the variation of two terms in objective function (19) to clearly state the influence of different weight parameters, where the total delay time at origin station and dwelling time at all intermediate stations, respectively, refer to as Eqs. (18,17) (i.e., T_{delay} and T_{dwell}). Consequently, the total delay and total dwelling time in near-optimal solutions take different variant tendencies for individual parameter pairs $\{\gamma_1, \gamma_2\}$ (i.e., they are not comonotonic with respect to either γ_1 or γ_2), since different values of these two parameters in essence represent distinctive decision strategies. For instance, when γ_2 is increased from 0.5 to 0.6, the total delay at origin station increases from 0 to 2 min and the dwelling time decreases from 394 to 392 min. Moreover, when γ_2 is respectively set as 0.6, 0.8, 0.9 and 1.0, the total delay between optimized and expected departure times are respectively 2, 7, 5 and 86 min with the same dwelling time 392 min, which also illustrates the importance of considering the different coefficients in the objective. In addition, from Table 9, we can see that the computational time of CPLEX solver finding the near-optimized solution is influenced by the values of these two coefficients. In extreme cases, no solution can be found by CPLEX solver for the first and third implementations. Thus, more efficient and stabilized heuristic algorithms should be designed in our further research. If there are no additional explanations in the following experiments, the weight coefficients of the total delay and dwelling time are respectively set as 0.1 and 0.9 for computational convenience.

To show the influence of different weight coefficients on the generated stop plans, the train stop plan for ITT with weight coefficients $\gamma_1 = \gamma_2 = 0.5$ is also displayed in Fig. 15. Typically, in this case, the stop plans for individual trains and the trains

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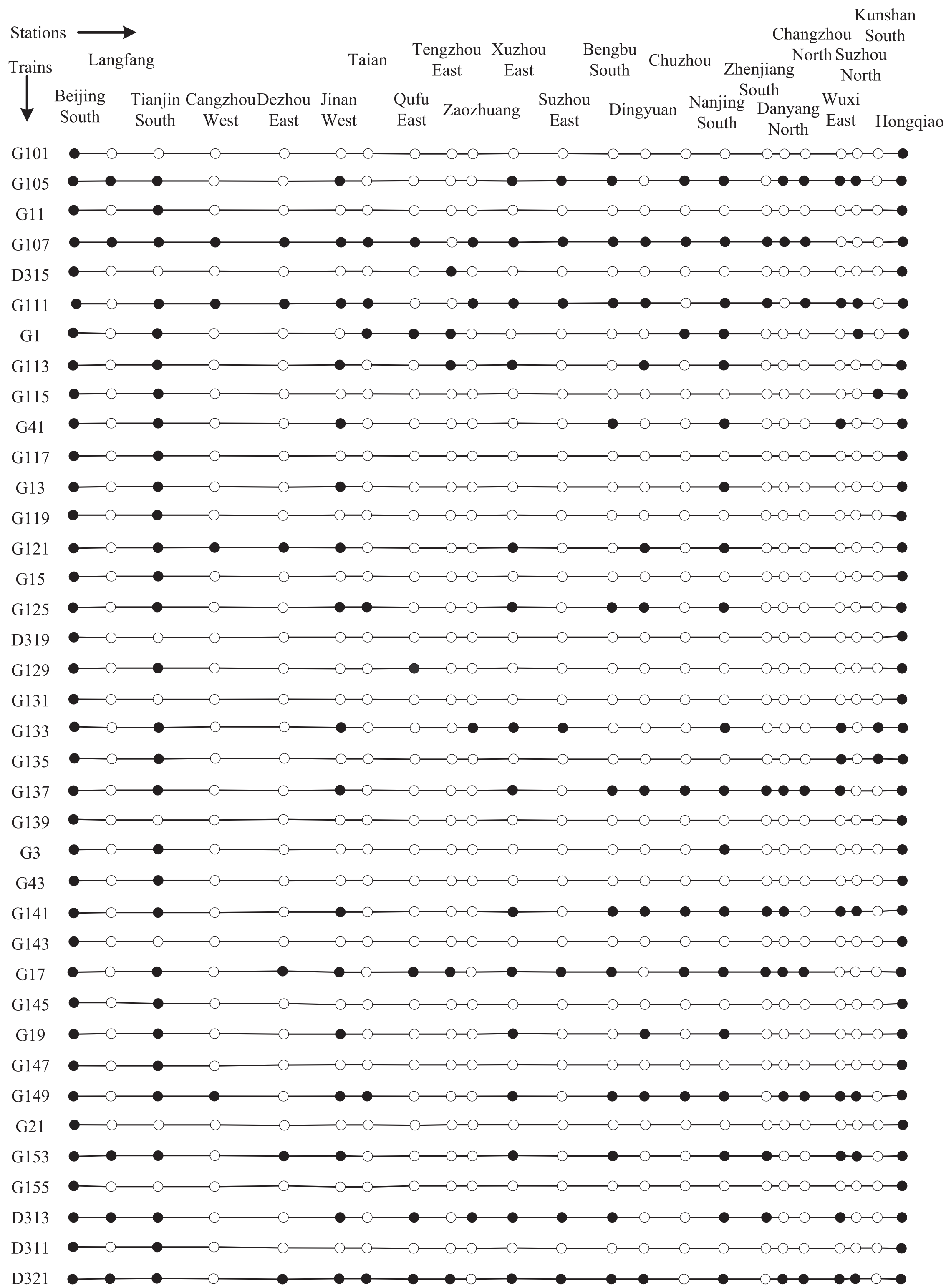

Fig. 14. Train stop plan of Beijing–Shanghai high-speed railway corridor for ITT. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

stopping at each station are different from those of Fig. 14 where weight coefficients are set as $\gamma_1 = 0.1$ and $\gamma_2 = 0.9$. For example, trains G105, G107, G153, D313 and D321 stopping at Langfang station in Fig. 14 are replaced by trains G107, G111, G149, D313 and D321 in Fig. 15. Except for the initial and terminal stations, train G133 is scheduled to stop at other 12 stations (i.e., Tianjin South, Cangzhou West, Jinan West, Taian, Qufu East, Xuzhou East, Suzhou East, Bengbu South, Dingyuan, Chuzhou, Nanjing South and Danyang North stations) in the stop plan with weight coefficients $\gamma_1 = \gamma_2 = 0.5$, while for the case of $\gamma_1 = 0.1, \gamma_2 = 0.9$, the scheduled stops are changed as 8 stations (i.e., Tianjin South, Jinan West, Zaozhuang, Xuzhou East, Suzhou East, Nanjing South, Wuxi East

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Table 8
Computational results with different OPTCR.

OPTCR	Comput. time	Total delay (min)	Dwelling time (min)	Returned obj. (min)	Estimated best obj. (min)	Relative gap
0.5	$9''946$	59.00	457.00	417.20	349.20	0.162991
0.1	$16''178$	51.00	423.00	385.80	349.20	0.094868
0.05	$35''750$	14.00	406.00	366.80	349.20	0.047983
0.01	$8'52''696$	5.00	392.00	353.30	349.90	0.009623
0.005	$38'25''270$	11.00	390.00	352.10	351.00	0.003124
0.001	$2:14'03''689$	5.00	390.00	351.50	351.15	0.000996

Table 9
Optimal objectives and computational times with different weight coefficients.

γ_1	γ_2	Comput. time	Total delay (min)	Dwelling time (min)	Returned obj. (min)	Relative gap
1.0	0.0	Failed to find the solution				
0.9	0.1	$1:27'00''716$	0.00	394.00	39.40	0.008883
0.8	0.2	Failed to find the solution				
0.7	0.3	$1:11'21''405$	0.00	394.00	118.20	0.008883
0.6	0.4	$19:58'44''562$	0.00	394.00	157.60	0.009994
0.5	0.5	$15'59''766$	0.00	394.00	197.00	0.00992
0.4	0.6	$1:39'52''557$	2.00	392.00	236.00	0.008413
0.3	0.7	$11'29''233$	6.00	390.00	274.80	0.006550
0.2	0.8	$12'11''874$	7.00	392.00	315.00	0.009524
0.1	0.9	$8'52''696$	5.00	392.00	353.30	0.009623
0.0	1.0	$9'23''496$	86.00	392.00	392.00	0.009720

and Kunshan South stations). In addition, the extra stop of train G129 in Fig. 14 at Qufu East station is avoided in Fig. 15 through adjusting the weight parameters.

(iii) In the formulation, we particularly relax the expected departure time at origin station to a time interval with the largest allowable fluctuation time ΔT. This treatment can expectedly cancel the conflicts in case that the expected departure time is presetted improperly. In the following, the sensitivity analysis of the near-optimal objectives will be performed with respect to different fluctuation time ΔT, where the ITT is used as input data, and the fluctuation time ΔT is set as 0, 5, 10, 15 and 20, respectively. In Table 10, the corresponding computational results are displayed in detail.

Clearly, as each train has more free time to choose its departure time with large fluctuation time, it is easy to see that the total dwelling time at intermediate stations decreases with the increase of fluctuation time ΔT, which also coincides with the practical situations since enhancing the parameter ΔT necessarily enlarges the feasible region of the proposed model, leading to a relative small optimal objective value. Moreover, although the objective value only has little variation with the increase of fluctuation times, the total and maximum delays (i.e., the last column of Table 10) between the scheduled and expected departure times of all trains are changed greatly. Additionally, we also notice that an exception actually occurs for $\Delta T = 20$, where both the objective and total delay at origin station are greater than those of $\Delta T = 15$. This is mainly because that CPLEX solver can only find a near-optimal solution within the predetermined gap instead of the exact optimal solution (OPTCR = 0.05 in all implementations). If we further reduce the predetermined gap, for instance letting OPTCR = 0.04, we can find an optimal objective 363.10 for the case of $\Delta T = 20$, which is typically better than that of $\Delta T = 15$. Thus, the choice of this parameter should be treated carefully in practical applications. Note that the fluctuation time is impossible to be set as a too large constant in reality. Thus for other experiments, ΔT is always set as 5 min for each train in the process of optimizing stop plans and schedules.

(iv) Next, we aim to test the computational performance of CPLEX solver with respect to different numbers of trains. Specifically, we set 6 trains as a wave for operations on the corridor, in which there are 5 *G*-trains and 1 *D*-train (this is a fixed-order traversing mode). A total of eight experiments are implemented with 12, 24, 36, 48, 60, 72, 84 and 96 trains, respectively, in which the passenger demands at each station are assumed to increase by the same ratio of the initial demands. For instance, in Table 6, the total passenger demand at Beijing South station is 22 800 for 38 trains. Then in this set of experiments, the passenger demand at this station is set to be 7200 for 12 trains (i.e., $22\,800 \times 12/38 = 7200$), 14 400 for 24 trains and 21 600 for 36 trains, and so on. Similar to the passenger demand, the trains scheduled to stop at each intermediate station is no less than 1, 2, 3, 4, 5, 6, 7 and 8, respectively, in this set of experiments. The expected departure interval from the initial station is set as 30 min, and the other data are supposed to be the same as those experiments for ITT.

Clearly, in our proposed model, if we do not provide the departure order from the initial station, a binary variable z_i^G can be used to determine whether train i is a *G*-train or *D*-train. To compare the difference of these two methods, we also implement numerical experiments with the same number and type of trains, respectively, without specifying the detailed departure sequence (i.e., free-order traversing mode).

To depict the computational characteristics, we give Fig. 16 to show the variation tendencies of computational times for fixed-order and free-order traversing modes associated with different numbers of trains. It follows from the computational results shown in Table 11 and Fig. 16 that the computational time of solving these models will enhance with the increase of the number of trains for both traversing modes, because increasing number of trains significantly raises the complexity of decision variables and constraints.

To compare the near-optimal solutions for fixed-order and free-order modes, the differences of the corresponding results for these two modes are also displayed in Table 11, which are defined as the values of the free-order results minus the corresponding results of the fixed-order solutions. As expected, when we use the model to generate the departure sequence of trains from origin station, it needs more time to solve this model in comparison to the case of giving the departure order in advance. Additionally, time increment of the free-order traversing mode seems to increase more drastically along with the increasing number of trains compared to the fixed-order case. The free-order traversing mode can get a more optimal result in most situations. However, when we consider 96 trains in the experiment, CPLEX solver can not find the optimal solution for free-order mode in this experiment due to the limited memory of our computer (Windows 7 workstation with two Intel Core i3-4130M CPUs and 4G RAM). Thus, although the GAMS software demonstrates its efficiency in solving the medium-scale optimal problem, it is still not time-efficient enough in

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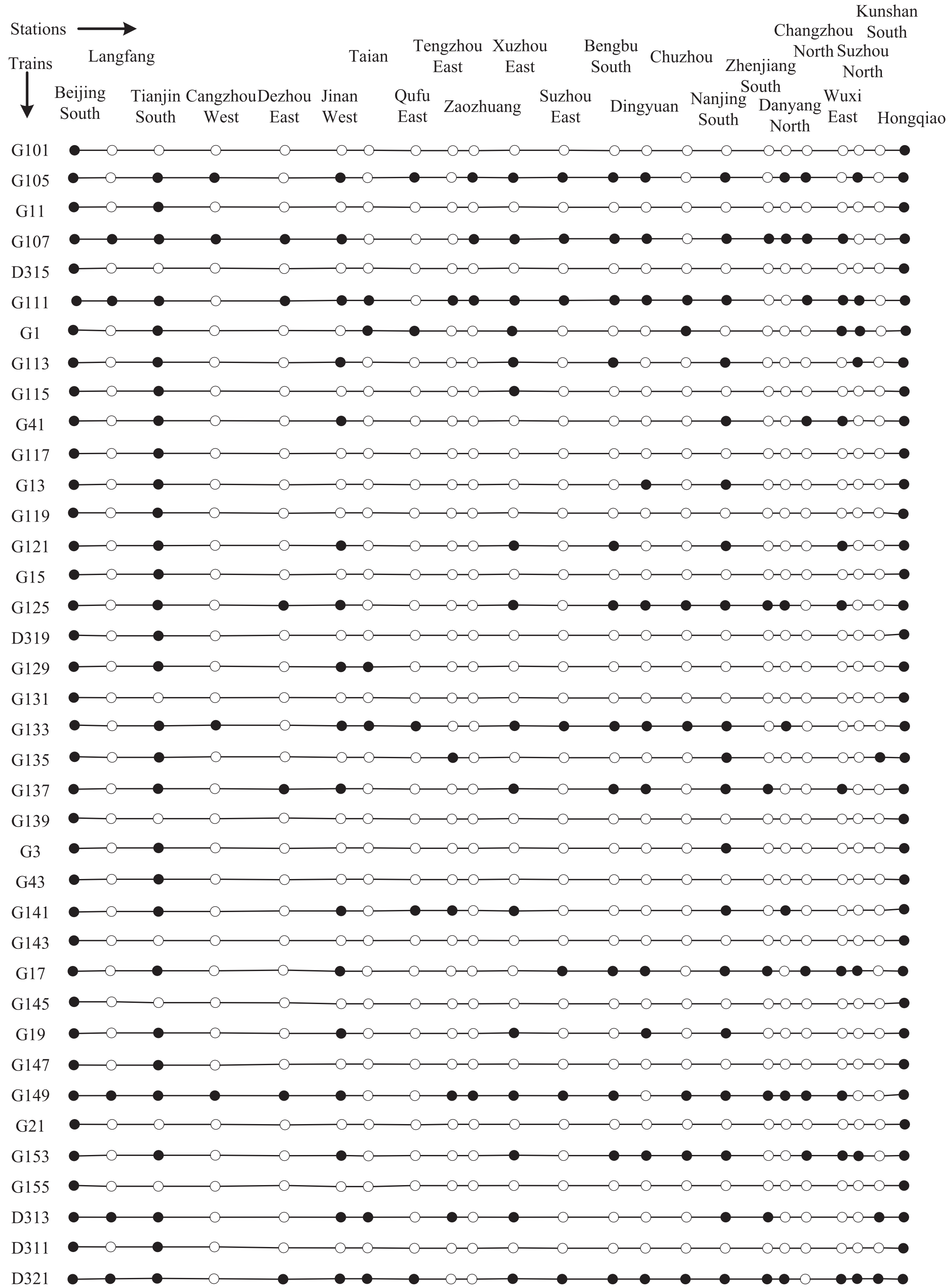

Fig. 15. Train stop plan for ITT with weight coefficients $\gamma_1 = \gamma_2 = 0.5$.

solving large-scale problems (with more trains). Future research should focus on designing efficient heuristic algorithms to solve the real-world large-scale problems.

(v) To the best of our knowledge, the mixed train flow with different velocities will lead to the reduction of passing capability of railway lines. In order to analyze the influence of *D*-trains in the traffic system (as the allowable highest speed is lower than that of *G*-trains), we particularly take 0, 5, 10, 15, 20, 25, 30, 35 and 38 *D*-trains in numerical experiments on Beijing–Shanghai high-speed railway corridor for ITT, in which the final departure orders from the initial station are automatically generated in the process of solving models. From Table 12, the computational time for solving this problem increases with the increase of the number of *D*-trains, and meanwhile the decreasing tendency occurs when the number of *D*-trains is over 20. The similar variation tendency also appears for the total stops at intermediate stations. This fact

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means that the computational time and total stops for only one type of trains on the railway line are both less than those of two types of trains for most situations. For instance, when there are no *D*-trains in the experiment, the computational time turns out to be 14″266 with 194 total stops at intermediate stations; if there are no *G*-trains in the experiment (i.e., the number of *D*-trains is 38), the computational time turns out to be 36″160 with 194 stops. However, when both *G*-trains and *D*-trains have relatively large percentage of the total trains (e.g., the numbers of *D*-trains and *G*-trains are 20 and 18, respectively), the computational time will be enhanced up to 36′23″197 with 195 stops at intermediate stations.

Table 10
Computational results with different fluctuation times.

Fluctuation ΔT (min)	Comput. time (min)	Total delay (min)	Dwelling time (min)	Returned obj. (min)	Relative gap	Max delay (min)
0	4′31″127	0.00	408.00	367.20	0.049020	0
5	35″750	14.00	406.00	366.80	0.047983	5
10	35″475	55.00	402.00	367.30	0.049279	10
15	32″206	34.00	400.00	363.40	0.039075	13
20	41″842	50.00	400.00	365.00	0.043288	13

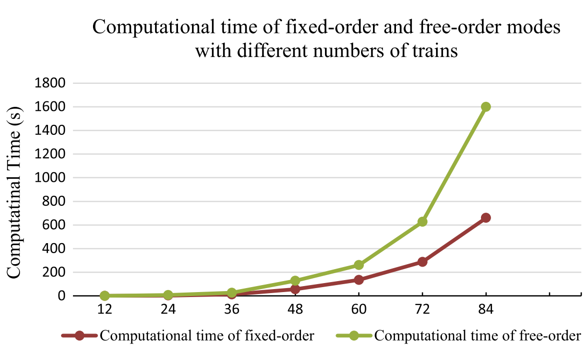

Fig. 16. Computational time of two modes with different numbers of trains.

In addition, note that although the total stops of all trains at intermediate stations are all 194 or 195 times to guarantee the passenger demands when the number of *D*-trains increases, the detailed stop plans are actually different for various experimental data. For example, trains 6, 12, 20, 26, and 30 and trains 4, 10, 14, 22, and 28 are respectively stop at Langfang station to provide the necessary services for the passengers when there are 0 and 38 *D*-trains; however, for cases of 5 *D*-trains and 10 *D*-trains, these trains will be replaced by trains 22, 30, 32, 36, and 38 and trains 14, 26, 28, 30, and 32, respectively (here, the train index refers to its index of departure sequence). It is also interesting to mention that the maximum number of extra stops in this set of experiments is 3 with a relatively large parameter $\gamma_2 = 0.9$ in the implementations. Here, an extra stop refers to the stop without loading/unloading passengers operations, which is generated in the scheduling process only for safe operation concerns. The number of extra stops is thus the difference between the total number of stops in the schedule and total number of stops for loading/unloading passengers (i.e., $\Sigma_{i=1}^{|T|}\Sigma_{s=1}^{|S|}x_{is}$).

(vi) In the last set of experiments, we are especially interested in comparing the computational performance of different optimization solvers. With this concern, the same experiments with different number of trains for the free-order traversing mode are also implemented by XPRESS solver. The detailed computational results are listed in Table 13.

To compare the performance of CPLEX and XPRESS solvers clearly, the differences of the corresponding results for these two solvers are also displayed in Table 13, which are defined as the values that XPRESS results minus the corresponding results of CPLEX solutions. Clearly, since the increasing number of trains significantly raises the complexity of decision variables and constraints, the computational time of solving these models will raise with the increase of the number of trains for both of these two solvers. In comparison, the variation tendency by XPRESS solver increases much more drastically than that by CPLEX solver. When the number of involved trains is added up to 48, the XPRESS solver fails to return a solution within 10 h. Moreover, compared with XPRESS, the CPLEX solver can find a relatively better solution in most situations. Although XPRESS solver also has a relatively good

Table 11
The comparison of different number trains for fixed-order and free-order traversing modes.

Train numbers	Comput. time			Total delay (min)		
	Fixed-order	Free-order	Difference	Fixed-order	Free-order	Difference
12	0″549	1″154	0″605	2.00	0.00	−2.00
24	2″629	7″037	4″408	10.00	13.00	3.00
36	12″902	27″004	14″102	9.00	12.00	3.00
48	56″701	2′08″170	1′11″469	11.00	14.00	3.00
60	2′15″465	4′20″692	2′5″227	25.00	25.00	0.00
72	4′47″656	10′26″544	5′38″888	24.00	33.00	9.00
84	10′59″465	26′38″317	15′38″852	35.00	56.00	21.00
96	19′23″883			42.00		

Train numbers	Dwelling time (min)			Returned obj. (min)		
	Fixed-order	Free-order	Difference	Fixed-order	Free-order	Difference
12	146.00	142.00	−4.00	131.60	127.80	**−3.80**
24	268.00	270.00	2.00	242.20	244.30	2.10
36	388.00	378.00	− 10.00	350.10	341.40	**−8.70**
48	518.00	504.00	−14.00	467.30	455.00	**−12.30**
60	637.00	628.00	−9.00	575.80	567.70	**−8.10**
72	755.00	748.00	−7.00	681.90	676.50	**−5.40**
84	883.00	882.00	−1.00	798.20	799.40	1.20
96	1010.00			913.20		

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Table 12
Computational results for different numbers of *D*-trains with ITT.

D-trains number	Comput. time	Total delay (min)	Dwelling time (min)	Returned obj. (min)	Relative gap	Total stops	Extra stops
0	14″266	13.00	406	366.70	0.047723	194	0
5	52″111	12.00	407.00	367.50	0.049796	194	3
10	5′40″602	15.00	404.00	365.10	0.043550	194	1
15	1′21″472	12.00	406.00	366.60	0.047463	194	1
20	**36′23″197**	3.00	404.00	363.90	0.040396	195	0
25	19′20″508	17.00	406.00	367.10	0.048761	195	3
30	17′25″519	8.00	407.00	367.10	0.048761	195	2
35	1′7″425	18.00	406.00	367.20	0.049020	194	0
38	36″160	11.00	406.00	366.50	0.047203	194	0

Table 13
The comparison of CPLEX and XPRESS solvers associated with different numbers of trains.

Train numbers	Comput. time			Total delay (min)		
	CPLEX	XPRESS	Difference	CPLEX	XPRESS	Difference
12	1″154	8″892	**7″738**	0.00	2.00	2.00
24	7″037	31″777	**24″740**	13.00	5.00	− 8.00
36	27″004	15′47″874	**15′20″870**	12.00	1.00	− 11.00
48	2′08″170			14.00		

Train numbers	Dwelling time (min)			Returned obj. (min)		
	CPLEX	XPRESS	Difference	CPLEX	XPRESS	Difference
12	142.00	148.00	6.00	127.80	133.40	**5.60**
24	270.00	268.00	− 2.00	244.30	241.70	− 2.60
36	378.00	384.00	6.00	341.40	345.70	**4.30**
48	504.00			455.00		

performance for solving medium-scale optimal problems, the CPLEX solver is typically more steady for the proposed model with the high-quality returned solutions.

5. Conclusions and further works

Aiming to provide a system-optimization framework for railway planning, this study first integrated the train stop planning and train scheduling problems together into a fundamental collaborative optimization model on a single-track high-speed railway corridor. To set up the connection between the train stop plan and train schedule, a binary variable was introduced to determine whether a train is scheduled to stop at a station or not. Through minimizing the total delay at origin station and dwelling time at intermediate stations, the problem was formulated as a mixed integer linear programming model, where the passenger demands constraints are used to guarantee the necessary service levels. In order to show the effectiveness and efficiency of the proposed approaches, two sets of examples were implemented. The first set of experiments demonstrated the applications of the produced model. Then, a real case study was performed on the Beijing–Shanghai high-speed railway corridor with the practical operation data. The computational results showed that the GAMS software with CPLEX solver can efficiently solve the medium-scale problem with the reasonable computational time.

It is worth mentioning that our proposed model is an initial collaborative optimization model for handling both train scheduling and train stop planning simultaneously, which has more generalization spaces for some particular requirements. For instance, in our proposed model, it is possible to fix the train departure order from initial station for some special purposes, or generate the best departure order through introducing a binary variable in the model to indicate the train types, according to the real-life requirements. Besides, the operational zone-based services can also be integrated into the formulation through suitably defining the OD pairs of trains on the railway corridor for meeting various passenger demands (short-distance or long-distance trips). Typically, all of these modelling generalizations can be expected to fit the real-life operations as much as possible, and then provide system optimization-based near-optimal operational strategies by designing efficient solution methods.

Further research will focus on the following several aspects. (1) Besides the dwelling time and delay time, more evaluation indexes can be explicitly analyzed and taken into consideration in the further studies because the problem of interest is typically related to a multi-objective decision making process. (2) An efficient heuristic algorithm can be developed to speed up the searching process as the GAMS software has a relatively low time-efficiency in solving large-scale problems (e.g., when over 96 trains are considered, the computational times for some experiments will be more than one hour or the memory limit will be broken). (3) On the operational level, it is meaningful to explore the train scheduling with specific passenger choice behaviors through tracking the number of passengers in each train (see Canca et al. [2]). Thus, the topic, which explicitly considers the train stop planning, train scheduling and passenger micro-choice behaviors, can also be investigated in our future research.

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