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## Labor market frictions and optimal steady-state inflation

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## ABSTRACT

In central theories of monetary non-neutrality, the Ramsey optimal steady-state inflation rate varies between the negative of the real interest rate and zero. This paper explores how the interaction of nominal wage and search and matching frictions affect the policy prescription. We show that adding the combination of such frictions to the canonical monetary model can generate an optimal inflation rate that is significantly positive. Specifically, for a standard U.S. calibration, we find a Ramsey optimal inflation rate of 1.16 percent per year.

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## 1. Introduction

In leading theories of monetary non-neutrality, the policy prescription for the optimal steady-state inflation rate varies between the negative of the real interest rate (the Friedman rule) and zero (price stability); see [Schmitt-Grohé and Uribe \(2010\)](#), for an overview. In this paper we explore a new channel where the interaction of nominal wage and labor market search and matching frictions affects the planner's trade-off between the welfare costs and benefits of inflation. We show that the combination of such frictions can in fact generate a Ramsey optimal inflation rate that is significantly positive. Importantly, this is the case even in the presence of a monetary friction, which drives the optimal inflation rate towards the Friedman rule of deflation.

The mechanism we have in mind arises in a model with search frictions when nominal wages are not continuously rebargained and some newly hired workers enter into an existing wage structure.<sup>1</sup> In this case, we show in a stylized model that inflation not only affects real-wage profiles over a contract spell, but also redistributes surplus between workers and firms, since incumbent workers impose an externality on new hires through the entry wage. Specifically, this affects the wage-bargaining outcome through the workers' outside option and hence the expected present value of total labor costs for a match as well as firms' incentives for vacancy creation. We derive a Hosios condition for the stylized model and show that the Ramsey planner has incentives to increase inflation if employment and vacancy creation are inefficiently low in order to push the economy towards the efficient allocation.

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<sup>1</sup> Data supports that a non-negligible share of newly hired workers enter into an existing wage structure. First, micro-data evidence on wages does not indicate that wages are more sensitive to labor market conditions at the beginning than later in the span of a match (once the variation in the composition of firms and match quality over the cycle is controlled for); see [Gertler and Trigari \(2009\)](#). Secondly, survey evidence, like [Bewley \(1999, 2007\)](#) for the U.S. and the study performed within the Eurosystem Wage Dynamics Network (WDN), reported by [Galuscak et al. \(2012\)](#), present strong evidence that the wages of new hires are tightly linked to those of incumbents.

It is worth noting that this incentive vanishes at an efficient allocation, as in [Thomas \(2008\)](#) where the calibration is chosen so that the Hosios condition holds, and hence that search and wage setting externalities cancel out in steady-state (and the reverse occurs if employment is inefficiently high). Moreover, the model of [Erceg et al. \(2000\)](#) does not feature this mechanism either, since there is no extensive margin on the labor market in that model and hence the Ramsey planner has no leverage on job creation through the channel outlined above. However, and similar to our model, the [Erceg et al. \(2000\)](#) model features a markup in wage-setting where the actual markup can be different from the flexible price markup because of [Calvo \(1983\)](#)-style wage stickiness. The planner has incentives to tilt the real-wage profile in order to lower the actual markup and increase labor input. Note though, since the model lacks a leverage on job creation there is much less of a motive for the planner to use this channel and the effect of using inflation to affect the average markups in the economy is tiny.<sup>2</sup> The mechanism also vanishes if search frictions vanish since the Ramsey planner loses any leverage over vacancy/job creation. Thus, models without an extensive margin on the labor market lack the mechanism described here. Furthermore, the Ramsey planner loses the ability to affect real-wage costs via inflation if all new workers get to rebargain their wage. In this case, the full effect of inflation on entry wages is internalized in the wage bargain, and firm and worker surpluses, as well as real wage costs, become neutral to inflation.

Overall, the key insight from the stylized model is that if both search and wage-setting externalities are present, there is an incentive for the Ramsey planner to vary the inflation rate to increase welfare through its effect on job creation and unemployment.

To quantitatively evaluate the relative strength of this mechanism, we introduce it into a full-fledged model encompassing leading theories of monetary non-neutrality. The model we outline features a non-Walrasian labor market with search frictions as in [Mortensen and Pissarides \(1994\)](#), [Trigari \(2009\)](#) and [Christoffel et al. \(2009\)](#). Moreover, there are impediments to continuous resetting of nominal prices and wages modeled along the lines of [Dotsey et al. \(1999\)](#), where adjustment probabilities are endogenous. Finally, the model features a role for money as a medium of exchange, as in [Khan et al. \(2003\)](#) and [Lie \(2010\)](#).

In the quantitative model, variation in the average inflation rate will have several effects on welfare. First, inflation will affect the opportunity cost of holding money, pushing the optimal inflation rate towards the Friedman rule. Second, because of monopolistic competition and nominal frictions, inflation causes relative price distortions, which drive the optimal inflation rate towards zero. Finally, the mechanism presented above is introduced, i.e., search frictions combined with new hires entering into an existing wage structure, where the inflation rate affects equilibrium real-wage costs and, in turn, job creation.

In a standard U.S. calibration of the model, implying that employment is 1.87 percentage points lower than in the efficient allocation, the Ramsey optimal inflation rate is 1.16 percent per year. Moreover, varying the share of new hires receiving rebargained wages has a substantial effect on the optimal inflation rate. If all newly hired workers receive rebargained wages, thus shutting down the interaction effect between nominal wage frictions and search and matching frictions, the optimal inflation rate is  $-0.76$  percent.<sup>3</sup> When none [50 percent, the baseline] (all, as in [Gertler and Trigari, 2009](#)) of the newly hired workers enter into an existing wage structure, the optimal inflation rate is  $-0.76$  [1.16] (1.44) percent. Thus, only a moderate share of new workers entering into an existing wage structure is needed to obtain a significantly positive optimal inflation rate.

When shutting down the monetary distortion and looking at the cashless economy, as analyzed in [Woodford \(2003\)](#), the Ramsey optimal inflation rate increases to 1.51 percent. Thus, the monetary distortion has a moderately negative effect on the optimal policy prescription.

The results reported above are conditional on agents optimally choosing when to change prices and wages. It is then interesting to study the effect of shutting down the endogenous response of the adjustment probabilities to variations in inflation and let the agents face a fixed adjustment hazard. In contrast to [Lie \(2010\)](#), we find that endogenizing adjustment probabilities matters for the quantitative analysis. Specifically, exogenous price and wage adjustment hazards give a Ramsey optimal inflation rate of 3.02 percent, thus an increase of almost two percentage points relative to the case with endogenous adjustment hazards.

All in all, the combination of search and wage-setting externalities within the canonical monetary model introduces an important link between inflation and welfare and hence potentially a large difference in prescribed policy.

For clarity, the quantitative model outlined in this paper does not encompass all mechanisms that can affect the Ramsey optimal steady-state inflation rate. Papers studying the effect of other mechanisms on the Ramsey optimal steady-state inflation are [Schmitt-Grohé and Uribe \(2010\)](#) using inflation as an indirect tax to address tax evasion, [Schmitt-Grohé and Uribe \(2012a\)](#) analyzing foreign demand of domestic currency, [Schmitt-Grohé and Uribe \(2012c\)](#) studying quality bias, [Adam and Billi \(2006\)](#) and [Billi \(2011\)](#) looking into the effect of the zero lower bound, and [Kim and Ruge-Murcia \(2011\)](#) addressing downward nominal wage rigidity. Of these, only a substantial foreign demand of domestic currency and a planner that only cares about the well-being of the home country may lead to a significantly positive inflation rate. Moreover, all of these

<sup>2</sup> See e.g., [Amano et al. \(2009\)](#) or [Kim and Ruge-Murcia \(2011\)](#).

<sup>3</sup> This is the same rate as if all wage contracts are continuously rebargained in the model (not only those of the new hires). These cases are the same due to the fact that wages are not allocative in the search-matching framework we rely on, or more specifically, a relative-wage dispersion across firms does not give rise to a dispersion of labor supply across individuals working at different firms.

features are, if anything, likely to drive up the Ramsey optimal steady-state inflation rate. Thus, in this sense the results presented here can be viewed as a lower bound.

This paper is outlined as follows: In [Section 2](#) the basic mechanism is presented, in [Section 3](#), we outline the framework for the quantitative evaluation, including a description of the optimal Ramsey policy, in [Section 4](#) the calibration and the quantitative results are presented. Finally, [Section 5](#) concludes.

## 2. The mechanism

To set ideas, it is helpful to first focus on a stylized stationary equilibrium model of the labor market featuring the interaction mechanism we have in mind.<sup>4</sup> Let firms and workers sign contracts with a fixed (nominal) wage,  $W$ , that with certainty lasts for two periods. Letting  $P$  denote the price level in the first period of the contract and  $\pi$  the gross inflation rate, the real wage in the first and second periods of the contract, respectively, are then  $w = \frac{W}{P}$  and  $w' = \frac{W}{\pi P} = \frac{w}{\pi}$ . This captures the first component we need, i.e. nominal wage frictions. Note that gross inflation is positive, i.e.,  $\pi > 0$ . Secondly, there are search and matching frictions captured by a constant returns matching function, giving rise to a surplus to be bargained over. Specifically, the number of matches,  $\mu$ , is given by the constant-returns matching function in [Den Haan et al. \(2000\)](#);

$$\mu = \frac{u\nu}{(u^{\sigma_a} + \nu^{\sigma_a})^{\frac{1}{\sigma_a}}}, \quad (1)$$

where  $u$  is unemployment and  $\nu$  vacancies. The probability that a worker is matched to a firm is  $s = \frac{\mu}{u}$  and the probability that a vacancy is filled is  $q = \frac{\mu}{\nu}$ . The surplus of the firm (worker) is denoted  $J^i (H^i)$  with  $i \in \{0, 1\}$  where  $i=0$  denotes that the wage is rebargained and  $i=1$  that the wage is not rebargained. The surplus for the firm in a period when wages are rebargained ( $J^0$ ) and not rebargained ( $J^1$ ) are then

$$\begin{aligned} J^0 &= y - w + \beta \rho J^1 \\ J^1 &= y - \frac{w}{\pi} + \beta \rho J^0, \end{aligned} \quad (2)$$

where  $y$  is the (real) marginal revenue for the firm,  $\beta$  is the discount factor and  $\rho$  is the fixed probability that the match survives into the next period. Similarly, the surpluses for the worker are

$$\begin{aligned} H^0 &= w - z + \beta [\rho H^1 - sH^a] \\ H^1 &= \frac{w}{\pi} - z + \beta [\rho H^0 - sH^a], \end{aligned} \quad (3)$$

where  $z$  is (real) income received when unemployed,  $s$  the probability of finding a job and  $H^a$  the average value of being employed across all firms in the economy. Wages are determined in bargaining and are given by the Nash Bargaining Solution (NBS);  $\varphi J^0 = (1 - \varphi)H^0$ . Note that variations in  $H^a$  affect the workers' outside option in the bargain. All renegotiating firms set the same wage  $w$  and the wage path for a rebargaining firm is  $\{w, \frac{w}{\pi}, w, \frac{w}{\pi}, \dots\}$  and for a non-rebargaining firm, the wage path is  $\{\frac{w}{\pi}, w, \frac{w}{\pi}, w, \dots\}$ . Relying on the same notation, note also that job creation is

$$c = \beta q(\theta)J^i \quad (4)$$

for  $i=0,1$ . Thus, it is the wages of newly hired workers that matters for equilibrium outcomes (echoing [Pissarides, 2009](#)).

For simplicity, this section focuses on the two boundary cases where  $H^a = H^0$  or  $H^a = H^1$ . Thus, either new hires get rebargained wages or they enter into the second period of the wage contract. Letting  $\tilde{w}^0 = \frac{w + \beta \rho \frac{w}{\pi}}{1 + \beta \rho}$  and  $\tilde{w}^1 = \frac{\frac{w}{\pi} + \beta \rho w}{1 + \beta \rho}$  denote the present value of wages scaled by  $1 - \beta \rho$  in case of rebargaining or not in the initial period, respectively, Nash bargaining implies that, when renegotiating, the parties share the surplus so that the present value of wages shares the present value of surpluses according to the bargaining power  $\varphi$  and we get

$$\tilde{w}_i^0 = \varphi y + (1 - \varphi)(z + \beta s(\theta)H^a), \quad (5)$$

where subscript  $i = \{n, e\}$  denotes when  $H^a = H^0$  and  $H^a = H^1$ , respectively. That is  $\tilde{w}_n^0$  is the wage when all new hires get a new rebargained wage and  $\tilde{w}_e^0$  is the wage when all new hires enter into an old contract, i.e., enter into an existing wage structure. In the latter case, the present value of the wage sequence is lower when  $\pi > 1$ , since the worker first gets the deflated wage and then the rebargained wage  $w$ . Furthermore, the higher the inflation rate, the lower the present value of wages when new hires enter into an existing contract relative to in a renegotiated match. We denote this ratio by  $\delta(\pi) \equiv \frac{\tilde{w}_e^1}{\tilde{w}_e^0} = \frac{\frac{w}{\pi} + \beta \rho w}{\frac{w}{\pi} + \beta \rho \frac{w}{\pi}}$  for  $i = \{n, e\}$ . Note also that  $\delta'(\pi) < 0$ ,  $\lim_{\pi \rightarrow 0} \delta(\pi) = 1/\beta \rho$ ,  $\delta(1) = 1$  and  $\lim_{\pi \rightarrow \infty} \delta(\pi) = \beta \rho$ .

When  $H^a = H^0$ , using that we have  $H^0 = \frac{\varphi}{1 - \varphi} \frac{c}{q(\theta)J^0}$  from the NBS and job creation (4) and  $s(\theta) = \theta q(\theta)$  we get

$$\tilde{w}_n^0 = \varphi(y + c\theta) + (1 - \varphi)z. \quad (6)$$

Straightforward arguments show that the Hosios condition is  $\eta(\theta) = \varphi$ .

<sup>4</sup> See the Online Appendix for a more detailed description of the model.

When  $H^a = H^1$ , using the value functions, the NBS and that vacancy creation implies  $J^1 = \frac{c}{q(\theta)\beta}$  to solve for  $H^1$  and then using the solution for  $H^1$  in (5) gives

$$\tilde{w}_e^1 = \delta(\pi) \frac{\varphi(y + c\theta) + (1 - \varphi)z}{1 + \frac{\beta s(\theta)}{1 + \beta\rho} \left(1 - \frac{1}{\pi}\right)}. \quad (7)$$

Note that whenever  $\pi > 1$  and hence  $\delta(\pi) < 1$  the wage when new hires get rebargained wages is higher than when new hires enter into an existing wage structure, i.e.  $\tilde{w}_n^0 > \tilde{w}_e^1$ . The reason for this is that employed workers take into account that an increase in inflation affects the wage profile for their own contract when bargaining, but they do not take into consideration the effect the wage profile has on the entry wages of new hires, i.e.  $w/\pi$ . Thus, incumbent (bargaining) workers impose an externality on the entry wage of new hires. This, in turn, leads to a lowering of the present value of the wage sequence for new hires, and in turn to a worsening of the outside option for workers when bargaining and a reduction in the equilibrium wage.

Now consider the relationship to the flexible wage economy and the Hosios condition. First, when  $H^a = H^0$ , it can be shown that  $\tilde{w}_n^0$  is equal to the flexible wage; see the Online Appendix for details. Hence, the firm surplus  $J_n^0$  in Eq. (4) is the same as in the economy with flexible wages. This in turn implies that job creation, employment and unemployment are the same. Moreover, the Hosios condition is the standard  $\eta(\theta) = \varphi$ . Let  $\varphi_{H,n}$  denote the bargaining power satisfying the Hosios condition and let  $w_{H,n}$  denote the wage in (6) corresponding to this bargaining power, i.e., the wage that implements the planner allocation.

Second, if  $H^a = H^1$  when  $\pi > 1$  we have  $\tilde{w}_e^1 < \tilde{w}_n^0$  since  $\delta < 1$  and when  $\pi < 1$  we have  $\tilde{w}_e^1 > \tilde{w}_n^0$ . Focusing on the case where  $\pi > 1$ , the equilibrium average wage in a non-renegotiating match  $\tilde{w}_e^1$  does not share the surplus according to the bargaining power  $\varphi$  as in (6), but instead gives the worker a lower wage as in (7). Thus, for an unchanged surplus, the firm would get a larger share of the surplus. Moreover, from (2)–(3) the total surplus for new matches is  $[y - (z + \beta s(\theta)H^a)] / (1 - \beta\rho)$ . Since  $H_n^0 = \frac{\tilde{w}_n^0 - z}{1 - \beta(\rho - s(\theta))} > H_e^1 = \frac{\tilde{w}_e^1 - z}{1 - \beta(\rho - s(\theta))}$ , the total surplus for new matches is larger when  $H^a = H_e^1$  than when  $H^a = H_n^0$ . Thus, when new hires get non-renegotiated wages, the firm gets a larger share of a larger surplus, both of which increase the firm value of a new hire. Since  $\tilde{w}_e^1 < \tilde{w}_n^0$  we have  $J_e^1 > J_n^0$ , implying that firms will post more vacancies than in the case where new workers get rebargained wages. Hence, employment will be higher and unemployment lower. Furthermore, by subtracting and adding  $\tilde{w}_n^0$  in the numerator for the value of  $J_e^1$  in Eq. (4), and proceeding as in the flexible wage case, the Hosios condition is

$$\eta(\theta) = \varphi - \frac{1 - \beta\rho + \varphi\beta\theta q(\theta)}{(1 - \beta\rho)(y + c\theta - z)} (\tilde{w}_n^0 - \tilde{w}_e^1), \quad (8)$$

where the first two terms on the right-hand side are as in the Hosios condition when  $H^a = H^0$ , but the last term is new and is due to the fact that the wage paid to new hires is different from the wage in Eq. (6). Let  $\varphi_{H,e}$  denote the value of  $\varphi$  satisfying the above equation.

Note that Eq. (8) depends on  $\pi$  through  $\tilde{w}_e^1$ . As shown in the Online Appendix, since  $\delta$  is decreasing in  $\pi$  and the job creation condition (4) implicitly determines equilibrium tightness  $\theta$  as a function of  $\delta$  and  $\varphi$ , we can write  $\tilde{w}_e^1$  as a (decreasing) function of  $\delta$  and an (increasing) function of  $\varphi$ ;  $\tilde{w}_e^1(\delta, \varphi)$ . Then, in contrast to the flexible wage case and the case where new hires get new wages, there can be several values of  $\varphi$  for which the planner solution can be implemented by an appropriate choice of  $\delta$ . To see this, note first that, since  $\delta$  is in the open set  $(\beta\rho, 1/\beta\rho)$  and since  $\tilde{w}_e^1(\delta, \varphi)$  is increasing in  $\delta$ , the set of feasible wages,  $W_e$ , is open where the lower and upper bounds are  $\tilde{w}_e^1(\beta\rho, \varphi)$  and  $\tilde{w}_e^1(1/\beta\rho, \varphi)$ , respectively. Furthermore, as  $\beta\rho \rightarrow 1$  both the upper and lower bounds converge to  $w_{H,n}$ . Whether  $w_{H,n} \in W_e$  or not determines if the planner solution can be achieved. Note first that when  $\varphi = \varphi_{H,n}$  the planner solution can be implemented by setting  $\pi = 1$ , implying  $\delta(\pi) = 1$  and hence  $\tilde{w}_e^1(1, \varphi_{H,n}) = w_{H,n}$  and  $\tilde{w}_e^1(\beta\rho, \varphi) < w_{H,n} < \tilde{w}_e^1(1/\beta\rho, \varphi)$ . Second, since the bounds are continuous in  $\varphi$ , we have  $w_{H,n} \in W_e$  for  $\varphi$  close to  $\varphi_{H,n}$  implying that there is a  $\pi \in (0, \infty)$  that implements the planner solution.<sup>5</sup> Third, if  $w_{H,n} < \tilde{w}_e^1(\beta\rho, \varphi)$  or  $w_{H,n} > \tilde{w}_e^1(1/\beta\rho, \varphi)$  the planner solution cannot be implemented and the policymaker has incentives to either create hyper-inflation ( $\pi \rightarrow \infty$ ) or hyper-deflation ( $\pi \rightarrow 0$ ) depending on which boundary is relevant.<sup>6</sup> Thus, there is a set of values for the bargaining power

$$\Omega = \left\{ \varphi \in [0, 1] \mid \exists \delta \in (0, \infty) \text{ s.t. } \tilde{w}_e^1(\delta, \varphi) = w_{H,n} \right\}, \quad (9)$$

where the planner solution can be implemented by appropriately choosing  $\pi$ .

Recall that the Ramsey planner chooses inflation subject to the constraints from private sector behavior. Then it follows from above that the Ramsey optimal inflation rate either does not exist or is the rate that implements the planner solution. Below we develop a richer model adding additional frictions for the Ramsey planner to consider when designing optimal policy (i.e., price adjustment frictions and money demand). These frictions introduce additional trade-offs that eliminate the

<sup>5</sup> Here we do not consider a subsidy to goods production as an instrument for the Ramsey policy maker. For a discussion on tax-policy implementation of the first-best allocation in a model with nominal rigidities and labor market frictions see [Ravenna and Walsh \(2012\)](#).

<sup>6</sup> That the planner solution cannot be implemented for some parameter values follows from noting that when  $\varphi > \varphi_{H,n}$  we have  $\tilde{w}_e^1(1, \varphi) > w_{H,n}$  and, if we in addition let  $\beta\rho \rightarrow 1$ , we also have  $\tilde{w}_e^1(\beta\rho, \varphi) > w_{H,n}$ .

nonexistence problems associated with hyper-inflation/deflation. Specifically, price adjustment tends to push the Ramsey optimal inflation rate to zero, while the Friedman rule tends to push it towards the negative of the real interest rate.

Note also that, if search frictions vanish, i.e., when the job finding probability  $s \rightarrow 1$ , due to  $c \rightarrow 0$ , the competitive equilibrium converges to the planner solution. To see this, note that the planner solution has  $q \rightarrow 0$  and  $s \rightarrow 1$  when  $c \rightarrow 0$ ; see the Online Appendix for details. In the competitive economy, the equilibrium also has  $q \rightarrow 0$  and  $s \rightarrow 1$  in the case where  $c \rightarrow 0$ . This follows from that the surplus of a match  $\beta J_e^1$  is strictly positive and hence we have  $q \rightarrow 0$  from Eq. (4), in turn implying  $s \rightarrow 1$ . Thus, when search frictions vanish, the competitive equilibrium allocation converges to the planner solution, eliminating any incentives to use the mechanism above.<sup>7</sup>

The key insight here is that if both search and wage-setting externalities are present, this mechanism is active. In this case, a Ramsey planner has incentives to vary the inflation rate in order to increase welfare through its effect on equilibrium wages through  $\delta$ , in turn affecting job creation and unemployment.

In relation to earlier literature it is first worth noting that this mechanism is not at work in Thomas (2008) due to the fact that the calibration implies that the Hosios condition holds and hence that search and bargaining externalities balance each other out. Moreover, the model of Erceg et al. (2000) does not feature this mechanism either. This is due to the fact that there is no extensive margin on the labor market in that model and hence no room for search frictions. Thus, the Ramsey planner has no leverage on job creation through the channel outlined above. However, and similar to our model, the Erceg et al. (2000) model features a markup in wage-setting where the actual markup can be different from the flexible price markup because of Calvo (1983)-style wage stickiness. Thus, in both models the planner has incentives to tilt the real-wage profile in order to lower the actual markup and increase labor input. Note though, since the model lacks a leverage on job creation there is much less of a motive for the planner to use this channel as shown by Amano et al. (2009).<sup>8</sup>

The model in Kim and Ruge-Murcia (2011) is similar to Amano et al. (2009), but differs in that it instead relies on wage-setting frictions along the lines of Rotemberg (1982). This, however, does not seem to be important for the Ramsey optimal inflation rate.

### 3. A model for quantitative evaluation

The next step in our analysis attempts to realistically evaluate the quantitative importance of the mechanism outlined above by embedding it in the canonical monetary model. The basic framework for the quantitative evaluation shares many elements of standard models. There is a monopolistically competitive intermediate goods sector where producers set prices facing a stochastic fixed adjustment cost as in Dotsey et al. (1999), thus introducing a markup distortion to the model. The intermediate goods sector buys a homogeneous input from the wholesale sector, which, in turn, uses labor in the production of this input. The market for this homogeneous input is characterized by perfect competition.

In contrast to previous papers studying the Ramsey optimal steady-state inflation rate, our model features search and matching frictions and endogenous staggered wage bargaining. Specifically, the wholesale sector posts vacancies on a search and matching labor market similar to Christoffel et al. (2009) and Trigari (2009). Wages are bargained between a representative family and wholesale firms in a setting with stochastic impediments to rebargaining, akin to how price setting is modeled. The representative family construct, composed of many workers as in Merz (1995), is introduced to ensure complete consumption insurance. The representative family then supplies labor, bargains wages and assures equal consumption across workers within the family. In addition, unemployed workers receive unemployment benefits paid by the government that are financed via lump-sum taxes. The unemployment benefits introduce another distortion in the model. Finally, notation is simplified by assuming a flexible-price retail sector that repacks the intermediate goods in accordance with consumer preferences and sells them to the representative family on a competitive market. We also add a monetary friction along the lines of Dotsey et al. (1999).

In terms of the model in Section 2, the markup distortion reduces the equilibrium price for the firm and the unemployment benefits increases the wage, both reducing the surplus of a match. To implement the planner solution, the bargaining power needs to be lower than  $\eta(\theta)$  (when  $H^a = H^0$ ) or lower than in (8) (when  $H^a = H^1$ ), in order to reduce the wage so that firm job creation implements the planner solution.<sup>9</sup>

#### 3.1. Intermediate-goods firms

The intermediate-goods firm chooses whether to adjust prices or not. Let the probability of adjusting prices in a given period be denoted by  $\alpha_t^j$ , given that the firm last adjusted its price  $j$  periods ago. For technical reasons, we assume that there is some  $J > 1$  such that  $\alpha^{J-1} = 1$ . Note that we follow standard notation and label the  $J$  cohorts from 0 to  $J-1$ .

<sup>7</sup> As long as there is not too much deflation. For the case of too much deflation, the entry wage  $\tilde{w}_e^1(\delta, \varphi)$  is larger than the gross surplus, for job finding rates close to one. Then the limit equilibrium job finding probability is instead the value of  $s$  where  $J_e^1 = 0$ , leading to an inefficient outcome.

<sup>8</sup> In Table 1 of Amano et al. (2009), the optimal inflation rate (without productivity growth) is 0.03%. Since only taking into account markup variations across households would imply an optimal inflation rate of zero, the effect of using inflation to affect the average markups in the economy is tiny.

<sup>9</sup> In the Online Appendix, details can be found on how the Hosios condition in the simple example above is affected by this modification.

### 3.1.1. Prices

Given that an intermediate-goods firm last reset prices in period  $t - j$ , the maximum duration of the price contract is then  $J - j$ , where  $J$  is the maximum price contract duration and  $\alpha_t^j$  is the adjustment probability  $j$  periods after the price was last reset. We assume that  $\alpha_t^{J-1} = 1$  for some  $J > 1$ . The intermediate-goods firms buy a homogeneous input from the wholesale firms at the (real) price  $p_t^w$ . As in Khan et al. (2003), an intermediate producer chooses the optimal price  $P_t^0$  so that

$$v_t^0 = \max_{P_t^0} \left[ \frac{P_t^0}{P_t} - p_t^w \right] Y_t^0 + E_t \Lambda_{t,t+1} \beta \left( \alpha_{t+1}^1 v_{t+1}^0 + (1 - \alpha_{t+1}^1) v_{t+1}^1 \left( \frac{P_t^0}{P_{t+1}} \right) - p_{t+1}^w \Xi_{t+1}^1 \right) \quad (10)$$

where  $Y_t^j = \left( \frac{P_t^j}{P_t} \right)^{-\sigma} Y_t$  and where  $P_t$  is the aggregate intermediate goods price level and  $\beta$  the discount factor. Moreover,  $\Lambda_{t,t+1}$  is the ratio of Lagrange multipliers in the problem of the consumer tomorrow and today. Finally,  $\Xi_{t+1}^1$  is the expected adjustment cost. Note that the term within the square brackets is just the firm's per unit profit in period  $t$ .

The values  $v_t^j$  evolve according to

$$v_t^j \left( \frac{P_t^j}{P_t} \right) = \left[ \frac{P_t^j}{P_t} - p_t^w \right] Y_t^j + E_t \Lambda_{t,t+1} \beta \left( \alpha_{t+1}^{j+1} v_{t+1}^0 + (1 - \alpha_{t+1}^{j+1}) v_{t+1}^{j+1} \left( \frac{P_t^j}{P_{t+1}} \right) - p_{t+1}^w \Xi_{t+1}^{j+1} \right) \quad (11)$$

We model price adjustment probabilities as in Dotsey et al. (1999) and others. Thus, adjustment probabilities are chosen endogenously by the firm and are one if  $c_{p,t}^j < \frac{v_t^0 - v_t^j}{p_t^w}$  and zero if  $c_{p,t}^j > \frac{v_t^0 - v_t^j}{p_t^w}$ . Adjustment costs are drawn from a cumulative distribution function  $G_P$  with upper bound  $\Omega_P$ . The maximal cost  $c_{p,t}^{j,\max}$  for a cohort  $j$  at time  $t$  that induces price changes is then  $c_{p,t}^{j,\max} = \frac{v_t^0 - v_t^j}{p_t^w}$  and we can thus express the expected adjustment costs as  $\Xi_t^j = \int_0^{c_{p,t}^{j,\max}} c_p dG_P(c_p)$ . The share of firms among those that last adjusted the price  $j$  periods ago that adjusts the price today is then given by  $\alpha_t^j = G_P(c_{p,t}^{j,\max})$ .

The share of firms with duration  $j$  since the last price change is denoted by  $\omega_t^j$ . For  $j \geq 1$  the shares evolve as  $\omega_t^j = (1 - \alpha_t^j) \omega_{t-1}^j$  and the share of firms with newly set prices ( $\omega_t^0$ ) in period  $t$  will be  $\omega_t^0 = \sum_{j=1}^J \alpha_t^j \omega_{t-1}^{j-1}$ .

### 3.2. Retailers

The retail firm buys intermediate goods and repackages them as final goods. We follow Erceg et al. (2000) and Khan et al. (2003) and assume a competitive retail sector selling a composite good. The composite good is combined from intermediate goods in the same proportions as families would choose. Given intermediate goods output  $Y_t^j$ , produced by intermediate-goods firms in each cohort  $j$ , the amount of the composite good  $Y_t$  is

$$Y_t = \left[ \sum_{j=0}^{J-1} \omega_t^j \left( Y_t^j \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (12)$$

where  $\sigma > 1$  and  $\omega_t^j$  is the share of retail firms producing  $Y_t^j$  at price  $P_t^j$ .

As in Khan et al. (2003), the retailers need to borrow to finance current production and choose  $\{Y_t^j\}_{j=0}^J$  to minimize costs for a given amount  $Y_t$  of final goods created. Thus, retailers solve

$$\min_{\{Y_t^j\}_{j=0}^J} (1 + R_t) \sum_{j=0}^{J-1} \omega_t^j P_t^j Y_t^j, \quad (13)$$

where  $(1 + R_t)$  is the gross nominal interest rate, subject to (12). The price level of the retailers is then  $\bar{P}_t = (1 + R_t) P_t$  where  $P_t$  is the intermediate goods price level and hence  $\bar{p}_t = \bar{P}_t / P_t = (1 + R_t)$ .

### 3.3. Families

To introduce a demand for money in the model, we follow Khan et al. (2003) and assume that agents use either credit or money to purchase consumption goods. Specifically, families purchase a fraction  $\xi_t$  of consumption with credit goods. Using credit requires paying a stochastic fixed time cost, drawn from a cumulative distribution  $G_c$ , with upper bound  $\Omega_c$ , and hence  $\xi_t = \int_0^{\bar{c}} dG_c(x)$ , where  $\bar{c}$  is the maximal credit cost paid by the family for a consumption good (for a detailed discussion see Khan et al., 2003). The amount of labor used in obtaining credit is denoted  $h_t^c$ . The total time cost of credit for the family is then  $h_t^c = \int_0^{\bar{c}} x dG_c(x)$ .

Families have preferences

$$E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ u(c_t) + \sum_{j^w=0}^{J_w-1} n_t^{j^w} z \frac{(1 - \bar{h} - h_t^c)^{1-\phi}}{1-\phi} + (1 - n_t) z \frac{(1 - h_t^c)^{1-\phi}}{1-\phi} \right], \quad (14)$$

where  $\bar{h}$  denotes the workers' hours worked at a wholesale firm,  $c_t$  consumption,  $n_t^{j^w}$  the number of employees in wage cohort  $j_w$  and  $n_t$  aggregate employment. Families hold an aliquot share of all firms. The budget constraint of the family is

given by

$$m_t + \frac{1}{1+R_t} b_{t+1} \geq \frac{b_t - d_t}{\pi_t} - \tau_t + \sum_{j_w=0}^{J_w-1} n_t^{j_w} \frac{W_t^{j_w}}{P_t} \bar{h} + (1-n_t) b_r, \quad (15)$$

where  $m_t$  is real money balances,  $b_{t+1}$  real bond holdings,  $d_t$  real debt,  $\tau_t$  real lump-sum transfers,  $W_t^{j_w}/P_t$  the workers' real wage in wage cohort  $j_w$ ,  $b_r$  the unemployment benefits and  $\pi_t = \frac{P_t}{P_{t-1}}$  is the gross inflation rate between period  $t-1$  and  $t$ . Since agents purchase a fraction  $1-\xi_t$  of consumption goods with money, the demand for money is  $m_t = (1-\xi_t)\bar{p}_t c_t$ . Then the real credit debt to be paid in period  $t+1$  is  $d_{t+1} = \xi_t \bar{p}_t c_t$ . The credit cost is realized after the family has decided on the amount of a product to buy but before choosing between credit or money as the mean of payment. Here, credit is defined as a one-period interest rate-free loan that needs to be repaid in full the next period. Families then choose to use credit as long as the gain,  $R_t c_t$ , is larger than the cost of credit.<sup>10</sup>

### 3.4. Search and matching, the hiring decision and employment flows

The model outlined here extends analysis, relative to the previous literature, to allow for the combination of search and matching frictions and wage staggering where workers may enter into an existing wage structure. To introduce search and matching frictions, we model the matching function as in Eq. (1) and the job finding and vacancy filling probabilities are as defined in Section 2.

As in Christoffel et al. (2009), where firms are modeled as having one employee, new matches may enter into an existing wage structure. Vacancies are determined as usual by the equation of the vacancy cost of an employee and the expected value of the worker to the firm. Thus, hiring is determined by

$$c = q_t \beta E_t \left[ (1-s^{new}) \sum_{j_w=0}^{J_w-1} \varpi_t^{j_w} J_{t+1}^{j_w} (w_{t+1}^{j_w}) + s^{new} J_{t+1} (w_{t+1}^0) \right], \quad (16)$$

where  $c$  is the cost of posting a vacancy,  $s^{new}$  the share of new hires that receive a rebargained wage,  $\varpi_t^{j_w}$  the share of employed workers in cohort  $j_w$  and  $J_t^{j_w} (w_t^{j_w})$  the value of the firm in cohort  $j_w$ , described in detail below. Note that this formulation builds on the assumption that filled vacancies become productive and receive a wage in the next period. Thus, with probability  $(1-s^{new})\varpi_t^{j_w}$  a firm is randomly assigned to cohort  $j_w > 0$  and with probability  $(1-s^{new})\varpi_t^0 + s^{new}$  to cohort 0.

The employment flow between categories  $n_t^{j_w}$  is given by

$$n_t^0 = \sum_{j_w=1}^{J_w-1} \rho \alpha_t^{j_w} n_{t-1}^{j_w-1} + (s^{new} + (1-s^{new})\varpi_t^0) \mu_t, \quad (17)$$

and, for  $j > 0$ ,

$$n_t^j = \rho (1-\alpha_t^j) n_{t-1}^{j-1} + (1-s^{new}) \varpi_t^j \mu_t, \quad (18)$$

where  $\alpha_t^j$  is the wage adjustment probability  $\alpha_t^j$  in the  $j$ th period following the last rebargain. We assume that  $\alpha_t^{j_w-1} = 1$  for some  $J_w > 1$ . Also,  $\varpi_t^{j_w}$  is the share of workers in the  $j_w$ th cohort.

Aggregate employment is  $n_t = \sum_{j_w=0}^{J_w-1} n_t^{j_w}$  and the number of unemployed workers is  $u_t = 1 - n_t$ .

### 3.5. Value functions

The expected net surplus for the family to have a worker employed in a wholesale firm that last rebargained wages  $j_w$  periods ago is

$$H_t^{j_w} (w_t^{j_w}) = w_t^{j_w} \bar{h} - b_r - z \frac{(1-\bar{h}-h_t^c)^{1-\phi}}{(1-\phi)\lambda_t} + z \frac{(1-h_t^c)^{1-\phi}}{(1-\phi)\lambda_t} + \beta E_t \lambda_{t,t+1} \left[ \rho \alpha_{t+1}^{j_w+1} H_{t+1}^0 (w_{t+1}^0) + \rho (1-\alpha_{t+1}^{j_w+1}) H_{t+1}^{j_w+1} (w_{t+1}^{j_w+1}) - s_t H_{t+1}^a \right], \quad (19)$$

where

$$H_t^a = s^{new} H_t^0 (w_t^0) + (1-s^{new}) \sum_{j_w=0}^{J_w-1} \varpi_t^{j_w} H_t^{j_w} (w_t^{j_w}). \quad (20)$$

is the net value of getting a job in an average wholesale firm. Thus, as in the stylized model above in Section 2, whether newly hired workers get new rebargained wages or enter into a given wage structure of the firm affects the value of  $H_t^a$  and hence the family's outside option.

<sup>10</sup> That is, the real discounted net gain of placing the transaction amount in a bond for a period and repay the transaction amount the next period. To see this, combine the household first-order condition with respect to  $\xi_t$  together with the household Euler equation.

The wholesale firm in cohort  $j_w$  uses labor as input to produce output, using a constant returns technology. The value is then

$$J_t^{j_w}(w_t^{j_w}) = p_t^w Z \bar{h} - w_t^{j_w} \bar{h} + \beta E_t \Lambda_{t,t+1} \alpha_{t+1}^{j_w+1} (\rho J_{t+1}^0(w_{t+1}^0)) + \beta E_t \Lambda_{t,t+1} (1 - \alpha_{t+1}^{j_w+1}) \rho J_{t+1}^{j_w+1}(w_{t+1}^{j_w+1}) \quad (21)$$

with  $Z$  being a level shifter of productivity.

### 3.6. Wage bargaining

To incorporate staggered state-dependent wage bargaining into the canonical monetary model, we model wage determination in the spirit of [Haller and Holden \(1990\)](#) and [Holden \(1994\)](#). However, in order to end up in a wage-setting formulation that is comparable to standard search and matching models we slightly modify their set-up. That is, instead of having conflicts as in [Haller and Holden \(1990\)](#) we have a probability of breakdown.<sup>11</sup> The nominal wage  $W_t^0$ , when wages are rebargained (i.e., changed), is chosen such that it maximizes the Nash product, noting that  $w_t^0 = \frac{W_t^0}{P_t}$  and that  $\varphi$  denotes the bargaining power of the family. Otherwise the work continues according to the old contract as in [Holden \(1994\)](#).

#### 3.6.1. Wage adjustment probabilities

In the bargaining game, to get a new rebargained wage, one of the parties must find it credible to threaten with disagreement, which is costly.<sup>12</sup> The disagreement costs, drawn at the start of time period  $t$ , for the firm, denoted  $c_j$ , follows the cumulative distribution function  $G_j$  and the cost  $c_H$  of the family follows the cumulative distribution function  $G_H$  with upper bounds  $\Omega_j$  and  $\Omega_H$ , respectively. The difference in the firm's value between adjusting the wage or not is  $dJ_t^{j_w}(w_t^{j_w}) = J_t^0(w_t^0) - J_t^{j_w}(w_t^{j_w})$  and similarly for the family  $dH_t^{j_w}(w_t^{j_w}) = H_t^0(w_t^0) - H_t^{j_w}(w_t^{j_w})$ . The firms have incentives to call for rebargaining whenever  $c_j < dJ_t^{j_w}(w_t^{j_w})$  and the worker when  $c_H < dH_t^{j_w}(w_t^{j_w})$ . Adjustment probabilities  $\alpha_t^{j_w}$  can then be computed by using  $dJ_t^{j_w}(w_t^{j_w})$  and  $dH_t^{j_w}(w_t^{j_w})$  and the disagreement cost distributions  $G_j(dJ_t^{j_w}(w_t^{j_w}))$  and  $G_H(dH_t^{j_w}(w_t^{j_w}))$ . See the Online Appendix for a detailed description of how these objects are computed.

### 3.7. The aggregate resource constraint and government budget constraint

The aggregate resource constraint can be written as

$$\sum_{j=0}^{J-1} \omega_t^j \left( \frac{p_t^j}{P_t} \right)^{-\sigma} (c_t + c_{\nu t}) = \sum_{j_w=0}^{J_w-1} n_t^{j_w} Z \bar{h} - \sum_{j=0}^{J-1} \omega_t^j \Xi_t^j. \quad (22)$$

The government uses lump-sum taxes to finance unemployment benefits. Thus,  $\tau_t = (1 - n_t)b_t$ .

### 3.8. Optimal policy

As discussed above, the policy maker needs to take several distortions into account when designing optimal policy. First, there is imperfect competition in the product market. There is also a distortion due to money demand and the cost of using credit. Furthermore, there are relative price and wage distortions. Finally, there are distortions in the hiring decision on the labor market. Here, we focus on the Ramsey policy as discussed by [Schmitt-Grohe and Uribe \(2004\)](#), maximizing welfare, subject to the constraints given by optimizing agents in the economy, i.e., for example first-order and market-clearing conditions.

The policymaker then maximizes (14) subject to the constraints (1), the job finding and filling probabilities, (10), (11), price setting adjustment probabilities, the first-order conditions to problem (10), the firm cohort shares, the intermediate goods price level, the flow equation of prices  $p_t^j = p_{t-1}^{j-1} / \pi_t$ , the household first-order conditions to maximizing (14), expressions (16)–(18), the relationship between cohort employment and aggregate employment, the definition of unemployment, (19)–(21), the first-order condition to the Nash product, the wage derivative of firm and worker value functions in (19)–(21), wage setting adjustment probabilities as described in the Online Appendix, the flow equation of wages  $w_t^j = w_{t-1}^{j-1} / \pi_t$ , and the aggregate resource constraint (22).

<sup>11</sup> Specifically, the conflict subgame in Fig. 1 in [Haller and Holden \(1990\)](#) is replaced by a subgame where there is a positive probability of breakdown.

<sup>12</sup> Note that threats of conflict in wage bargaining will not be exercised along the equilibrium path, but is a credible threat to enforce a new wage offer. Hence, these costs are not paid in equilibrium, in contrast to costs associated with price setting.



**Table 1**  
Baseline calibration of the model.

Parameters		
$\beta$	Time preference	0.9928
$\sigma$	Product market substitutability	10
$\phi$	Disutility of work curvature parameter	1
$\rho$	Match-retention rate	0.9
$\varphi$	Family bargaining power	0.5
$\sigma_a$	Matching function parameter	1.27
$Z$	Productivity shifter	5
$1 - s^{new}$	Share of workers entering into existing wage structure	0.5
$\bar{h}$	Hours worked	0.2
$z$	Disutility of work parameter	2.4035
$c$	Vacancy cost	0.0486
$b_r$	Income when unemployed	0.3529
$a_l^p$	Beta left parameter (prices)	2.1
$a_r^p$	Beta right parameter (prices)	1
$\Omega_p$	The largest fixed cost (prices)	0.0024
$a_l^w = a_l^h$	Beta left parameter (wages)	2.1
$a_r^w = a_r^h$	Beta right parameter (wages)	1
$\Omega_j = \Omega_H$	The largest fixed cost (wages)	0.0396
$a_l^c$	Beta left parameter (credit)	2.806
$a_r^c$	Beta right parameter (credit)	10.446
$\Omega_C$	The largest fixed cost (credit)	0.0342
$\hat{\xi}$	Mass of goods with positive credit cost	0.361

#### 4. Quantitative evaluation

We now turn to the calibration of the model and the quantitative evaluation of the mechanism.

##### 4.1. Calibration

For our quantitative evaluation, we assume log preferences in consumption and leisure. The baseline calibration of the structural parameters is chosen to represent the U.S. economy on a quarterly basis and is presented in Table 1.

We set  $\beta$  to 0.9928 as in Khan et al. (2003). This generates a real interest rate of slightly below 3 percent and is motivated by data on one-year T-bill rates and the GDP deflator. Note that this is a key parameter for governing the strength of the monetary distortion. For  $\sigma$  we use a baseline value of 10, generating a markup of around 11 percent. Our calibration of  $\sigma$  is based on work of e.g. Basu and Fernald (1995, 1997), Basu and Kimball (1997), and Basu (1996), and is the same value chosen by for example Chari et al. (2000). However, markup estimates from the industrial organization literature is typically larger (see e.g. Berry et al., 1995). We will return to the markup in robustness exercises. We set the bargaining power  $\varphi=0.5$ , implying symmetrical bargaining in the baseline calibration. For the job separation rate  $1-\rho$ , we follow Hall (2005) and set  $\rho=0.9$ . The value of  $\sigma_a$  is set to 1.27 following Den Haan et al. (2000). We set hours worked to 0.2 and  $Z$  to 5 in order to normalize output per employee to unity.

To calibrate the share of new hires that get rebargained wages, there are several sources of evidence. Micro-data studies, summarized in Pissarides (2009), seem to indicate that newly hired workers' wages are substantially more flexible than incumbents' wages speaking against the idea that a large share of entrants enter into an existing wage structure. However, the studies summarized in Pissarides (2009) generally fail to control for effects stemming from variations in the composition of firms and match quality over the cycle. Thus, it might be that the empirical evidence just reflects that workers move from low-wage firms (low-quality matches) to high-wage firms (high-quality matches) in boom periods and vice versa in recessions. The approach taken to address this issue is to introduce job-specific fixed effects in a regression of individual wages on the unemployment rate and the interaction of the unemployment rate and dummy variable indicating if the tenure of the worker is short, see Gertler and Trigari (2009). This dummy structure controls for composition effects in workers, firms and match quality. Importantly, the results reported by Gertler and Trigari (2009) no longer indicate that wages are more sensitive to labor market conditions at the beginning than later in the span of a match, contrasting Pissarides (2009). This finding is thus in line with a low calibration of  $s^{new}$ .<sup>13</sup> Moreover, if we turn to survey evidence, like Bewley (1999, 2007) for the U.S. and the study performed within the Eurosystem Wage Dynamics Network (WDN) covering about 15,000 firms in 15 European countries, we see strong evidence that the wages of new hires are tightly linked to those of incumbents. As reported by Galuscak et al. (2012), about 80% percent of the firms in the WDN survey respond that

<sup>13</sup> As discussed in Gertler and Trigari (2009), additional findings on employment effects of wage contracting presented in Card (1990) and Olivei and Tenreyro (2007, 2010) provide further evidence in line with a low calibration of  $s^{new}$ .

**Table 2**  
Yearly optimal inflation rate under the Ramsey policy.

Model variations	$\pi$
No price or wage rigidities	–2.85
State dependent prices only	–0.76
State dependent prices and wages (baseline)	1.16
No monetary frictions (cashless)	1.51
Exogenous adjustment probabilities	3.02

internal factors (like the internal pay structure) are more important in driving wages of new hires than market conditions. Taken together the results points towards a non-negligible share of new hires that enters into an existing wage structure. However, lacking any sharp evidence on the exact value of this parameter we set  $s^{new}$  to 0.5 in the baseline calibration and vary the parameter between 0 and 1 in the robustness exercises.

For credit costs, a fraction  $1 - \hat{\xi}$  of the goods costs zero. Then

$$G_c(v) = (1 - \hat{\xi}) + \hat{\xi} G_C(v; a_i^j, a_r^i, \Omega_i). \quad (23)$$

The cost cumulative distribution functions  $G_p$ ,  $G_H$ ,  $G_J$  and  $G_C$  are beta distributed;

$$g_i(v; a_i^j, a_r^i, \Omega_i) = \frac{1}{\Omega_i} g^{beta}\left(\frac{v}{\Omega_i}; a_i^j, a_r^i\right) \quad (24)$$

for  $i \in \{P, H, J, C\}$ . Except for  $\Omega_p$  and  $\Omega_C$ , the parameters for  $G_p$  and  $G_C$  are calibrated following Lie (2010) closely.<sup>14</sup> For the parameters for the disagreement cost distributions  $G_H$  and  $G_J$  we set  $a_i^H = a_i^J = 2.1$  and  $a_r^H = a_r^J = 1$  (similar to the values for  $a_i^P$  and  $a_r^P$  taken from Lie, 2010). We also set  $\Omega_J = \Omega_H$  and then choose the parameters  $z$ ,  $c$ ,  $b_r$ ,  $\Omega_H$ ,  $\Omega_p$  and  $\Omega_C$  so that the sticky price and wage model under two percent inflation has vacancy costs of one percent of output, a replacement rate (rr) of 40 percent as in Hall (2005), a matching function elasticity ( $\eta$ ) of 0.6, which is the midpoint of the interval 0.5–0.7 as suggested by Petrongolo and Pissarides (2001), a mean duration of wage contracts of a year as in Taylor (1993), a mean duration of prices of a year in line with Nakamura and Steinsson (2008) and that the flexible wage model has an optimal policy steady-state deflation rate of 0.76, as in Khan et al. (2003). The results are presented in Table 1. We set  $J=5$  and  $J_w=8$  in order to avoid price/wage setting cohorts without mass.

To solve for the efficient allocation we maximize family welfare, as described in (14), given that the Friedman rule holds, subject to the matching function (1), the flow equation of employment  $n_t = \rho n_{t-1} + \mu_t$  and the aggregate resource constraint

$$c_t + cv_t = n_t Z \bar{h}. \quad (25)$$

## 4.2. Quantitative results

To solve for the Ramsey optimal steady-state inflation rate, we follow Schmitt-Grohé and Uribe (2012b).<sup>15</sup> In Table 2 the Ramsey optimal steady-state inflation rates implied by our model is presented. In the absence of price or wage rigidities, the Ramsey optimal inflation rate is –2.85 percent per year, in line with previous literature. In other words, with no frictions to price or wage setting, the model replicates the finding of Friedman (1969) that deflation is optimal when there is a role for money as a medium of exchange.

When introducing price rigidities, the Ramsey optimal inflation rate increases, but remains below zero, as previously pointed out by Khan et al. (2003) and Schmitt-Grohé and Uribe (2010). In the baseline model, also introducing impediments to continuous wage rebargaining, the Ramsey optimal inflation rate is 1.16 percent.<sup>16</sup> Thus, the Ramsey optimal inflation rate increases by almost two percentage points in a calibration that implies that employment is 1.87 percentage points lower than the efficient allocation.

<sup>14</sup> The paper by Lie (2010) analyzes the Ramsey policy in a model akin to the model in this paper but with a competitive labor market. The reason for our modification of  $\Omega_C$  is that other variables enter the household money-demand first-order condition in a slightly different way as compared to Khan et al. (2003) and Lie (2010), thus motivating a change so that optimal inflation under flexible wages is in line with their model. Also,  $\Omega_p$  is modified because intermediate goods producer costs are slightly different in this model.

<sup>15</sup> The first-order condition in the Ramsey optimal policy consists of the derivatives of the objective with respect to the control and state variables and derivatives with respect to the Lagrange multipliers for the constraints. Note that the derivative with respect to the Lagrange multipliers is just the equation system defining the competitive equilibrium for a given inflation rate. Following along the lines of Schmitt-Grohé and Uribe (2012b), we posit an inflation rate  $\pi^0$  and then solve for the competitive equilibrium given the inflation rate. Then a candidate for the Lagrange multiplier vector is computed using the remaining first-order conditions (i.e. with respect to the control and state variables) as if these hold, relying on a least-squares method. The candidate Lagrange multiplier is then used to compute the squared residuals from these first-order conditions. A standard iterative minimization routine can then be used to find the value of inflation that leads to the sum of squared residuals of these first-order conditions being zero.

<sup>16</sup> Experimenting with introducing capital accumulation, as in Gertler and Trigari (2009), yields a very similar Ramsey optimal inflation rate of 0.88 percent.

**Table 3**  
Yearly optimal inflation rate under the Ramsey policy.

Baseline	$rr=0.2$	$\sigma=20$	$s^{new}=1$	$\varphi=0.4$	$\eta=0.7$
	0.93	0.99	-0.76	1.23	0.94
	$rr=0.4$	$\sigma=10$	$s^{new}=0.5$	$\varphi=0.5$	$\eta=0.6$
	1.16	1.16	1.16	1.16	1.16
	$rr=0.5$	$\sigma=6$	$s^{new}=0$	$\varphi=0.6$	$\eta=0.5$
	1.23	1.18	1.44	1.13	1.38

$rr$  denotes the replacement rate,  $\sigma$  the demand elasticity,  $s^{new}$  the share of new hires bargaining over wages,  $\varphi$  worker bargaining power and  $\eta$  the matching elasticity.

Removing the monetary friction by eliminating credit costs and looking at the cashless economy as often done in the monetary policy literature, see [Woodford \(2003\)](#), increase inflation to 1.51 percent. Thus, the monetary distortion has a moderately negative effect on the optimal policy.

Furthermore, the importance of endogenous price and wage adjustment probabilities is analyzed by fixing the price and wage adjustment probabilities to the values when steady-state inflation is two percent. Then we solve for the Ramsey policy under these exogenous adjustment probabilities. The optimal inflation rate increases by slightly less than two percentage points in this case, as compared to the case with endogenous adjustment probabilities. Thus, the ability of agents to self-select into adjustment has strong effects on the Ramsey planner's choice. This result contrasts with [Lie \(2010\)](#), who finds that endogenizing adjustment probabilities is not important in a model with flexible wages.

To explore how the quantitative results change in response to substantial changes in key parameters or targets, we vary the replacement rate, the markup, the bargaining power, the match elasticity, the share of new workers getting new rebargained contracts.<sup>17</sup> The results from this exercise can be seen in [Table 3](#). When the replacement rate is increased (decreased), the optimal inflation rate increases (decreases) by 0.07 (0.23) percentage points. The intuition is that an increase in the replacement rate makes the economy less efficient due to an increasing wage, thus increasing the net gain for the Ramsey planner to use inflation to move the economy towards the efficient allocation, as can also be seen in the Hosios condition (8). Note also that, even if the replacement rate is zero, optimal inflation is 0.16 and thus significantly larger than the flexible wage optimal inflation rate.

An increase in the markup of the intermediate goods producers to 20% from the baseline value of 11%, by decreasing  $\sigma$  to 6, increases the optimal inflation rate somewhat to 1.18, while a decrease in the markup to around 5%, by setting  $\sigma$  to 20, pushes down the optimal inflation rate to 0.99. The reason is that an increase (decrease) in the markup pushes the economy further away (closer to) from the efficient allocation, inducing the planner to use inflation more (less) aggressively.

Varying the share of new hires receiving rebargained wages has big effects. If all new hires get rebargained wages ( $s^{new}=1$ ), the optimal inflation rate is, as expected, the same as when wages are flexible, while when all workers enter into an existing wage structure as in [Gertler and Trigari \(2009\)](#) ( $s^{new}=0$ ), the optimal inflation rate is 1.44 percent.<sup>18</sup>

Varying the bargaining power has moderate effects on the results; increasing the worker bargaining power to 0.6 decreases optimal inflation to 1.13 percent, while reducing it to 0.4 leads to inflation of about 1.23 percent. The reason for this perhaps surprising result is that the change in bargaining power also leads to a change in  $z$ , as can be seen in [Table 1](#) in the Online Appendix. The decrease in bargaining power tends as expected to reduce the real wage, pushing the economy closer to the efficient allocation. However, in order to match e.g., the replacement rate,  $z$  increases. The increase in  $z$  makes wage changes have a larger effect on the Hosios condition, as can be seen in [Eq. \(8\)](#), giving the planner stronger incentives to use inflation to affect the wage. This second effect is larger than the direct effect from the reduction in bargaining power in our experiment.

Finally, varying the matching elasticity  $\eta$ , also affecting the degree of efficiency in the economy via the Hosios condition (8), has a moderate effect on the optimal steady-state inflation rate. When the matching elasticity is 0.7, i.e., the upper bound suggested in [Petrongolo and Pissarides \(2001\)](#), the optimal inflation rate is 0.94 while it increases to 1.38 when the matching elasticity is at the lower bound of 0.5.

Note that we assume that firms have one employee, in order to describe the relationship between search and wage-setting externalities in a simple way. Relaxing this assumption, thus allowing for large firms, may or may not affect the outcome, depending on how the wage setting of large firms is modelled. If the large firms have different types of employees that rebargain at different points in time, then the results may be unchanged. Specifically, if the firms have the same share of rebargaining workers in each time period, then all firms are identical with respect to wages for new hires and in turn to vacancy posting, preserving the representative-firm assumption in job creation in the one-worker firm model. Alternatively, if all workers in an existing firm rebargain at the same time, then, since different firms rebargain at different times, wages for new jobs will vary across firms, creating an additional incentive to push inflation towards zero in order to avoid wage dispersion and, in turn, vacancy dispersion across firms.

<sup>17</sup> Thus, we refit the parameters  $z$ ,  $c$ ,  $b_r$ ,  $\Omega_H$ ,  $\Omega_P$  and  $\Omega_C$ . See the Online Appendix for details and the resulting calibrations.

<sup>18</sup> Note that the share of workers in the first cohort is always larger than  $s^{new}$ , except when  $s^{new}=1$ , since some of the workers entering an existing wage structure will enter into the first cohort. In the baseline calibration, the share of new hires entering into the first cohort and hence getting rebargained wages is 0.61 while it is 0.22 when  $s^{new}=0$ .

## 5. Concluding discussion

This paper explores how the interaction of nominal wage and labor market search and matching frictions can affect the planner's trade-off when choosing the Ramsey optimal inflation rate. In a stylized model with search frictions where some newly hired workers enter into an existing wage structure we show that inflation not only affects real-wage profiles over a contract spell, but also redistributes surplus between workers and firms since incumbent workers impose an externality on new hires through the entry wage. This affects the wage-bargaining outcome through its effect on the workers' outside option and hence the expected present value of total labor costs for a match and thus also firms' incentives for vacancy creation and, in turn, employment. Moreover, models without an extensive margin on the labor market lack the mechanism described here (as e.g. in Erceg et al., 2000).

Overall, the key insight from the model is that if both search and wage-setting externalities are present, there is an incentive for the Ramsey planner to vary the inflation rate to increase welfare through its effect on job creation and unemployment.

In the baseline quantitative evaluation, featuring many of the aspects that have been deemed important in determining the optimal inflation rate, we find that the Ramsey optimal inflation rate is 1.16 percent per year. When comparing with a model with flexible wages for new hires, this is an increase of almost two percentage units, confirming that the mechanism is important in an elaborate general equilibrium context.

In conclusion, we show that the combination of nominal wage and search externalities can provide a mechanism that helps reconcile theories of monetary non-neutrality with observed inflation targets of central banks.

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## Appendix A. Supplementary data

Supplementary data associated with this paper can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2016.01.002>.

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