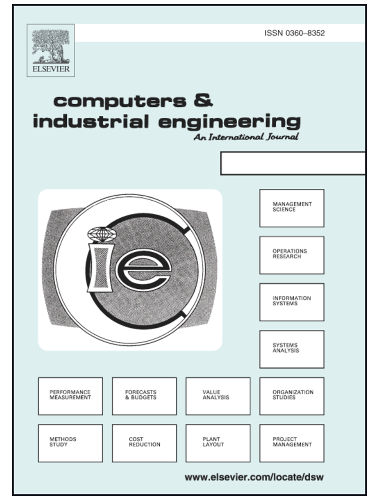

Supplier Selection under Failure Risk, Quantity and Business Volume Discounts

Christian Bohner, Stefan Minner

PII: S0360-8352(16)30460-0
DOI: http://dx.doi.org/10.1016/j.cie.2016.11.028
Reference: CAIE 4544

To appear in: *Computers & Industrial Engineering*

Received Date: 17 May 2016
Accepted Date: 24 November 2016

Supplier Selection under Failure Risk, Quantity and Business Volume Discounts

Christian Bohner, Stefan Minner
TUM School of Management, Technische Universität München,
Arcisstr. 21, 80333 Munich, Germany

Abstract

We consider a supply chain problem with simultaneous supplier selection and order allocation for multiple products. The suppliers offer quantity and business volume discounts, and they are subject to failure. The buyer aims at minimizing total expected costs. We consider both all-units and incremental quantity discounts and find optimal solutions through mixed-integer linear programming. We discuss the trade-off between economies of scale and failure risk and show the cost reduction of our exact approach compared to a previously proposed heuristic.

Key words: supplier selection; order allocation; quantity discount; business volume discount; failure risk; mixed-integer linear programming

1 Introduction

Natural disasters not only cause horrific human tragedies; they also shatter the economy and influence businesses all around the globe. Japan's 2011 "quadruple disaster" (The Economist, 2011) is one recent example of a supply chain disruption that highlights the importance to hedge supply chains against supplier outage. Multi-sourcing, i.e. having different suppliers for the same item, is one way of approaching this issue. However, this leads to a more complex supply chain (e.g. setting up order policies is more difficult), the contract negotiations with different suppliers may be rather time-consuming, or the costs per item may increase due to lower quantity discounts compared to single-sourcing. Balancing the advantages and disadvantages of multi-sourcing is a key issue when it comes to the design of the supply chain and a major challenge for production/inventory systems.

When sourcing an item, a buyer has to consider the risk of a supply chain disruption, which has a certain probability. Dealing with this risk, the buyer needs to decide upon the supplier selection and the allocation of orders between suppliers. Ruiz-Torres and

Supplier Selection under Failure Risk, Quantity and Business Volume Discounts

author names
blinded

Abstract

We consider a supply chain problem with simultaneous supplier selection and order allocation for multiple products. The suppliers offer quantity and business volume discounts, and they are subject to failure. The buyer aims at minimizing total expected costs. We consider both all-units and incremental quantity discounts and find optimal solutions through mixed-integer linear programming. We discuss the trade-off between economies of scale and failure risk and show the cost reduction of our exact approach compared to a previously proposed heuristic.

Key words: supplier selection; order allocation; quantity discount; business volume discount; failure risk; mixed-integer linear programming

---

1 Introduction

Natural disasters not only cause horrific human tragedies; they also shatter the economy and influence businesses all around the globe. Japan's 2011 "quadruple disaster" (The Economist, 2011) is one recent example of a supply chain disruption that highlights the importance to hedge supply chains against supplier outage. Multi-sourcing, i.e. having different suppliers for the same item, is one way of approaching this issue. However, this leads to a more complex supply chain (e.g. setting up order policies is more difficult), the contract negotiations with different suppliers may be rather time-consuming, or the costs per item may increase due to lower quantity discounts compared to single-sourcing. Balancing the advantages and disadvantages of multi-sourcing is a key issue when it comes to the design of the supply chain and a major challenge for production/inventory systems.

When sourcing an item, a buyer has to consider the risk of a supply chain disruption, which has a certain probability. Dealing with this risk, the buyer needs to decide upon the supplier selection and the allocation of orders between suppliers. Ruiz-Torres and

Mahmoodi (2006) studied a problem with multiple suppliers subject to failure risk and used an enumeration to decide about supplier selection and order allocation. Meena et al. (2011) considered disruptions due to catastrophic events and determined the optimal number of suppliers. Sawik (2014) compared single sourcing to dual sourcing strategies in make-to-order systems. Silbermayr and Minner (2016) considered multi-sourcing with different speeds and costs per supplier and examined the trade-off between economies of scale and supply disruptions. Tang (2006), Snyder et al. (2016) and Fahimnia et al. (2015) reviewed supply chains subject to disruptions and their risk management. Thomas and Tyworth (2006) presented a critical literature review on order splitting. Minner (2003) provided an overview of multi-sourcing problems.

Individual contracts with suppliers often allow for quantity discounts, i.e. in addition to considering the disruption risk of a supplier, there are incentives for the buyer to allocate larger orders to suppliers with larger quantity discounts. *Incremental* discounts only apply to items exceeding a certain order quantity while *all-units* discounts apply to all items. Burke et al. (2008) modeled incremental quantity discounts without supplier failure risk. Munson and Rosenblatt (1998) reviewed quantity discounts and Munson and Jackson (2015) provided a more recent, extensive overview on quantity discounts. Another regularly observed price reduction comes from business volume discounts, i.e. the supplier offers a discount on the total amount of sales generated by a buyer. Sadrian and Yoon (1994) and Xia and Wu (2007) considered supplier selection under business volume discounts. An overview on multi-criteria supplier selection was presented in Ho et al. (2010).

Our research is based on Meena and Sarmah (2013) who considered order allocation with supplier failure risk, all-units quantity discounts and deterministic demand among a pre-selected, fixed set of suppliers. The authors proposed a genetic algorithm for solving the optimization problem. However, formulating the problem as a mixed-integer linear program (MILP), we find optimal solutions to all their instances in negligible computation time. Their heuristic solutions deviate from our optimal solutions by up to 4%. We further extend the problem to include integrated supplier selection and multiple products. We model both incremental and all-units quantity discounts and include business volume discounts. In a numerical study, we show that our MILP-approach solves realistic problem sizes.

The paper is structured as follows: Section 2 provides an introduction to the problem and presents the optimization model. In Section 3, we present numerical results that consist of a) a discussion of the results of Meena and Sarmah (2013) and b) a numerical study of the extended problem under a full factorial design. In Section 4, we provide conclusions and ideas for further research.

2 Model

We consider a risk-neutral vendor of multiple products who faces deterministic demand and has to decide about the supplier selection and order allocation for a single period. There is a pool of suppliers where every supplier offers both quantity discounts per product and business volume discounts depending on the total sales volume. All suppliers may disrupt and have certain failure probabilities that may be correlated. Further, a super-event that hits all suppliers simultaneously can occur. Each supplier has a maximum capacity for each product and delivers products in fixed lot sizes. From this supplier pool, we want to find a selection of suppliers such that the total expected costs, consisting of procurement costs, supplier management costs, and shortage penalty costs, are minimized.

If a supplier is selected, fixed supplier management costs that are independent of the number of products ordered from this supplier, arise. For each product ordered at a supplier, a certain minimum share of the total demand has to be allocated. If a failure of one of the selected suppliers occurs, the remaining selected suppliers try to compensate for this loss by producing up to their respective capacity limits. Note that for each product only those suppliers who have been selected to deliver the product in the first place can compensate a loss.

The sequence of events is as follows: the buyer selects suppliers and allocates orders. Selected suppliers might fail; their quantities are reallocated to suppliers who can compensate for the loss. The available share of the orders is delivered, the rest is lost and incurs a penalty cost.

The quantity discounts offered by the suppliers have different levels; the start of one level is denoted as a price break quantity. For all-units discounts, the attained quantity

Table 1. Notation.

Sets and parameters	
$p \in P$	products
$i \in N$	suppliers
$j \in J$	quantity discount levels
$b \in B$	business volume discount levels
$\mathcal{P}(N)$	set of supplier selections | power set of N
$m \in \mathcal{B}(N)$	enumeration of selections | $\mathcal{B}(N) = \{1, ..., \|\mathcal{P}(N)\|\}$
T_{pm}	supplier selection m for product p | $T_{pm} \in \mathcal{P}(N)$
$\mathcal{P}(T_{pm})$	power set of T_{pm}
D_p	deterministic demand for product p
C_p	base price for product p
L_p	per unit penalty cost for not satisfying the demand for product p
F	management costs per supplier (identical for all suppliers $i \in N$)
Q_{pi}	lot size for product p at supplier i
$Q_{pi}^{\max}$	capacity for product p at supplier i
$Q_{pi}^{\min}$	minimum order quantity for product p at supplier i (if supplier i is selected for product p)
Q_{pij}	price break quantity for product p at supplier i at quantity discount level j
d_{pij}	relative quantity discount at price break quantity Q_{pij}
$\tilde{Q}_{ib}$	price break volume at supplier i for business volume discount level b
$\tilde{d}_{ib}$	relative business volume discount at price break volume $\tilde{Q}_{ib}$
d_{pijb}	relative combined quantity and volume discounts, $d_{pijb} = 1 - (1 - d_{pij}) \cdot (1 - \tilde{d}_{ib})$
p^*	probability of super-event that all suppliers fail
X_i	binary random variable | $X_i = 1$ if supplier i fails
a_i, c_m	auxiliary parameters (explained below)
M	large number
Decision variables	
q_{pijb}	quantity of product p at supplier i with quantity discount j and volume discount b
q_{pij}	quantity of pruduct p at supplier i with quantity discount j
q_{pi}	quantity of product p at supplier i
$\tilde{q}_{pib}$	quantity of product p at supplier i with volume discount b
k_{pi}	integer for determining multiple of lot size of product p at supplier i, $q_{pi} = k_{pi} \cdot Q_{pi}$
x_{pij}	indicator whether quantity $q_{pij} > 0$
x_{pi}	indicator whether supplier i is selected for product p, i.e. if $q_{pi} > 0$
x_i	indicator whether supplier i is selected, i.e. $\exists q_{pi} > 0$
$\tilde{x}_{ib}$	indicator whether volume discount b applies at supplier i
b_{pm}	indicator whether supplier selection T_{pm} is chosen
z_p	auxiliary decision variable (explained below)
$u_{pm}, \bar{u}_{pm}$	auxiliary binary decision variables (explained below)

discount is applied to all units ordered, whereas in the case of incremental discounts, the respective discount level only applies to those units exceeding the price break quantity.

A supplier offers a business volume discount if the total sales volume of a buyer exceeds a certain threshold. That is, the buyer's total costs reduce by a certain percentage. For business volume discounts we use the term price break volumes, analogously to price break quantities. Table 1 introduces the notation that is used to develop the model.

The total expected costs have three components: (i) purchasing cost, (ii) supplier management costs, and (iii) expected total penalty costs. First, we have purchasing costs

$$PC = \sum_{p \in P} C_p \cdot \sum_{i \in N} \sum_{j \in J} \sum_{b \in B} q_{pijb} \cdot (1 - d_{pijb}). \tag{1}$$

The total quantity of product p ordered at supplier i, q_{pi}, can be uniquely mapped to one attained quantity discount level j and one attained business volume discount level b. For this combination of quantity and business volume discounts, it holds $q_{pijb} = q_{pi}$, while for all other combinations we have $q_{pijb} = 0$, as will be ensured in the constraints. Therefore, first summing over all quantity and business volume discounts yields the desired summands for finally summing over products and suppliers. Multiplication with $(1 - d_{pijb})$ applies the combined quantity and volume discount per product and supplier.

We further have supplier management costs, given through

$$SMC = F \cdot \sum_{i \in N} x_i. \tag{2}$$

These are linear in the number of suppliers and apply if a supplier is selected for at least one product, regardless of the number of products and the respective quantities.

The last component of the total costs are the expected total penalty costs (ETP). For a fixed set of suppliers $S \subseteq N$, these are obtained by

$$ETP = \sum_{p \in P} L_p \cdot \left(p^* \cdot D_p + (1 - p^*) \cdot \sum_{A \in \mathcal{P}(S)} \mathbb{P}(A) \cdot \left[D_p - \min\left(D_p, \sum_{l \in S \setminus A} Q_{pl}^{\max}\right) \right] \right), \tag{3}$$

where

$$\mathbb{P}(A) = \mathbb{P}(X_j = 1, \forall j \in A, X_l = 0, \forall l \in S \setminus A). \tag{4}$$

For a given supplier selection, A is a subset of failed suppliers. $\mathbb{P}(A)$ represents the joint probability of having a failure of all suppliers in A and no failure by the remaining suppliers $S \setminus A$. If a supplier fails, the remaining suppliers try to compensate the loss through extra production. Following Meena and Sarmah (2013), the purchasing prices for the additional production lots equal the prices of the failed suppliers, i.e. there are no additional costs for the buyer to consider. We explain ETP in more detail but will do so for our broader setting where the supplier selection is part of the optimization, rather than given. Here, the expected total penalty costs depend on the number of selected suppliers, i.e. they are non-linear. We deal with this non-linearity by preprocessing the *expected penalty costs of product p and selection m*

$$V_{pm} = L_p \cdot \sum_{A \in \mathcal{P}(T_{pm})} \mathbb{P}(A) \cdot \Big[D_p - \min \big(D_p, \sum_{l \in T_{pm} \setminus A} Q_{pl}^{\max} \big) \Big] \tag{5}$$

for all possible selections $T_{pm} \in \mathcal{P}(N)$, which resembles the last term in the above definition of ETP. For obtaining the expected penalty costs V_{pm} of selection T_{pm}, we sum over all possible sets of jointly failed suppliers $A \in \mathcal{P}(T_{pm})$. For each of these sets A, we calculate the joint failure probability of having failures by suppliers $i \in A$ ($X_i = 1$) and of having no failures by suppliers $i \in T_{pm} \setminus A$ ($X_i = 0$). The resulting probability is multiplied by the total lost demand for product p arising from failure set A, which is the difference between the demand and the sum of the capacities of the non-failed suppliers. With this definition of expected penalty costs V_{pm}, we find the linear expression

$$ETP = \sum_{p \in P} \left(p^* \cdot L_p \cdot D_p + (1 - p^*) \cdot \sum_{m \in \mathcal{B}(N)} b_{pm} \cdot V_{pm} \right), \tag{6}$$

with $b_{pm} = 1$ for exactly one selection T_{pm} per product p and zero otherwise. The ETP per product then consist of the probability of the super-event that hits all suppliers, p^*, where all demand is lost, plus the probability of the super-event not happening multiplied by the corresponding expected penalty costs of the particular product and supplier selection for that product. Finally, we sum over all products.

2.1 All-Units Discount

The MILP formulation is given through

$$\min \quad TEC = PC + SMC + ETP \tag{7}$$

$$\text{s.t.} \quad Q_{pi}^{\min} \cdot x_{pi} \leq q_{pi} \leq Q_{pi}^{\max} \cdot x_{pi}, \qquad \forall\, p \in P,\ i \in N, \tag{8}$$

$$q_{pi} = k_{pi} \cdot Q_{pi}, \qquad \forall\, p \in P,\ i \in N, \tag{9}$$

$$\sum_{i \in N} q_{pi} = D_p, \qquad \forall\, p \in P, \tag{10}$$

$$x_{pi} \leq x_i, \qquad \forall\, p \in P,\ i \in N, \tag{11}$$

$$Q_{pij} \cdot x_{pij} \leq q_{pij} < Q_{pi,j+1} \cdot x_{pij}, \qquad \forall\, p \in P,\ i \in N,\ j \in J, \tag{12}$$

$$\sum_{j \in J} q_{pij} = q_{pi}, \qquad \forall\, p \in P,\ i \in N, \tag{13}$$

$$\sum_{j \in J} x_{pij} = x_{pi}, \qquad \forall\, p \in P, i \in N, \tag{14}$$

$$\tilde{Q}_{ib} \cdot \tilde{x}_{ib} \leq \sum_{p \in P} C_p \cdot \tilde{q}_{pib} < \tilde{Q}_{i,b+1} \cdot \tilde{x}_{ib}, \qquad \forall\, i \in N,\ b \in B, \tag{15}$$

$$\sum_{b \in B} \tilde{q}_{pib} = q_{pi}, \qquad \forall\, p \in P,\ i \in N, \tag{16}$$

$$\sum_{b \in B} \tilde{x}_{ib} = x_i, \qquad \forall\, i \in N, \tag{17}$$

$$\sum_{b \in B} q_{pijb} = q_{pij}, \qquad \forall\, p \in P,\ i \in N,\ j \in J, \tag{18}$$

$$\sum_{j \in J} q_{pijb} = \tilde{q}_{pib}, \qquad \forall\, p \in P,\ i \in N,\ b \in B, \tag{19}$$

$$z_p = \sum_{i \in N} a_i \cdot x_{pi}, \qquad \forall p \in P, \tag{20}$$

$$z_p - c_{pm} \leq M \cdot u_{pm}, \qquad \forall\, p \in P,\ m \in \mathcal{B}(N), \tag{21}$$

$$z_p - c_{pm} \geq -M \cdot \bar{u}_{pm} \qquad \forall\, p \in P, m \in \mathcal{B}(N), \tag{22}$$

$$b_{pm} = 1 - (u_{pm} + \bar{u}_{pm}), \qquad \forall\, p \in P,\ m \in \mathcal{B}(N), \tag{23}$$

$$u_{pm} + \bar{u}_{pm} \leq 1, \qquad \forall\, p \in P,\ m \in \mathcal{B}(N), \tag{24}$$

$$\sum_{m \in \mathcal{B}(N)} b_{pm} = 1, \qquad \forall\, p \in P, \tag{25}$$

$$b_{pm}, u_{pm}, \bar{u}_{pm}, x_{pij}, x_{pi}, \tilde{x}_{ib}, x_i \in \{0,1\}, \quad \forall\, p \in P,\ i \in N,\ j \in J,\ b \in B,\ m \in \mathcal{B}(N), \tag{26}$$

$$q_{pijb}, q_{pij}, \tilde{q}_{pib}, q_{pi}, k_{pi} \in \mathbb{N}, \qquad \forall\, p \in P,\ i \in N,\ j \in J,\ b \in B, \tag{27}$$

$$z_p \geq 0, \qquad \forall\, p \in P. \tag{28}$$

Constraint (8) ensures that the quantity q_{pi} of product p ordered from supplier i, if selected, is larger than the minimum quantity and does not exceed the supplier's capacity. (9) ensures that q_{pi} is a multiple of the lot size given by supplier i, (10) requires the demand for all products to be exactly met. Note that the demand for product p has to be a linear combination of the lot sizes of the selected suppliers, otherwise constraints (9) and (10) might cause an infeasibility. This is trivially satisfied for all demands if the lot size is one. (11) makes sure that, if a supplier is selected for at least one product p, supplier management costs for this supplier arise. (12)-(14) determine the quantity discount at supplier i for product p. The quantity q_{pij} is only positive if supplier i is selected for product p and discount level j is attained. (15)-(17) determine the business volume discount at supplier i using the same logic. For $\bar{b} = \sup B$ and $\bar{j} = \sup J$, we define $\tilde{Q}_{i,\bar{b}+1} = \sum_{p \in P} C_p \cdot D_p,\ \forall\ i$, and $Q_{pi,\bar{j}+1} = Q_{pi}^{\max}$. Note that sets B and J are defined such that $\tilde{Q}_{i,1} = 0,\ Q_{pi,1} = 0$, where there might be either no discounts in the first interval, i.e. $d_{pi,1,1} = 0\ \forall\ p, i$, or differences in base prices among suppliers modeled through positive discount levels $d_{pi,1,1} > 0$ for some suppliers. (18)-(19) determine the quantity of product p that is ordered at supplier i at discount levels j and b. $\tilde{q}_{pib}$ and q_{pij} are positive for a maximum of one b and j, respectively. Together, these constraints ensure that supplier i delivers product p at exactly one discount level combination of b and j.

In Table 1, we introduced *auxiliary* parameters that are explained in the following. One of the crucial elements of the MILP model is the use of preprocessing to avoid non-linearities. In order to determine the ETP, we computed the expected penalty for every supplier selection of every product, V_{pm}. Now we have to make sure that the respective value of ETP in the objective function for the chosen sets of suppliers is added. That is, we only add one V_{pm} for every product p in equation (6), since we only have one selection per product. In other words, $b_{pm} = 1$ for exactly one selection m. Constraints (20)-(25) make sure that we have exactly one positive decision variable b_{pm} for each product p. In order to achieve this, the parameters a_i are assigned to suppliers $i \in N$ and parameters

c_{pm} are assigned to selections $T_{pm} \in \mathcal{P}(N)$ such that

$$c_{pm} = \sum_{i \in T_{pm}} a_i.$$

a_i must be chosen such that $c_{pm_1} \neq c_{pm_2},\ \forall\, m_1, m_2 \in \mathcal{B}(N),\ m_1 \neq m_2$, e.g. $a_i = \frac{1}{2^{i-1}}$. The constraints ensure that z_p from (20) equals c_{pm} for exactly one selection $m \in \mathcal{B}(N)$, and for this selection m, the binary decision variable b_{pm} is set to one.

2.2 Incremental Discount

When it comes to incremental discounts, different quantity discounts apply to different shares of the purchased quantity (as opposed to all-units discounts, where one discount applies to the entire quantity). Taking this into consideration, we have to determine which discount applies to which share. To do so, we introduce new decision variables, additional constraints and an additional summand in the objective function. Constraints (29) - (32) ensure that if $q_{pi\tilde{j}b} \geq Q_{pi\tilde{j}}$, the indicator δ_{pijb} is set to one for all $j \in \{2, ..., \tilde{j}\}$. w_{pijb} gives the quantity of product p ordered from supplier i at volume discount level b with lower quantity discount than $d_{pi\tilde{j}b}$. ϵ is a fixed small positive real number.

$$\tilde{q}_{pib} - Q_{pij} \geq -M \cdot (1 - \delta_{pijb}), \qquad \forall\, p \in P,\ i \in N,\ j \in J \backslash \{1\},\ b \in B, \tag{29}$$

$$\tilde{q}_{pib} - Q_{pij} + \epsilon \leq M \cdot \delta_{pijb}, \qquad \forall\, p \in P,\ i \in N,\ j \in J \backslash \{1\},\ b \in B, \tag{30}$$

$$w_{pijb} = (Q_{pij} - 1) \cdot \delta_{pijb}, \qquad \forall\, p \in P,\ i \in N,\ j \in J \backslash \{1\},\ b \in B, \tag{31}$$

$$\delta_{pijb} \in \{0, 1\}, \qquad \forall\, p \in P,\ i \in N,\ j \in J \backslash \{1\}, \tag{32}$$

$$w_{pijb} \geq 0, \qquad \forall\, p \in P,\ i \in N,\ j \in J \backslash \{1\},\ b \in B. \tag{33}$$

The term IN is added to the objective function and accounts for the additional costs as compared to all-units discounts.

$$IN = \sum_{p \in P} C_p \sum_{i \in N} \sum_{j \in J \backslash \{1\}} \sum_{b \in B} w_{pijb} \cdot (d_{pijb} - d_{pi,j-1,b}). \tag{34}$$

2.3 Compensation Costs

The assumption made by Meena and Sarmah (2013) that other suppliers compensate for a supplier failure at the same price, including discounts, seems somewhat disputable. In the following, we present an approach for adjusting the model to higher compensation costs. Recall that $C_{pi} = C_p \cdot (1 - d_{pi,1})$ is the base price for product p at supplier i. We solve the following transportation problem (TPP) for all selections $T_{pm} \backslash A$ with $p \in P,\ m \in \mathcal{B}(N), A \in \mathcal{P}(T_{pm})$. The TPP does an optimal allocation of orders for the given (remaining) number of suppliers. It is part of the preprocessing and uses an additional artificial supplier $N+1$, to whom the lost demands are allocated at a cost of $C_{p,N+1} = L_p$.

$$\min \quad C_{T_{pm}\backslash A} = \sum_{i \in T_{pm}\backslash A \cup \{N+1\}} C_{pi} \cdot q_{pi} \tag{35}$$

$$s.t. \quad \sum_{i \in T_{pm}\backslash A \cup \{N+1\}} q_{pi} = D_p, \tag{36}$$

$$q_{pi} \leq Q_{pi}^{\max}, \qquad \forall\ i \in T_{pm} \backslash A \cup \{N+1\} \tag{37}$$

$$q_{pi} \in \mathbb{N}, \qquad \forall\ i \in T_{pm} \backslash A \cup \{N+1\}. \tag{38}$$

We set $Q_{p,N+1}^{\max} = D_p$. The objective (35) minimizes the total price paid under the selection $T_{pm} \backslash A$. Constraints (36)-(38) ensure that all demand is met, all suppliers only deliver up to their capacities and order quantities are non-negative integers. In (39), we subtract the optimal costs from the TPP without supplier failure from those with supplier failure. This difference serves as the combined failure and compensation cost of supplier selection T_{pm}.

$$r_{pm}^{A} = C_{T_{pm}\backslash A} - C_{T_{pm}}. \tag{39}$$

We replace the term V_{pm} in (6) by the *expected penalty and compensation costs of product p and selection m*

$$\tilde{V}_{pm} = \sum_{A \in \mathcal{P}(T_{pm})} \mathbb{P}(A) \cdot r_{pm}^{A}. \tag{40}$$

As we do not consider discounts in the TPP, $\tilde{V}_{pm}$ does not include exact compensation costs. Instead, it serves as a close approximation. In particular, it is more realistic than the initial assumption of no compensation costs at all. The computation time for one TPP is less than 0.1 seconds.

3 Numerical Results

3.1 Discussion of Meena and Sarmah (2013)

We consider the numerical study by Meena and Sarmah (2013) and compare their results with our exact approach. Their problem has a single product, all-units quantity discounts, and supplier failure risk. All of our computations are conducted using Xpress-MP 7.9 on an Intel(R) Core(TM) i7-3770, 3.4 GHz processor with 16 GB RAM.

3.1.1 Order Allocation

Meena and Sarmah (2013) consider only the order allocation part of the problem, i.e. they focus on finding the optimal allocation for a given set of suppliers. They use a genetic algorithm to find solutions that are not always optimal. In Table 2, we list the parameters of their base case and three instances from their sensitivity analysis where their order allocation was suboptimal. (In Meena and Sarmah (2013), these are the instances 3 and 6 in Table 5 and instance 4 in Table 6.) In these three instances, the problem is to find the best allocation among the 3 predetermined suppliers 8, 9 and 10. We have $C = 10,\ F = 20,\ L = 15,\ p^* = 0.01,\ Q_i^{\min} = 10,\ Q_i = 1\ \forall i$. As there is only one product, we need no product index p. Large deviations of their results from the optimal allocation, as observed in instances 2 and 3 of Meena and Sarmah (2013), cause a large increase in total costs.

Solving the problem introduced by Meena and Sarmah (2013) requires only a small special case of our model with little complexity, no preprocessing effort (as the supplier selection is already given) and negligible computation time. As exact solutions can be obtained that easily, we propose to always apply our exact approach, rather than the heuristic. The relative improvements in Table 2 are up to 4%. Using other examples may increase these improvements even further.

Table 2. Cases where Meena and Sarmah (2013) find suboptimal results.

$D = 100$			Base case parameters							
Supplier i	$Q_i^{\max}$	p_i	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$	$d_{i,1}$	$d_{i,2}$	$d_{i,3}$		
1	70	0.13	30	45	60	0.11	0.22	0.31		
2	93	0.09	40	55	70	0.07	0.19	0.29		
3	110	0.15	35	45	50	0.09	0.18	0.33		
4	90	0.17	50	60	80	0.14	0.19	0.25		
5	105	0.12	30	40	45	0.10	0.15	0.27		
6	80	0.19	37	52	60	0.17	0.21	0.30		
7	95	0.05	30	45	55	0.13	0.23	0.35		
8	115	0.14	45	50	65	0.10	0.29	0.37		
9	100	0.11	40	55	60	0.15	0.25	0.35		
10	140	0.16	50	60	70	0.20	0.27	0.46		
$D = 150$									$\boldsymbol{q_i}$ **(MS)**	$\boldsymbol{q_i}$ **(OPT)**
8	base case		base case			0.05	0.09	0.12	**15**	**15**
9						0.23	0.26	0.28	**60**	**65**
10						0.22	0.24	0.26	**75**	**70**
								TEC	**1249.9**	**1248.58**
$D = 300$										
8	base case		base case			0.05	0.09	0.12	**90**	**65**
9						0.23	0.26	0.28	**90**	**100**
10						0.22	0.24	0.26	**120**	**135**
								TEC	**2880.9**	**2839.44**
$D = 200$										
8	120	0.14	50	70	90	base case			**100**	**70**
9	90	0.11	70	80	100				**80**	**20**
10	110	0.16	60	90	100				**20**	**110**
								TEC	**1595.9**	**1533.15**

3.1.2 Supplier Selection and Order Allocation

We now apply supplier selection and order allocation simultaneously to the basic problem presented by Meena and Sarmah (2013) with 10 suppliers and 3 quantity discount levels. We find that the optimal solution is to order 10 units at supplier 7 and 90 units at supplier 10, with an objective value of 664.17. For this extended, simultaneous problem, the MILP needs 435 Simplex iterations, 97 integer nodes and 17.8 seconds of computation time with default solver settings.

Table 3. Selection and allocation for given number of suppliers.

No.	Selection	Allocation	Obj.	MIP nodes	Time (s)
1	10		812.6	1	0.2
2	**(7,10)**	$\boldsymbol{q_{10} = 90}$	**664.17**	**1**	**6.4**
3	(7,9,10)	$q_{10} = 80$	709.55	17	5.3
4	(5,7,9,10)	$q_{10} = 70$	773.6	117	8.5
5	(3,5,7,9,10)	$q_7 = 60$	905.0	0	3.8
6	(2,3,5,7,8,9)	$q_3 = 50$	970.0	1	2.7
7	(3,5,6,7,8,9,10)	$q_6 = 40$	1087.0	7	4.1
8	(1,2,3,4,5,6,7,8)	$q_7 = 30$	1136.0	1	1
9	(1,3,4,5,6,7,8,9,10)	$q_{10} = 20$	1195.0	0	0.1
10	(1,2,3,4,5,6,7,8,9,10)		1215.0	1	0.1

In order to compare the impact of different numbers of suppliers on the total costs, we fix the number of suppliers in the MILP. Table 3 shows the resulting selection and allocation decisions. Having few suppliers, e.g. only one, economies of scale arise from quantity discounts. However, this comes at the price of a higher failure risk. If you hedge against failure risk by selecting more suppliers, the economies of scale decrease. Figure 1 depicts this trade-off based on the optimal selections from Table 3. The decreasing line represents ETP, measured on the right scale, and the increasing line represents the sum of PC and SMC, measured on the left scale.

From this figure, it becomes clear that 2, 3, or 4 suppliers yield lower total costs than having only one supplier. This means that, in this problem, hedging disruption risk has a higher priority than realizing economies of scale.

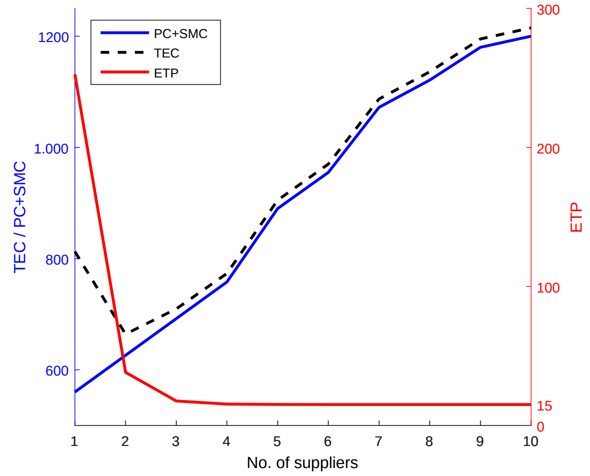

Figure 1. Economies of scale vs. failure risk.

3.2 Full Factorial Design

3.2.1 Data

As we want to evaluate our MILP model for multi-product cases that include business volume discounts, we conduct a numerical study under a full factorial design. We use 10 suppliers, which gives us 1,024 possible supplier selections. Increasing the number of suppliers to 15 would lead to 32,768 possible supplier selections, which is the number of binary variables per product required to determine ETP. Nevertheless, having a preselected pool of 10 suppliers before doing the final optimization of selection and allocation seems reasonable.

We have fixed the capacity for each supplier and each product as given in Table 4 and vary the parameters as follows.

$$|P| \in \{10, 15\}, |J| \in \{2, 4, 7\}, |B| \in \{0, 3, 5\},$$
$$D \in \{D_1, D_2\}, \mathbb{P} \in \{\mathbb{P}_1, \mathbb{P}_2\}, L \in \{50, 100\}, F \in \{50, 100\},$$

where D_1 and D_2 are deterministic demand vectors for all products. Demands and base prices (in monetary units) for the products are given in Table 4. We generated the data similar to Stadtler (2007).

In D_1, demands vary strongly among products ($\mu = 60,\ \sigma = 37$), while in D_2, demands are rather homogeneous ($\mu = 63,\ \sigma = 9$). We assume supplier failures to be independent

Table 4. Capacity, failure probability, demand and prices.

i	1	2	3	4	5	6	7	8	9	10			
$\mathbb{P}_1(i)$	0.05	0.1	0.15	0.02	0.07	0.12	0.04	0.08	0.14	0.1			
$\mathbb{P}_2(i)$	0.1	0.2	0.3	0.04	0.14	0.24	0.08	0.16	0.28	0.2			
p	$Q_{pi}^{\max}$										D_1	D_2	C_p
1	167	62	111	81	99	101	111	68	135	74	111	74	21
2	111	81	84	80	77	122	96	63	38	78	54	60	15
3	95	111	191	135	93	90	105	51	80	102	127	60	20
4	98	48	83	159	53	89	99	96	60	78	106	77	8
5	93	92	113	80	138	129	60	54	75	71	92	55	13
6	45	242	138	102	137	17	99	173	168	170	11	60	19
7	135	51	138	179	128	152	69	92	119	69	46	69	8
8	146	54	57	69	27	170	120	41	167	29	27	61	14
9	101	95	120	119	66	104	98	134	98	155	65	74	10
10	66	108	51	56	105	174	71	122	95	51	34	70	8
11	74	152	113	53	123	60	42	84	215	117	49	63	9
12	60	146	59	96	48	84	93	99	149	128	97	52	5
13	41	162	44	93	63	111	113	171	92	39	29	66	22
14	120	135	119	21	72	92	138	66	125	59	14	46	12
15	123	102	111	95	62	119	93	141	59	86	41	62	12

and the corresponding failure probabilities per supplier to be given through $\mathbb{P}_1$ and $\mathbb{P}_2$ (see Table 4). In particular, the failure probabilities under $\mathbb{P}_2$ are twice as high as under $\mathbb{P}_1$. We vary the number of products between 10 and 15. By 10 products we mean the first 10 products. The price breaks and quantity discounts are given in Tables A.1-A.2 in the Appendix. The business volume levels (in monetary units) and the corresponding relative discounts are given in Table 5 and kept similar to those presented by Xia and Wu (2007). We vary between 2, 4 and 7 quantity discount intervals and between 0, 3 and 5 business volume discount intervals. For both discount types, there is no discount in the first interval, i.e. the base price of a product is the same for all suppliers. The instances with more quantity or business volume intervals are extensions of the instances with fewer intervals, i.e. $|J| = 4$ means the first four intervals of the seven given in Tables A.1-A.2 are available. According to Munson and Jackson (2015), the maximum quantity discount is usually below 20%. For two and four quantity discount intervals, we stick to this value; for seven intervals, we allow for larger maximum discounts.

All suppliers allow for any order size within their capacity, i.e. $Q_{pi} = 1$ for all products as long as the order size exceeds the minimum order quantity, which is set to $Q_{pi}^{\min} = 5$ for all suppliers and products. The probability of a super-event hitting all suppliers is

Table 5. Business volume levels and discounts.

b	1	2	3	4	5
$\tilde{Q}_{ib}$	0	2000	4000	6000	8000
$\tilde{d}_{ib}$	0	0.05	0.15	0.2	0.25

set to $p^* = 0.01$. The penalty cost L_p per lost sale is identical for all products and is varied between 50 and 100. So are the supplier management costs. In order to remain in the setting of Meena and Sarmah (2013), we focus on the problem with costs for lost demands but without compensation costs.

3.2.2 Results

We ran all 288 instances for four hours. 58% of the instances were solved to optimality, 74% of the instances have a gap of less than 2%. The average number of visited MIP nodes is 31,000.

Supplier structure. For most products, there is one main supplier while the other suppliers serve as backup. While the main supplier delivers most of the units of an item, a backup supplier often only delivers the minimum quantity; her main function is to serve as insurance against a failure of the main supplier. However, mostly due to volume discounts, we sometimes see orders that are equally split among suppliers. This helps both suppliers towards attaining their respective discount levels.

Quantity discounts. Varying the number of quantity discount levels between two, four and seven, we observe a strong sensitivity of the total costs to these changes. If you have more quantity discounts, a decrease in the total costs is natural; however, the decrease from two to four intervals is about 14%, whereas the decrease from two to seven intervals is about 35%. i.e. quite significant. This effect is shown in Figure 2 (1). The explanation is that for a higher number of discount intervals (and thus, in this study, higher attainable discounts), the buyer's decision tends towards realizing economies of scale and reducing the focus on risk management. This is shown in Figure 2 (2), which depicts the proportion of expected total penalty costs in the total costs for the different numbers of discount intervals. For a higher number of intervals, the proportion of the expected total penalty costs grows. In Figure 3 (1), where we look at the sensitivity of the number of suppliers used per product to changes in the number of intervals, the effects underline the afore-

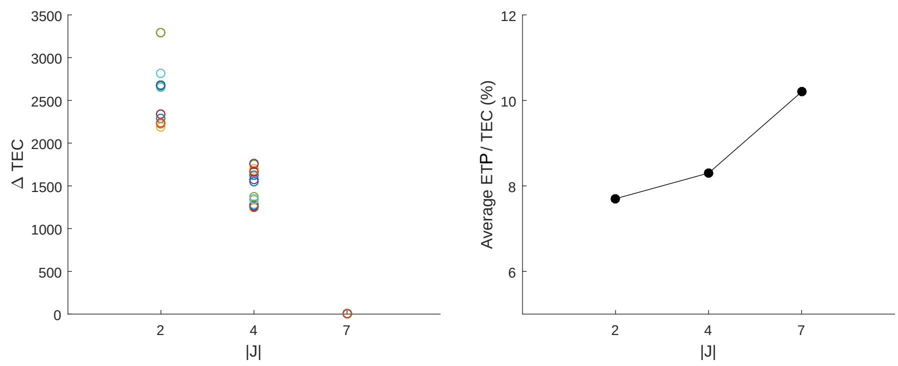

Figure 2. (1) Change of the total costs. | (2) Proportion of ETP in the total costs.

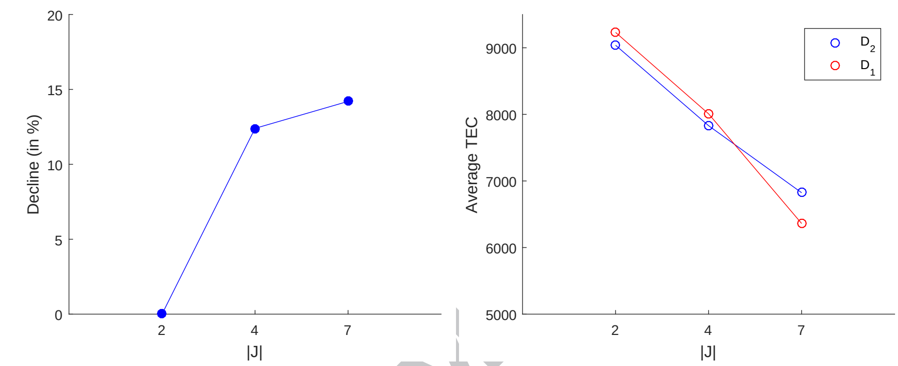

Figure 3. (1) Av. no. of suppliers/product. | (2) Av. total costs of demands.

mentioned findings. If we have more discount intervals, fewer suppliers are used, which increases the overall risk of suffering from supplier failure. In Figure 3 (2), we compare the two demand cases and find that the second demand pattern D_2 can benefit more if two to four intervals are offered, while the first demand pattern D_1 benefits more if seven intervals are offered. The second demand pattern has a lower variation and fewer products with low demands, i.e. more products reach higher discount levels in the first two cases. However, also having fewer high demands, the second demand pattern has fewer chances for fully exploiting discount levels five to seven – if they exist – than the first demand pattern.

Business volume discounts. Looking at Figure 4 (1), we find that the number of business volume discounts has a huge effect on the total costs, which is in line with the expectations. However, the effect is somewhat smaller than the effect obtained from quantity discounts, which is due to volume discounts being lower than quantity discounts. In Figure 4 (2), we see that D_1 benefits more from business volume discounts than D_2 in all cases. In particular, when having five business volume discount levels, the demand structure of D_1

allows for a better use of business volume discounts than D_2. Larger discount levels lead to a stronger focus on economies of scale and to more risk, i.e. the above analysis of quantity discounts also applies to volume discounts.

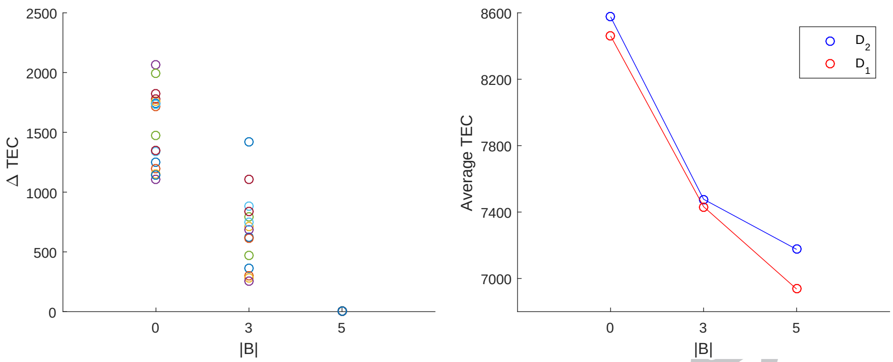

Figure 4. (1) Change of the total costs. | (2) Av. total costs per demand.

Penalty costs/Failure probabilities. If we double the penalty costs from 50 to 100, we see an increase in the total costs of about 400 monetary units on average (Fig. 5 (1)). If we double the failure probabilities from $\mathbb{P}_1$ to $\mathbb{P}_2$, the increase of the total costs is only 100 monetary units (Fig. 5 (2)). In particular, if we double failure probabilities instead of penalty costs, the increase in the total costs is much lower.

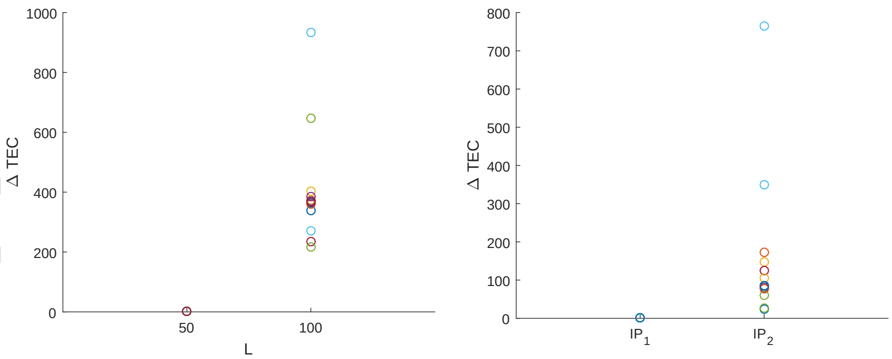

Figure 5. (1) Change of the total costs (L). | (2) Change of the total costs ($\mathbb{P}$).

Risk mitigation strategies. We observe two different strategies when risk (through L or $\mathbb{P}$) is increasing: (a) The buyer adds additional backup suppliers for several products as insurance against a failure by the main supplier. (b) The buyer does a complete reallocation and uses a less risky but more expensive main supplier. While (a) dilutes economies of scale, (b) reduces the sheer number of suppliers and focuses more strongly on economies of scale; however, with (b) one no longer has the cheapest main supplier. Effect (b) can be observed more often when penalty costs are increased, which explains the higher total costs in those instances compared to an increased failure probability.

Changing the number of products from 10 to 15 increases computation times significantly. Varying supplier management costs between 50 and 100 does not lead to different decisions. This is no surprise since economies of scale from quantity and volume discounts carry larger incentives for focusing on only a few suppliers. For incremental quantity discounts, the total price reduction in the same numerical setup is lower, therefore the sensitivity to the number of quantity discount levels decreases. Computation times increase due to an increased number of binary variables.

4 Conclusion

We considered the problem of simultaneous supplier selection and order allocation under quantity and business volume discounts and supplier failure risk. We formulated a mixed-integer linear program that solves realistic problem sizes. Considering the results of a previously published heuristic, we showed potential for improvement and derived the optimal solutions. In a numerical study, we further gained insights into the sensitivity of the optimal decisions with respect to input parameters as, e.g., penalty costs, failure probabilities, or the number of discount levels. We studied changes in total expected costs and expected total penalty costs and analyzed the trade-off between economies of scale and failure risk, finding different supplier selection strategies for an increasing risk.

There are several approaches one might wish to consider for further research. One is to extend the problem at hand and include multiple time periods. Another idea would be to extend the problem to random demand. As we consider a risk-neutral decision maker, another interesting research task would be to incorporate the decision maker's attitude towards risk.

References

Burke, GJ, SS Erenguc, and AJ Vakharia (2008). Optimal requirement allocation among quantity-discount quoting suppliers. *Operations Management Research* 1.1, pp. 53–60.

Fahimnia, B, CS Tang, H Davarzani, and J Sarkis (2015). Quantitative models for managing supply chain risks: A review. *European Journal of Operational Research* 247.1, pp. 1–15.

Ho, W, X Xu, and PK Dey (2010). Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research* 202.1, pp. 16–24.

Meena, PL and SP Sarmah (2013). Multiple sourcing under supplier failure risk and quantity discount: A genetic algorithm approach. *Transportation Research Part E: Logistics and Transportation Review* 50, pp. 84–97.

Meena, PL, SP Sarmah, and A Sarkar (2011). Sourcing decisions under risks of catastrophic event disruptions. *Transportation Research Part E: Logistics and Transportation Review* 47.6, pp. 1058–1074.

Minner, S (2003). Multiple-supplier inventory models in supply chain management: A review. *International Journal of Production Economics* 81, pp. 265–279.

Munson, CL and J Jackson (2015). Quantity Discounts: An Overview and Practical Guide for Buyers and Sellers. *Foundations and Trends (R) in Technology, Information and Operations Management* 8.1-2, pp. 1–130.

Munson, CL and MJ Rosenblatt (1998). Theories and realities of quantity discounts: An exploratory study. *Production and Operations Management* 7.4, pp. 352–369.

Ruiz-Torres, AJ and F Mahmoodi (2006). A supplier allocation model considering delivery failure, maintenance and supplier cycle costs. *International Journal of Production Economics* 103.2, pp. 755–766.

Sadrian, AA and YS Yoon (1994). A procurement decision support system in business volume discount environments. *Operations Research* 42.1, pp. 14–23.

Sawik, T (2014). Joint supplier selection and scheduling of customer orders under disruption risks: Single vs. dual sourcing. *Omega* 43, pp. 83–95.

Silbermayr, L and S Minner (2016). Dual sourcing under disruption risk and cost improvement through learning. *European Journal of Operational Research* 250.1, pp. 226–238.

Snyder, LV, Z Atan, P Peng, Y Rong, AJ Schmitt, and B Sinsoysal (2016). OR/MS models for supply chain disruptions: A review. *IIE Transactions* 48.2, pp. 89–109.

Stadtler, H (2007). A general quantity discount and supplier selection mixed integer programming model. *OR Spectrum* 29.4, pp. 723–744.

Tang, CS (2006). Perspectives in supply chain risk management. *International Journal of Production Economics* 103.2, pp. 451–488.

The Economist (2011). *Broken Links*. URL: `http://www.economist.com/node/18486015` (visited on 03/11/2015).

Thomas, DJ and JE Tyworth (2006). Pooling lead-time risk by order splitting: a critical review. *Transportation Research Part E: Logistics and Transportation Review* 42.4, pp. 245–257.

Xia, W and Z Wu (2007). Supplier selection with multiple criteria in volume discount environments. *Omega* 35.5, pp. 494 –504.

A Input: Quantity Discounts

i	$Q_{pi,1}$	$Q_{pi,2}$	$Q_{pi,3}$	$Q_{pi,4}$	$Q_{pi,5}$	$Q_{pi,6}$	$Q_{pi,7}$	$d_{pi,1}$	$d_{pi,2}$	$d_{pi,3}$	$d_{pi,4}$	$d_{pi,5}$	$d_{pi,6}$	$d_{pi,7}$
$p = 1$														
1	0	22	42	63	78	87	101	0	0.07	0.1	0.17	0.28	0.35	0.38
2	0	27	48	71	78	89	96	0	0.04	0.12	0.19	0.29	0.36	0.36
3	0	21	40	66	83	83	95	0	0.03	0.15	0.19	0.29	0.34	0.37
4	0	23	44	66	78	93	101	0	0.02	0.13	0.21	0.3	0.33	0.38
5	0	23	46	72	77	88	98	0	0.05	0.15	0.21	0.29	0.34	0.38
6	0	25	47	69	83	87	98	0	0.06	0.14	0.22	0.3	0.36	0.37
7	0	21	43	72	75	94	102	0	0.04	0.13	0.19	0.27	0.35	0.41
8	0	26	47	64	80	88	104	0	0.02	0.1	0.18	0.29	0.34	0.39
9	0	27	49	62	83	86	98	0	0.03	0.13	0.17	0.25	0.35	0.41
10	0	13	20	34	38	43	46	0	0.02	0.09	0.22	0.24	0.36	0.38
$p = 2$														
1	0	13	19	31	36	41	51	0	0.03	0.13	0.18	0.32	0.34	0.41
2	0	9	21	32	35	45	50	0	0.04	0.13	0.19	0.26	0.34	0.36
3	0	12	22	34	36	43	48	0	0.04	0.14	0.2	0.29	0.33	0.39
4	0	11	20	31	37	41	49	0	0.04	0.15	0.18	0.32	0.32	0.4
5	0	13	22	35	39	45	50	0	0.06	0.16	0.21	0.25	0.33	0.37
6	0	11	20	33	40	44	46	0	0.04	0.14	0.2	0.26	0.32	0.41
7	0	12	21	31	36	41	50	0	0.07	0.13	0.17	0.27	0.34	0.4
8	0	12	22	34	39	45	50	0	0.05	0.15	0.22	0.31	0.32	0.4
9	0	10	24	33	40	41	48	0	0.07	0.13	0.17	0.27	0.36	0.39
10	0	26	55	71	91	105	119	0	0.02	0.08	0.18	0.26	0.35	0.4
$p = 3$														
1	0	23	56	73	88	102	114	0	0.05	0.09	0.18	0.25	0.32	0.38
2	0	21	54	81	91	100	115	0	0.03	0.15	0.2	0.27	0.32	0.37
3	0	24	49	72	85	105	117	0	0.05	0.12	0.17	0.27	0.33	0.38
4	0	31	48	71	90	98	120	0	0.05	0.15	0.19	0.25	0.34	0.39
5	0	22	53	77	91	106	115	0	0.01	0.1	0.17	0.27	0.33	0.4
6	0	24	56	74	90	106	116	0	0.03	0.12	0.17	0.25	0.32	0.37
7	0	23	56	75	87	106	115	0	0.02	0.13	0.22	0.29	0.35	0.38
8	0	20	45	80	87	100	120	0	0.05	0.08	0.18	0.24	0.33	0.4
9	0	26	51	81	85	100	119	0	0.06	0.13	0.21	0.29	0.35	0.39
10	0	18	47	65	76	90	97	0	0.03	0.11	0.24	0.3	0.34	0.41
$p = 4$														
1	0	18	43	59	76	88	101	0	0.06	0.08	0.19	0.26	0.34	0.37
2	0	18	43	62	77	82	92	0	0.05	0.12	0.22	0.28	0.32	0.4
3	0	24	39	64	78	90	94	0	0.01	0.1	0.22	0.29	0.32	0.39
4	0	18	43	62	74	80	93	0	0.05	0.09	0.19	0.24	0.32	0.37
5	0	26	43	65	71	84	93	0	0.03	0.1	0.21	0.28	0.34	0.38
6	0	23	39	62	73	84	92	0	0.06	0.09	0.17	0.29	0.33	0.38
7	0	18	42	60	78	84	90	0	0.01	0.1	0.23	0.26	0.35	0.39
8	0	25	43	68	71	81	95	0	0.04	0.08	0.17	0.31	0.35	0.4
9	0	26	39	65	72	84	93	0	0.04	0.13	0.18	0.32	0.36	0.37
10	0	22	38	53	65	77	82	0	0.04	0.1	0.22	0.31	0.34	0.4
$p = 5$														
1	0	19	33	57	63	74	83	0	0.06	0.12	0.2	0.28	0.33	0.4
2	0	22	41	58	69	73	81	0	0.03	0.14	0.22	0.26	0.33	0.38
3	0	23	38	56	64	73	85	0	0.06	0.12	0.19	0.3	0.34	0.38
4	0	19	33	56	65	76	82	0	0.04	0.12	0.18	0.26	0.35	0.39
5	0	15	39	56	66	75	85	0	0.01	0.12	0.16	0.32	0.33	0.4
6	0	22	36	60	66	75	83	0	0.02	0.09	0.21	0.29	0.34	0.4
7	0	19	33	56	62	70	85	0	0.05	0.12	0.19	0.29	0.35	0.4
8	0	22	34	56	61	73	87	0	0.04	0.15	0.2	0.25	0.36	0.38
9	0	18	34	59	62	74	83	0	0.02	0.15	0.21	0.25	0.33	0.38
10	0	3	5	7	8	8	10	0	0.03	0.1	0.16	0.26	0.35	0.41
$p = 6$														
1	0	2	4	6	8	9	10	0	0.05	0.1	0.19	0.31	0.34	0.39
2	0	3	4	6	8	8	10	0	0.02	0.13	0.22	0.31	0.34	0.4
3	0	2	5	6	8	9	10	0	0.05	0.13	0.22	0.3	0.36	0.37
4	0	2	4	7	8	9	10	0	0.02	0.11	0.17	0.3	0.34	0.39
5	0	2	5	7	8	9	10	0	0.07	0.1	0.17	0.26	0.33	0.39
6	0	3	4	7	8	9	10	0	0.03	0.16	0.17	0.29	0.35	0.41
7	0	2	4	6	8	9	10	0	0.06	0.09	0.16	0.3	0.34	0.36
8	0	2	4	6	7	9	10	0	0.02	0.09	0.19	0.27	0.33	0.39
9	0	2	4	6	8	9	9	0	0.03	0.09	0.21	0.32	0.35	0.39
10	0	11	21	28	34	39	43	0	0.02	0.09	0.22	0.25	0.36	0.36
$p = 7$														
1	0	7	17	29	31	35	40	0	0.04	0.13	0.2	0.27	0.36	0.41
2	0	11	17	29	34	36	43	0	0.05	0.13	0.17	0.28	0.33	0.4
3	0	8	20	29	32	37	42	0	0.04	0.08	0.21	0.24	0.33	0.38
4	0	9	16	29	32	38	39	0	0.04	0.15	0.17	0.3	0.32	0.4
5	0	11	16	29	31	38	43	0	0.05	0.14	0.17	0.28	0.34	0.37
6	0	11	18	26	34	36	43	0	0.05	0.14	0.17	0.28	0.32	0.39
7	0	8	20	28	33	35	41	0	0.05	0.09	0.17	0.31	0.34	0.37
8	0	10	21	26	32	35	43	0	0.05	0.15	0.17	0.31	0.36	0.38
9	0	11	16	29	33	36	40	0	0.07	0.15	0.18	0.3	0.34	0.4
10	0	6	12	15	19	21	25	0	0.02	0.16	0.19	0.27	0.33	0.37
$p = 8$														
1	0	7	11	16	18	20	23	0	0.05	0.15	0.19	0.28	0.33	0.38
2	0	6	11	17	19	22	25	0	0.02	0.14	0.18	0.3	0.34	0.38
3	0	5	12	15	18	21	25	0	0.02	0.12	0.18	0.25	0.36	0.38
4	0	5	11	15	18	23	23	0	0.05	0.09	0.23	0.25	0.32	0.4
5	0	5	11	17	20	21	25	0	0.04	0.11	0.22	0.26	0.34	0.41
6	0	5	10	15	20	23	25	0	0.04	0.09	0.2	0.28	0.35	0.37
7	0	5	10	17	19	22	26	0	0.05	0.08	0.17	0.32	0.32	0.37
8	0	7	12	17	19	22	24	0	0.06	0.16	0.18	0.3	0.35	0.4
9	0	6	11	15	18	22	25	0	0.03	0.1	0.17	0.26	0.36	0.38
10	0	14	28	41	42	49	58	0	0.05	0.1	0.23	0.26	0.33	0.41

Table A.1. Price breaks and discount levels (1).

i	$Q_{pi,1}$	$Q_{pi,2}$	$Q_{pi,3}$	$Q_{pi,4}$	$Q_{pi,5}$	$Q_{pi,6}$	$Q_{pi,7}$	$d_{pi,1}$	$d_{pi,2}$	$d_{pi,3}$	$d_{pi,4}$	$d_{pi,5}$	$d_{pi,6}$	$d_{pi,7}$
$p = 9$														
1	0	10	24	37	45	51	58	0	0.03	0.11	0.22	0.31	0.33	0.41
2	0	11	28	39	49	52	60	0	0.06	0.12	0.2	0.31	0.35	0.39
3	0	15	24	36	43	53	61	0	0.06	0.13	0.19	0.29	0.36	0.4
4	0	15	28	37	47	55	56	0	0.03	0.08	0.17	0.26	0.34	0.38
5	0	10	23	39	46	55	60	0	0.05	0.15	0.21	0.25	0.36	0.39
6	0	13	27	41	46	49	59	0	0.04	0.12	0.24	0.31	0.33	0.41
7	0	11	26	37	45	53	61	0	0.04	0.15	0.17	0.29	0.35	0.39
8	0	16	25	41	46	53	56	0	0.06	0.11	0.18	0.32	0.33	0.41
9	0	12	24	40	45	52	57	0	0.03	0.12	0.19	0.24	0.34	0.4
10	0	5	15	19	24	27	30	0	0.03	0.08	0.17	0.29	0.33	0.38
$p = 10$														
1	0	7	15	21	25	26	30	0	0.02	0.09	0.21	0.26	0.35	0.39
2	0	8	14	19	23	26	29	0	0.07	0.13	0.19	0.31	0.34	0.4
3	0	6	12	19	25	26	30	0	0.05	0.11	0.24	0.26	0.35	0.37
4	0	6	14	19	23	29	31	0	0.04	0.15	0.19	0.28	0.35	0.38
5	0	6	13	22	23	28	30	0	0.05	0.09	0.21	0.27	0.32	0.41
6	0	7	15	22	24	27	31	0	0.04	0.16	0.17	0.31	0.36	0.38
7	0	8	13	19	25	28	31	0	0.05	0.12	0.19	0.29	0.34	0.39
8	0	7	14	20	24	28	32	0	0.04	0.14	0.17	0.26	0.35	0.41
9	0	6	14	19	22	28	29	0	0.05	0.16	0.22	0.27	0.35	0.41
10	0	10	17	32	35	39	45	0	0.04	0.1	0.23	0.25	0.35	0.39
$p = 11$														
1	0	10	21	28	33	39	44	0	0.07	0.11	0.19	0.29	0.33	0.37
2	0	8	21	30	35	39	42	0	0.02	0.12	0.21	0.29	0.34	0.36
3	0	10	19	29	34	41	45	0	0.02	0.14	0.18	0.25	0.34	0.39
4	0	8	18	27	36	38	45	0	0.02	0.15	0.2	0.25	0.36	0.39
5	0	10	19	31	35	41	45	0	0.01	0.09	0.23	0.28	0.35	0.41
6	0	11	21	29	34	37	43	0	0.03	0.09	0.21	0.31	0.36	0.4
7	0	10	21	31	33	38	44	0	0.04	0.11	0.19	0.28	0.34	0.39
8	0	10	20	27	34	41	45	0	0.03	0.08	0.18	0.24	0.34	0.39
9	0	8	22	28	36	38	46	0	0.06	0.12	0.2	0.24	0.35	0.37
10	0	15	37	62	63	76	89	0	0.05	0.11	0.19	0.3	0.32	0.41
$p = 12$														
1	0	16	35	59	69	74	91	0	0.06	0.09	0.19	0.28	0.36	0.39
2	0	21	40	56	64	74	89	0	0.07	0.1	0.2	0.27	0.34	0.36
3	0	22	36	56	67	76	92	0	0.07	0.15	0.22	0.3	0.32	0.39
4	0	24	42	58	68	76	92	0	0.02	0.13	0.19	0.27	0.35	0.39
5	0	16	41	62	63	80	85	0	0.02	0.12	0.19	0.28	0.33	0.36
6	0	18	38	62	64	77	89	0	0.05	0.15	0.17	0.3	0.33	0.4
7	0	21	34	58	72	77	90	0	0.02	0.09	0.16	0.31	0.34	0.38
8	0	18	35	59	68	74	87	0	0.04	0.14	0.18	0.27	0.33	0.37
9	0	23	37	60	67	81	91	0	0.04	0.14	0.19	0.29	0.33	0.37
10	0	5	12	17	19	25	25	0	0.06	0.12	0.21	0.32	0.34	0.38
$p = 13$														
1	0	5	11	18	20	25	27	0	0.04	0.09	0.24	0.25	0.34	0.37
2	0	4	11	17	20	25	27	0	0.03	0.13	0.23	0.29	0.34	0.39
3	0	5	12	17	20	22	25	0	0.05	0.1	0.2	0.27	0.34	0.36
4	0	6	13	19	19	25	25	0	0.05	0.09	0.18	0.26	0.33	0.37
5	0	6	13	19	19	23	25	0	0.04	0.1	0.22	0.26	0.33	0.37
6	0	5	12	18	19	22	25	0	0.03	0.15	0.22	0.3	0.32	0.4
7	0	5	13	18	22	23	26	0	0.02	0.09	0.22	0.32	0.33	0.38
8	0	4	13	19	21	24	26	0	0.05	0.1	0.22	0.25	0.35	0.4
9	0	7	10	17	21	24	26	0	0.03	0.08	0.17	0.3	0.33	0.39
10	0	3	5	9	10	12	13	0	0.01	0.12	0.21	0.26	0.35	0.4
$p = 14$														
1	0	3	5	9	10	11	12	0	0.06	0.08	0.2	0.32	0.35	0.37
2	0	3	6	9	10	11	13	0	0.02	0.15	0.18	0.3	0.35	0.41
3	0	2	5	9	10	12	13	0	0.04	0.1	0.17	0.27	0.35	0.4
4	0	2	5	9	10	11	13	0	0.05	0.09	0.23	0.3	0.33	0.36
5	0	2	6	8	10	12	13	0	0.03	0.1	0.17	0.28	0.33	0.38
6	0	2	5	8	10	11	12	0	0.05	0.12	0.17	0.3	0.34	0.38
7	0	3	6	9	10	11	13	0	0.03	0.09	0.21	0.27	0.33	0.39
8	0	2	6	8	9	11	12	0	0.05	0.16	0.23	0.25	0.35	0.37
9	0	3	6	8	10	11	13	0	0.05	0.11	0.2	0.29	0.35	0.39
10	0	9	14	24	28	31	37	0	0.04	0.1	0.22	0.31	0.34	0.39
$p = 15$														
1	0	7	15	23	30	34	37	0	0.01	0.08	0.17	0.26	0.33	0.39
2	0	9	14	25	31	31	38	0	0.03	0.1	0.24	0.27	0.35	0.38
3	0	8	15	25	29	31	37	0	0.04	0.08	0.2	0.3	0.33	0.36
4	0	10	14	24	27	32	39	0	0.03	0.12	0.21	0.24	0.34	0.39
5	0	7	17	26	27	31	39	0	0.02	0.14	0.16	0.32	0.34	0.38
6	0	6	16	26	29	34	39	0	0.06	0.13	0.22	0.3	0.34	0.37
7	0	7	15	24	28	34	36	0	0.04	0.09	0.22	0.28	0.36	0.38
8	0	6	15	25	28	32	36	0	0.06	0.09	0.17	0.25	0.35	0.38
9	0	10	15	24	29	31	38	0	0.03	0.14	0.2	0.28	0.36	0.39
10	0	9	22	35	38	45	50	0	0.06	0.15	0.19	0.28	0.33	0.4

Table A.2. Price breaks and discount levels (2).

Highlights

- Supplier selection and order allocation under supplier failure risk
- Model includes quantity and business volume discounts
- Exact model: Solves realistic problem sizes to optimality
- Improves previously published heuristic from 2013 by 4%