# **Accepted Manuscript**

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PII: DOI: Reference:	S0165-1765(16)30527-4 http://dx.doi.org/10.1016/j.econlet.2016.12.018 ECOLET 7454
To appear in:	Economics Letters
Revised date :	7 November 2016 4 December 2016
Accepted date :	9 December 2016

Please cite this article as: Hanson, A., Phan, T., Bubbles, wage rigidity, and persistent slumps. *Economics Letters* (2016), http://dx.doi.org/10.1016/j.econlet.2016.12.018

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# Highlights

- Highlight the interaction between the collapse of rational bubbles and downward wage rigidity
- Analytically characterize depth and duration of post-bubble recessions
- Higher credit growth during bubble episodes leads to deeper and longer recessions

# Bubbles, Wage Rigidity, and Persistent Slumps

Andrew Hanson & Toan Phan\*

04/12/2016

#### Abstract

We embed downward wage rigidity into a rational bubble model. We analytically characterize how the collapse of bubbles can interact with wage rigidity to generate deep and protracted recessions with involuntary unemployment, such as those in Japan or Spain.

JEL classification: E2; E3; E4 Keywords: rational bubbles; wage rigidity; unemployment; long recession

#### 1 Intro

The collapse of asset and credit bubbles often precedes financial crises and protracted recessions (Jordà et al., 2015), such as the "lost decades" following the collapse of Japan's housing bubble in 1991. However, understanding post-bubble crises and recessions remains an open question for the general equilibrium bubble literature (see surveys by Barlevy, 2012 and Miao, 2014), as most existing models predict a relatively benign post-bubble transition. A standard prediction is that while bubbles give rise to economic booms, their collapse simply precedes a gradual reversion to the pre-bubble trend while the economy retains full employment (e.g., Hirano and Yanagawa, 2016).

Our contribution is a tractable model where the collapse of bubbles can lead to a protracted slump, or even a "hysteresis" – periods in which investment, output, and employment are persistently below the prebubble trend. We embed downward wage rigidity (real or nominal, à-la Schmitt-Grohé and Uribe, 2016) into an otherwise relatively standard rational bubble model (à-la Martin and Ventura, 2012, 2016). Despite being a standard friction in New Keynesian models, wage rigidity has been largely absent from bubble models. We show its presence leads to drastically different post-bubble dynamics. As usual, bubbles in our model crowd in lending and investment. However, their collapse causes a slump in which wages cannot flexibly fall and firms have to cut employment, causing involuntary unemployment and a drop in net worth. This process amplifies and propagates the effects of collapse. Figure 1's top panel illustrates the model's dynamics (versus the standard dynamics with flexible wage).

Our model allows for analytical characterizations of the depth and duration of the slump. While a higher rate of credit creation leads to bigger bubbles and economic booms, it also causes a deeper and longer recession after bubbles collapse. Additionally, we show that post-bubble inflation can reduce the depth and duration of the recession, but deflation would exacerbate them. Overall, by combining the rational-bubble and New Keynesian frameworks, our paper provides a new and complementary perspective on long recessions (see, inter alia, Krugman, 1998, Christiano et al., 2015).

The model's predictions are largely consistent with Jordà et al. (2015)'s empirical evidence that the combination of credit and asset bubbles increases crisis risks. They are also consistent with the experiences of Japan in 1991 and Spain in 2008 (figure 2). Both countries enjoyed sizable increases in employment and real wages during the boom-phase of asset prices (in Japan's late 1980s and Spain's early 2000s). However, when the booms turned into busts, real wages remained rigidly high for many years while the the economies fell into recessions and unemployment substantially increased.

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#### 2 Real Model

We start with a real model. Consider a deterministic overlapping generations economy where agents live for two periods (called *young* and *old*).<sup>1</sup> Agents consume when old and are risk-neutral. Young agents earn labor income, borrow/lend, and accumulate capital with a technology that transforms each unit of consumption good to  $a^i$  units of capital, where the productivity  $a^i$  can take two possible values  $a^H > a^L > 0$ . The fraction of agents with productivity  $a^H$  (the "high type") in each generation is fixed at  $\mu \in (0, 1)$ . Agents supply labor and capital to competitive firms which produce the consumption good using a standard technology  $y_t = k_t^{\alpha} l_t^{1-\alpha}$ ,  $\alpha \in (0, 1)$ . For simplicity, assume capital depreciates completely, the population of each generation is fixed at one, and there is no exogenous TFP growth.

To model bubbles' expansionary effect on capital, we assume bubble creation as in Martin and Ventura (2016). Let  $B_t \ge 0$  denote the market value of the portfolio consisting of all bubbles in t. Some of these bubbles, whose aggregate market value denoted by  $B_t^{old}$ , are old since they were initiated by previous generations and purchased by agents of generation (born in) t - 1. Some, whose aggregate market value denoted by  $B_t^{old}$ , are old since they are market value denoted by  $B_t^{new}$ , are new since they are initiated by agents of generation t - 1. Thus,

$$B_t = B_t^{new} + B_t^{old}.$$

For simplicity, assume only the high type create new bubbles. Let

$$b_t^{new,H}\equiv \frac{B_t^{new}}{\mu},\ b_t^{new,L}\equiv 0$$

denote the value of new bubbles generated by each individual of generation t - 1.

Given labor income  $w_t l_t$ , rental rate of capital  $R_{t+1}^k$ , interest rate  $R_{t+1}$ , and value of new bubbles  $b_{t+1}^{new,i}$ , an agent of generation t with productivity  $a^i \in \{a^H, a^L\}$  chooses capital accumulation  $k_{t+1}^i \ge 0$ , a share  $x_t^i \ge 0$  of the portfolio of all existing bubbles, and net borrowing  $d_t^i$  to solve:

$$\max_{\substack{k_{t+1}^i \ge 0, \ x_t^i \ge 0, \ d_t^i}} c_{t+1}^i \tag{1}$$

subject to budget constraints:

$$\begin{aligned} \dot{x}_{t+1}^{i} + B_{t}x_{t}^{i} &= w_{t}l_{t} + d_{t}^{i} \\ c_{t+1}^{i} &= R_{t+1}^{k}k_{t+1}^{i} + B_{t+1}^{old}x_{t}^{i} + b_{t+1}^{new,i} - R_{t+1}d_{t}^{i} \end{aligned}$$

and a collateral constraint:<sup>2</sup>

$$R_{t+1}d_t^i \le b_{t+1}^{new,i}.$$
(2)

Hence bubble creation increases the high type's borrowing capacity, and can represent credit creation. We focus on equilibria in which the relative size of new bubbles is constant:

$$B_t^{new} = nB_t,$$

where  $n \in (0,1)$  represents an exogenous rate of bubble (or credit) creation, and where (2) binds for the high type.

Finally, we impose a downward real wage rigidity:<sup>3</sup>

$$w_t \ge \gamma w_{t-1},$$
 (DWR)

where  $\gamma \in [0,1]$  governs the degree of rigidity ( $\gamma = 0$  implies full flexibility). Downward real wage rigidity has been empirically documented (e.g., Babecký et al., 2010), and was present in Japan and Spain (figure 2).

 $<sup>^{1}</sup>$  We interpret the overlapping generations structure as representing the entry and exit of entrepreneurs/investors, and the duration of a period as the length of a loan contract.

<sup>&</sup>lt;sup>2</sup> The result does not change if the collateral also includes old bubbles.

<sup>&</sup>lt;sup>3</sup> Schmitt-Grohé and Uribe (2016) impose a similar form of rigidity on local-currency wage and a fixed exchange rate, which amount to a form of real rigidity as in (DWR).

The presence of rigid wages implies that the labor market does not necessarily clear. In each t, each young agent inelastically supplies one unit of labor, but the equilibrium employment  $l_t$  (symmetric across agents) satisfies:

$$l_t \le 1,\tag{3}$$

and complementary-slackness:

$$(1 - l_t)(w_t - \gamma w_{t-1}) = 1.$$
(4)

(4) states that involuntary unemployment must be accompanied by a binding (DWR). Conversely, when (DWR) does not bind, the economy must be in full employment.

In equilibrium, competitive factor prices satisfy:

$$R_t^k = \alpha \left(\frac{l_t}{k_t}\right)^{1-\alpha}$$

$$w_t = (1-\alpha) \left(\frac{k_t}{l_t}\right)^{\alpha},$$
(5)

where  $k_t = \mu k_t^H + (1 - \mu) k_t^L$  is the aggregate capital stock; markets clear:

$$\mu d_t^H + (1 - \mu) d_t^L = 0 \mu x_t^H + (1 - \mu) x_t^L = 1 \text{ if } B_t > 0,$$

and portfolio allocations solve (1). Initial  $B_0$  and  $k_0$  are given.

## 2.1 Bubble-less Steady State (SS)

Without bubbles  $(B_t \equiv 0, \forall t)$ , agents cannot borrow and lend, and all labor income is invested in capital. Since (DWR) does not bind in SS, it is straightforward to characterize the bubble-less SS by:

$$l = 1,$$
  

$$\bar{w} = (1 - \alpha)\bar{k}^{\alpha},$$
  

$$\bar{R} = a^{L}\bar{R}^{k} = \frac{a^{L}}{\bar{a}}\frac{\alpha}{1 - \alpha},$$
  

$$\bar{k} = ((1 - \alpha)\bar{a})^{\frac{1}{1 - \alpha}},$$
(6)

where  $\bar{a} \equiv \mu a^H + (1 - \mu) a^L$ .

#### 2.2 Bubble SS

With bubbles, the high type can borrow (recall collateral constraint (2)). The equilibrium interest rate satisfies  $a^L R_{t+1}^k \leq R_{t+1} \leq a^H R_{t+1}^k$ , as  $R_{t+1}$  cannot be below the low type's return from capital (else the low type would not lend) or above the high type's return from capital (else the high type would not borrow). For simplicity, we focus on equilibria where  $a^L R_{t+1}^k = R_{t+1}$  (i.e., constraint  $k_{t+1}^L \geq 0$  does not bind).<sup>4</sup> Then the low type's optimization implies a no-arbitrage condition between bubble speculation, lending, and capital investment:

$$\underbrace{\frac{(1-n)B_{t+1}}{B_t}}_{\frac{B_{t+1}}{B_t}} = R_{t+1} = a^L R_{t+1}^k.$$
(7)

<sup>&</sup>lt;sup>4</sup> For a thorough analysis of equilibria where  $a^L R_{t+1}^k = R_{t+1}$  vs. equilibria where  $a^L R_{t+1}^k < R_{t+1}$ , see Ikeda and Phan (2015).

Since  $a^H R_{t+1}^k > R_{t+1}$ , the high type are credit-constrained and do not speculate in bubbles. Thus, assuming (DWR) does not bind, the capital stock evolves as:

$$k_{t+1}^{H} = a^{H}(w_{t} + \frac{1}{\mu} \frac{nB_{t+1}}{R_{t+1}})$$

$$k_{t+1}^{L} = a^{L}(w_{t} - \frac{1}{1-\mu} \frac{nB_{t+1}}{R_{t+1}} - \frac{1}{1-\mu}B_{t})$$

$$\Rightarrow k_{t+1} = \bar{a}w_{t} + \underbrace{(a^{H} - a^{L})\frac{nB_{t+1}}{R_{t+1}}}_{crowd-in} - \underbrace{a^{L}B_{t}}_{crowd-out}.$$
(8)

The last two terms in (8) show different effects of bubbles on capital. First, the *crowd-in* effect: a higher rate of bubble creation n increases the low type's ability to lend to the high type. Second, the *crowd-out* effect: bubble speculation crowds out the flow from savings to capital accumulation. We focus on expansionary bubbles by assuming

$$n > \frac{a^L}{a^H},\tag{9}$$

so that the crowd-in effect dominates.

As (DWR) does not bind in SS, the deterministic bubble SS is characterized by:

Б

$$l_b = 1,$$
  

$$w_b = (1 - \alpha)k_b^{\alpha},$$
  

$$R_b = a^L R_b^k = 1 - n,$$
  

$$k_b = \left(\frac{\alpha}{R_b^k}\right)^{\frac{1}{1-\alpha}} = \left(\frac{a^L \alpha}{1-n}\right)^{\frac{1}{1-\alpha}},$$

with aggregate bubble value:

$$B = \frac{k_b - \bar{a}w_b}{(a^H - a^L)\frac{n}{1-n} - a^L}$$

B > 0 if and only if  $k_b > \bar{a}w_b$ , or equivalently:

$$n > 1 - \frac{a^L}{\bar{a}} \frac{\alpha}{1 - \alpha}.$$

Hence, we impose the following necessary and sufficient condition for the existence of a SS with expansionary bubbles:

$$n > \max\{1 - \frac{a^L}{\bar{a}} \frac{\alpha}{1 - \alpha}, \frac{a^L}{a^H}\}.$$
(10)

## 2.3 Post-bubble dynamics

Suppose the economy is in the bubble SS, but bubbles unanticipatedly collapse at T ( $B_{T+s} = 0, \forall s \ge 0$ ). As bubbles were expansionary, the post-bubble capital stock and wage will decline towards the bubble-less SS levels.

Flexible wage: If wages are flexible ( $\gamma = 0$ ), then the labor market clears and the economy maintains full employment along the transition path from the bubble SS to the bubble-less SS:

$$\begin{cases} l_{T+s}^{f} = 1, \\ k_{T+s+1}^{f} = \bar{a}w_{T+s}^{f}l_{T+s}^{f}, \\ w_{T+s}^{f} = (1-\alpha)(k_{T+s}^{f})^{\alpha} \\ R_{T+s}^{f} = a^{L}R_{T+s}^{k,f} = a^{L}\alpha(k_{T+s}^{f})^{\alpha-1}, \forall s \ge 0. \end{cases}$$
(11)

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This full-employment transition path is illustrated by the dashed lines in figure 1's top panel.

*Rigid wage:* However, if the downward wage rigidity constraint binds, then wage cannot flexibly fall to clear the labor market, leading to involuntary unemployment. The contraction in employment reduces young agents' net worth, which in turn reduces their ability to accumulate capital. The wage rigidity thus amplifies and propagates the shock of bursting bubbles. We can analytically characterize the depth and duration of the post-bubble unemployment episode. Let

$$s^* \equiv \min\{s \ge 0 | L_{T+s} = 1\},\$$

then  $T + s^*$  is the first post-bubble period when full employment is recovered. If  $s^* > 0$ , then the economy is said to be in a *slump* between T and  $T + s^* - 1$ . Let

$$k^* \equiv \inf\{k_{T+s} \colon 0 \le s \le s^*\}$$

be the recession's *trough*.

**Proposition 1.** [Post-bubble slump]

1. During the slump, (DWR) binds:

$$w_{T+s} = \gamma^s w_b, \tag{12}$$

the capital stock is:

$$k_{T+s} = \eta^s \cdot \Gamma^{\frac{s(s+1)}{2}} \cdot k_b, \tag{13}$$

where  $\eta \equiv \frac{\bar{a}}{a^L} \frac{1-\alpha}{\alpha} (1-n) \in (0,1), \ \Gamma \equiv \gamma^{-\frac{1-\alpha}{\alpha}} \geq 1$ , and involuntary unemployment is positive:

$$1 - l_{T+s} = 1 - \left(\frac{1-\alpha}{\gamma^s w_b}\right)^{\frac{1}{\alpha}} k_{T+s} > 0.$$
(14)

After the slump  $(t \ge T + s^*)$ , the economy follows the full employment transition path to the bubble-less SS.

2. The slump's duration and trough are functions of  $\gamma$  and n:

$$s^* = \begin{cases} \left\lceil \frac{2\alpha}{1-\alpha} \log_{\gamma} \eta - \frac{3-\alpha}{1-\alpha} \right\rceil & \text{if } \gamma < 1\\ \infty & \text{if } \gamma = 1 \end{cases},$$
(15)

$$^* = e^{\zeta} \cdot k_b \tag{16}$$

where  $\lceil x \rceil$  is the ceiling function and  $\zeta(n, \gamma) \equiv \inf\{\frac{s(s+1)}{2}\ln\Gamma + s\ln\eta: 0 \le s \le s^*\}.$ 

Intuitively, the credit market shuts down after bubbles collapse, and the aggregate capital then evolves according to  $k_{t+1} = \bar{a}w_t l_t$ . However, wage is rigidly constrained by (12). Combined with firms' optimal hiring condition (5), this implies (13) and (14). The rest follows. Note in (15) that in the extreme rigidity case of  $\gamma = 1$ , the economy never recovers from unemployment.

Proposition 1 implies  $k^*$  and  $s^*$  are monotone functions of  $\gamma$  and n. An increase in  $\gamma$  makes wages more rigid, weakening the "recovery term"  $\Gamma^{\frac{s(s+1)}{2}}$  in (13). An increase in bubble creation rate n raises bubbles' size, leading to a larger necessary post-bubble adjustment, i.e., a larger "contractionary term"  $\eta^s$  in (13). Consequently, increases in n and/or  $\gamma$  will exacerbate the depth and duration of the slump. Moreover, the economy can fall into a *hysteresis* (defined as  $k^* < \bar{k}$ ), where capital, employment, and output fall below the pre-bubble levels. Formally:

#### Corollary 2. [Comparative statics]

1. Slump duration  $s^*$  is increasing in wage rigidity parameter  $\gamma$  and bubble/credit creation rate n.

- 2. Trough  $k^*$  is decreasing in  $\gamma$  and n.
- 3. Hysteresis  $(k^* < \bar{k})$  occurs if and only if  $\gamma$  and n are sufficiently large that  $\zeta(n, \gamma) < 0$ .

This result has an important implication. From an ex-ante perspective, a marginal increase in bubble/credit creation n raises aggregate activities in the bubble SS. However, from an ex-post perspective, the increase in n causes a deeper and longer recession. Hence, a macroprudential policymaker who can tax bubble creation or investment may face a time-inconsistency problem. Future work can explore this problem.

Figure 1 summarizes the main results.

### 3 Nominal Rigidity

Our insights carry through to an environment with *nominal* instead of real rigidity. To fix ideas, we introduce money through a cash-in-advance framework as in Krugman (1998). Suppose each agent i needs to hold cash  $(M_t^i)$  to finance a small fraction  $\epsilon$  of consumption expenditure:

$$\epsilon c_t^i \le \frac{M_t^i}{P_t},$$

where  $P_t$  denotes the price level. We focus on equilibria where this constraint is binding. To avoid distributional effects, we follow Asriyan et al. (2016) and assume that the monetary authority transfers the seignorage to a fiscal authority who uses it to finance useless spending. Thus, monetary policy simply consists of setting the money supply to achieve a desired inflation target.

Most importantly, we replace (DWR) with the following nominal rigidity:

$$P_t w_t \ge \gamma P_{t-1} w_{t-1},$$

$$w_t \ge \frac{\gamma}{\Pi_t} w_{t-1},$$
(17)

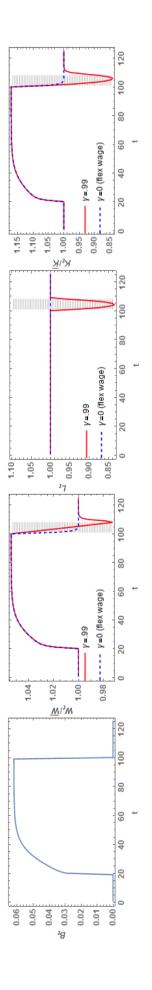
or equivalently:

where  $w_t$  denotes real wage and  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ . Constraint (17) implies that inflation ( $\Pi_t > 1$ ) can reduce the "effective" degree of rigidity  $\hat{\gamma}_t \equiv \gamma/\Pi_t$ . Formally, suppose for simplicity that the authority sets a constant inflation target  $\Pi^*$ . Then:

**Proposition 3.** Monetary policy can restore full employment transition (11) if:

$$\Pi^* \ge \max\left\{\gamma \cdot \eta^{\alpha}, \frac{1}{1-n}\right\}, \forall t.$$
(18)

Intuitively, by setting the inflation target above a certain threshold, the authority can prevent binding nominal wage rigidity and post-bubble slumps. Equation (18) also shows that a higher rigidity  $\gamma$  and bubble/credit creation rate n would raise the required threshold. In practice, many factors could hinder policymakers ability to raise inflation, such as deflationary pressures (e.g., Eggertsson and Krugman, 2012) that are absent from our simple model. A monetary analysis that combines our model and such deflationary pressures is left for future research.



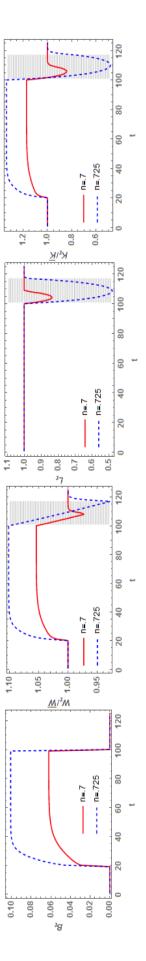


Fig 1. Transition dynamics of aggregate variables (relative to bubble-less steady state). The economy begins in the bubble-less SS. Then bubbles unanticipatedly arise at grey-dashed bar), as downwardly rigid wages cannot decline fast enough, and post-bubble economic activities dip below the pre-bubble trend. The bottom panel shows t=20. Capital, output and wage expand until the economy reaches the deterministic bubble SS. Then bubbles unanticipatedly collapse at t=100. If y=0, the economy flexibly transitions to the bubble-less SS (the dashed lines in top panels); but if y=0.99, the economy falls into a slump with involuntary unemployment (the how a larger n leads to a larger boom but a deeper and more persistent bust. Parameters:  $\alpha = \frac{1}{2}$ ,  $\alpha^{n} = 1$ ,  $\alpha^{n} = \frac{1}{2}$ ,  $\mu = \frac{1}{2}$ , and n=0.7 (top panel),  $\gamma = 0.99$  (bottom panel).

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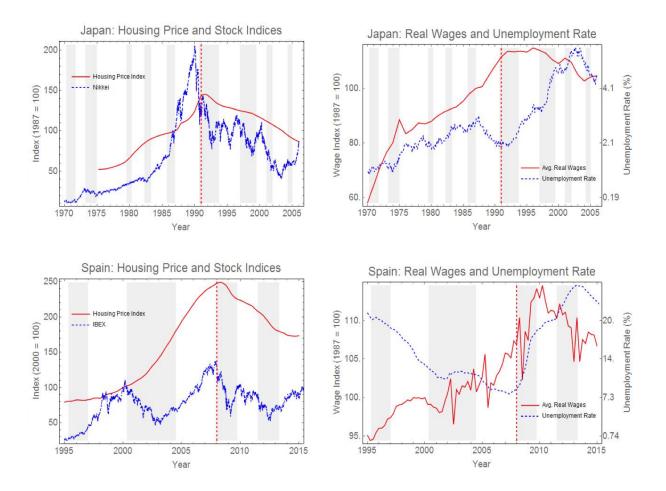


Fig 2. Asset prices (left panels) and employment and wages (right panels) in Japan and Spain. Dashed vertical lines indicate beginnings of post-bubble recessions (1991 for Japan and 2008 for Spain). Grey bars indicate recessions, according to OECD. Real wages calculated from nominal wages and consumer price indices. Sources: Statistics Bureau of Japan, OECD, FRB St. Louis, FRB Dallas.

# Appendix

Proof of Proposition 1.

1. (12) follows immediately from the definition of  $s^*$  and (4). Firms' first-order-condition (5) then implies

$$k_t/l_t = \left(\frac{w_t}{1-\alpha}\right)^{\frac{1}{\alpha}}, \ \forall \ T \le t \le T + s^* - 1.$$
(19)

Substitute (19) into aggregate capital's motion:

$$k_{t+1} = \bar{a}w_t l_t = \bar{a}w_t \frac{l_t}{k_t} k_t$$
  
=  $\bar{a}w_t \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} k_t$   
=  $\bar{a}(1-\alpha)^{\frac{1}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}} k_t, \ \forall \ T \le t \le T+s^*-1,$ 

Combining with (12) yields:

$$k_{t+1} = \gamma^{-\frac{1-\alpha}{\alpha}t} \left( \bar{a}(1-\alpha)^{\frac{1}{\alpha}} w_b^{-\frac{1-\alpha}{\alpha}} \right) k_t,$$

whose recursion leads to (13). Combined with (19), this implies (14). The rest of part 1 follows straightforwardly.

2. The definition of definition of  $s^*$  and (4) imply:

$$s^* = \min\{s \ge 0 | w_{T+s}^f(k_{T+s}) \ge w_{T+s}\}$$

where  $w_{T+s}^f(k_{T+s}) \equiv (1-\alpha)k_{T+s}^{\alpha}$  is the wage associated with full-employment given capital  $k_{T+s}$ . Equivalently:

$$s^* = \min\{s \ge 0 | (1 - \alpha)k_{T+s}^{\alpha} \ge \gamma^s k_b\}.$$

Combining with (13) yields:

$$s^* = \min\{s \ge 0 | \gamma^{-(1-\alpha)\frac{s(s+1)}{2}} \eta^{\alpha s} \ge \gamma^s\},$$

which leads to (15). Finally, (16) follows from (13) and (15)

Proof of Corollary 2. Immediate consequences of (6), (16) and (15)

Proof of Proposition 3. The full-employment transition is restored if and only if (17) does not bind, i.e.,

$$\Pi_{T+s} \ge \gamma \frac{w_{T+s-1}^f}{w_{T+s}^f}, \forall s \ge 0,$$
(20)

or equivalently:

$$\Pi^* \geq \gamma \cdot \sup_{s \geq 0} \left\{ \frac{w^f_{T+s-1}}{w^f_{T+s}} \right\}.$$

It is straightforward to verify that  $\frac{w_{T+s-1}^f}{w_{T+s}^f} = \left(\frac{K_{T+s-1}^f}{K_{T+s}^f}\right)^{\alpha} = \frac{1}{\bar{a}(1-\alpha)(K_{T+s}^f)^{\alpha-1}}$ , and  $\{K_{T+s}^f\}_{s\geq 0}$  is declining. Hence  $\sup_{s\geq 0}\left\{\frac{w_{T+s-1}^f}{w_{T+s}^f}\right\} = \frac{1}{\bar{a}(1-\alpha)(K_b)^{\alpha-1}} = \gamma \eta^{\alpha}$ . Finally to assume the cosh in solution of the term of the term of the term.

Finally, to ensure the cash-in-advance constraint binds, a sufficient condition is:

$$\Pi^* > \sup_{s \ge 0} \left\{ \frac{1}{R_{T+s}^f} \right\} = \frac{1}{R_b} = \frac{1}{1-n},$$

which guarantees that the return from lending dominates the return from holding cash.

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