Efficient black markets?

Carl Davidson [a,b], Lawrence Martin [a], John Douglas Wilson [a,*]

[a] *Department of Economics, Marshall-Adams Hall, Michigan State University, East Lansing, MI 48824, United States*
[b] *GEP, University of Nottingham, United Kingdom*

Abstract

This paper investigates analytically the welfare effects of black-market activities that firms undertake to evade taxes. The desirability of a black market is linked to the attributes of the goods supplied by black-market firms. The analysis identifies cases where a black market reduces (increases) the distortionary impact of taxation on the allocation of resources across the goods that the government is attempting to tax, leading to a welfare gain (loss).
© 2007 Published by Elsevier B.V.

1. Introduction

By their nature, black-market activities are difficult to measure. Nevertheless, there is widespread agreement that black-market activities account for a significant portion of GDP in many countries.[1] Less clear, however, are the welfare effects of such activities, particularly those motivated by tax evasion. In his leading undergraduate text on public finance, Rosen (2005, p. 353) states a second-

---

* Corresponding author. Department of Economics, Marshall-Adams Hall, Michigan State University, East Lansing, MI 48824, United States. Tel.: +1 517 432 3116; fax: +1 517 432 1068.
  E-mail address: wilsonjd@msu.edu (J.D. Wilson).

[1] Other terms for the "black market" are the "underground economy" and "shadow economy." As reported by Schneider and Enste (2000), estimates of the size of the shadow economy as a percentage of GDP in the 1990–93 period ranged from between 8–10% for the U.S., Austria, and Switzerland to 24–30% for Greece, Italy, Spain, Portugal and Belgium. But fractions above two-thirds of GDP have been calculated for Nigeria, Egypt and Thailand. There are several reasons for the growth of the shadow economy, but the authors single out tax evasion as one of the most important. For recent reviews of the tax evasion literature, see, Andreoni et al. (1998), Slemrod and Yitzhaki (2002).

best argument for why a black market, or an "underground economy," might be efficiency-enhancing in some cases: "Then under certain conditions, the existence of an underground economy raises social welfare. For example, if the supply of labor is more elastic to the underground economy than to the regular economy, optimal tax theory suggests that the former be taxed at a relatively low rate." But Slemrod and Bakija (2004, p. 179) expound an alternative view: "…because tax evasion depends on opportunities that are tied to particular activities, it provides an incentive—which is inefficient from a social point of view—to engage in those activities for which it is relatively easy to evade taxes."[2] The first view focuses on the potential for the black market to increase incentives to provide resources to taxed activities, whereas the latter view emphasizes the distorting effect of the tax system on the allocation of resources across different taxed activities.[3]

The relative merits of these two views might seem to be purely an empirical matter, but the current paper argues that surprisingly sharp results can be obtained by placing both views within a single model. Following Slemrod and Bakija, we model a set of activities that differ in the expected rewards from operating in the black market. In particular, our activities are distinguished by the levels of assets that the tax authority is able to seize in the event that tax evasion is discovered. Low-asset activities (per unit of output) self-select into the black market because the potential fine from detection is relatively low. For simplicity, the model abstracts from the myriad other considerations behind the decision to enter the black market; in particular, all firms are randomly audited for tax purposes. Following Rosen, we next assume that the tax system distorts the decision of whether to devote resources to any taxed activity. In particular, activities are ranked by a continuous parameter called "quality," interpreted here as the attribute of a good produced by firms. Each consumer purchases at most a unit of a variable-quality good, with choices based on a heterogeneous "taste" parameter. Recognizing the costs involved in administering a quality-differentiated tax system, we assume that the government's expenditure needs are met by taxing all variable-quality goods at a uniform statutory rate. Such a tax system causes consumers with low tastes for quality to drop out of the market — that is, they devote no resources to purchasing variable-quality goods, whereas those who remain reduce the qualities of the goods that they purchase.

With such a setup, we provide conditions that determine whether the black market consists of low- or high-quality goods. In the latter case, *neither* the Slemrod–Bakija nor Rosen arguments are relevant: a small black market (maintained through an appropriately low expected fine) does not distort consumption towards too much quality, because quality choices are already too low under a uniform tax; and it does not bring new consumers into the market for variable-quality goods. This case illustrates how a black market can be desirable, even when audits are costless, because it partially corrects the distorting effect that a uniform tax system has on the allocation of resources across taxed activities.

In stark contrast, *both* the Rosen and Slemrod-Bakija arguments appear relevant when the black market contains low-quality goods: allowing it to flourish brings some consumers back into the market for variable-quality goods, but it also distorts the choices of some existing consumers away from higher-quality "legal goods" and towards the lower-quality goods in the black market. But we demonstrate analytically that these conflicting welfare effects do not favor a black market.

---

[2] Many other studies emphasize the efficiency losses from tax evasion. See, for example, Yitzhaki (1987), Usher (1994), Slemrod and Yitzhaki (1995), Feldstein (1995), Palda (1998).

[3] Using U.S. data from 1980, Alm (1985) estimated the efficiency losses from the diversion of resources into the underground economy to lie between 100 billion and 220 billion dollars per year, where the latter figure represented nine percent of GDP. Both Alm's calculations and Kesselman's (1989) qualitative results about the extent and incidence of tax evasion are based on general equilibrium models in which tax evasion is associated with the production of particular goods. In our model, such goods are determined endogenously.

Thus, the potential for black markets to misallocate goods is found to depend systematically on where the black market is located. Black markets containing high-quality goods *improve* the allocation. With low-quality goods, the misallocation is so severe that it offsets any welfare gains from the ability of black markets to draw resources into taxed activities as a whole.

Rosen emphasizes the importance of black markets with low-income participants, noting that, "many observers believe that the underground economy is a crucial part of life in American inner cities." To the extent that low-wage labor is relatively elastic, his second-best argument then suggests that black markets may satisfy both efficiency and equity goals. Our model does not deal with this potential equity argument, because it follows the optimal commodity tax literature by focusing on efficiency issues (i.e., risk-neutral firms and consumers). But if our low-quality goods are viewed as goods consumed by low-income taxpayers, our results raise the possibility that a government's efficiency and equity goals may be at odds with regard to black markets.

In our model, a firm's assets consist of the capital used in production and proceeds from the sale of output. We assume that the fine levied on any firm caught evading taxes consists of a fraction of these assets. The government controls the size of the black market through its choice of the expected fine, equaling the product of the fine and audit rate. But a familiar result in the literature on crime and punishment is that fines should be set as high as possible, to minimize the resource costs necessary to achieve any given level of deterrence. The basic idea is that audits involve resource costs, but fines represent a socially costless income transfer.[4] By capping the maximum fines at the firm's total assets, we are essentially assuming that higher fines are precluded by either the economy's legal system (e.g., limited liability) or the excessive costs needed to obtain them. Such considerations may also reduce the share of physical capital that the government is able to seize, relative to the share of financial assets, but our results can be extended to the case where the former share is positive but less than the latter share. As a further extension, we show that these shares can always be chosen to induce only high-quality firms to self-select into the black market, thereby ensuring that the black market is welfare-improving. Whereas such a fine structure is not as administratively simple as the one in our initial model, neither does it appear to be administratively infeasible.

Our approach is distinct from that in several other papers that provide a rationale for illegal activity in an optimal tax system. Using a model in which the government taxes wage income to raise a given amount of revenue, Weiss (1976) shows that tax evasion may be desirable, because the resulting random taxes (e.g., fines for tax evasion with random audits) may reduce the deadweight loss from labor-supply distortions. The potential desirability of random commodity taxes is developed further by Stiglitz (1982a) and Chang (1986). Some papers point out that evasion can contribute to redistribution. Stiglitz (1982b) and Brito et al. (1995), for example, show that random taxes can relax self-selection constraints in the optimal nonlinear income tax problem, whenever high-ability taxpayers have different risk preferences than their lower ability counterparts. Kopczuk (2001), similarly, finds a role for evasion when the lower-ability taxpayers are more efficient at evasion.[5] In Boadway and Keen (1995), the government commits to lax enforcement and thereby reassures investors worried about its future incentive to impose high

---

[4] See Becker (1968). Polinsky and Shavell (2000) discuss exceptions, but the Becker result applies to the current model.

[5] There are now sizable literatures that incorporate tax evasion into models of optimal commodity taxation and optimal income taxation. See Slemrod and Yitzhaki (2002) for a review. For additional studies of optimal commodity taxation, see Yitzhaki (1979), Wilson (1989), Boadway et al. (1994), Cremer and Gahvari (1993), Kaplow (1990). In contrast to our work, none of these papers consider the division of risk-neutral firms between legal and black-market activities.

capital taxes. Finally, Andreoni (1992) builds an interesting model where tax evasion is a high-risk substitute for loans not otherwise available to liquidity-constrained agents. In his model, the government's ability to impose non-monetary penalties allows it to avoid adverse-selection problems that trouble private lending markets.

As described in the next section, our model departs from the optimal commodity tax literature by making adverse-selection problems a major part of the analysis. But unlike the optimal income tax literature, we assume risk-neutral agents and focus solely on efficiency issues. Section 3 uses the model to determine whether the black market contains high- or low-quality goods. Our main proposition on the benefits of black-market activities appears in Section 4. At the end of Section 4, we discuss some extensions of the analysis. The extension to a more complex fine structure is dealt with in Section 5, and Section 6 provides some concluding remarks.

2. The model

Consider an economy with a continuum of consumers, indexed by a taste parameter, α. Each consumer is endowed with E units of a composite commodity, or "endowment good," which may be interpreted as labor. To earn income, consumers supply this good to competitive private firms and the government at a price normalized to equal one. This income is used to purchase zero or one unit of a variable-quality good, at a price equal to $P(\theta)$ for a quality-θ good, and $E - P(\theta) > 0$ units of a homogeneous consumption good. In addition, all consumers receive the same G units of the public good. Utility is then given by

$$U(E - P(\theta), \theta, G; \alpha) = E - P(\theta) + \alpha v(\theta) + \sigma(G) \tag{1}$$

for a type-α consumer, where v is concave and σ is strictly concave. The parameter α possesses a continuous density function, $h(\alpha)$, on [0,1], and the population is normalized to equal one. From Eq. (1), higher values of α represent a greater marginal willingness to pay for quality. Quality is treated as a continuous variable, in which case utility maximization yields the following first-order condition at each $\theta > 0$ where $P(\theta)$ is differentiable:

$$\alpha v'(\theta) = P'(\theta). \tag{2}$$

By the second-order condition, the chosen θ is an increasing function of α. Under our subsequent assumptions about the cost structure, consumers with values of α below some positive level choose not to consume variable-quality goods.

The homogeneous consumption good is produced from labor via a linear technology. In contrast, there are two ways in which the variable-quality firms use the endowment good. First, it is sold directly to these firms as labor. Second, consumers may transform it into capital at a one-to-one rate, and then this capital is invested in these firms. For our purposes, the critical difference between labor and capital is that some of the capital remains after production has taken place (i.e., capital is durable), whereas an hour of labor services spent in production is an hour that is unavailable for other uses. Each unit of capital depreciates at rate δ, leaving $1 - \delta$ units that can be transformed into numeraire consumption at the end of the production process. The consumer must be indifferent between supplying labor or capital. With the wage rate equal to one, the payment for each unit of capital must also be one.

The variable-quality goods are produced using a fixed-proportions technology, with each unit of a quality-θ good requiring $W(\theta)$ units of labor and $A(\theta)$ units of capital. (We later allow for

substitutability in production.) Both $W(\theta)$ and $A(\theta)$ are increasing and convex in θ, and they also converge to positive numbers as θ goes to zero. Thus, some costs are independent of quality. After the production and sale of output, the firm is left with assets of $P(\theta)+(1-\delta)A(\theta)$. Since equilibrium profits equal zero under perfect competition, factor payments, $W(\theta)+A(\theta)$, exhaust these assets in the absence of taxes. It follows that $P(\theta)=W(\theta)+\delta A(\theta) \equiv C(\theta)$.[6] For simplicity, we are assuming a zero interest rate over the production period, in which case the user cost of capital is the rate of depreciation.

With some costs independent of quality, the average cost of utility, $C(\theta)/[\alpha v(\theta)]$, goes to infinity as the quality level goes to zero. It follows that nobody buys goods with qualities close to zero, and consumers with sufficiently low values of the taste parameter, α, choose not to consume variable-quality goods. We assume that $C(\theta)/v(\theta)$ is U-shaped in quality.

The manner in which $A(\theta)/W(\theta)$ varies with θ plays an important role in our analysis, for it determines which goods are drawn to the black market. There is no natural assumption to make concerning this issue. It is easy to think of examples in which the capital intensity of the production process is increasing in quality and others in which it is decreasing in quality. For an example of the former, consider the market for landscaping services, where it appears that capital intensity increases with the quality of the services provided. At the high end of the quality spectrum are the large outfits that come in and use heavy equipment to clear brush, grade a tract, and spread grass seed. At the other end of the spectrum are small teams of workers who rely largely on their own labor. In contrast, we would argue that the market for furniture provides an example in which capital intensity is decreasing in quality. Low-quality furniture is mass produced in large factories relying heavily on machinery and assembly lines, while high-quality furniture is often hand-crafted by artists who rely primarily on their own labor to produce the final product. Since there is no natural assumption to make concerning this issue, we consider both cases — that is, we examine the case in which $A(\theta)/W(\theta)$ is increasing in θ and the case in which it is decreasing in θ. We derive remarkably similar positive results in both cases, but starkly contrasting normative ones.

To introduce taxes into the model, assume that the government finances its expenditure needs by imposing a tax at a constant rate, $t<1$, on the revenue from the sale of variable-quality goods. A discriminatory tax scheme (in which t depends on θ) is assumed not to be available, perhaps because the informational requirements would be too costly. By auditing a firm, the government is assumed to learn the value of a firm's assets, but the value of θ cannot be deduced from this information because the government does not know the number of customers the firm has served.

By the zero-profit requirement, the price of a quality-θ good in the absence of tax evasion, or the "legal price," is given by

$$P_{\ell}(\theta)(1-t) = W(\theta) + \delta A(\theta). \qquad (3)$$

Firms may evade taxes by engaging in black-market activities, but they then risk detection and punishment.[7] Before capital suppliers are paid, the government audits a fraction π of firms and assesses monetary fines, calculated as a fraction of a firm's assets. Consistent with practice, we assume that the tax collector is first in line among the firm's creditors. The firm's scale of production is indeterminate, given the assumption of a linear transformation between the

---

[6] Zero profits would also hold if we instead assumed that firms are engaged in Bertrand competition, with more than one firm producing each good, or if we assumed a contestable market.

[7] The analysis could be generalized by assuming that an exogenous percentage of income is not reported.

endowment good and output. This scale therefore conveys no useful information to the government, and we may examine assets per unit of output sold. As described above, these assets consist of revenue and non-depreciated capital, $P_b(\theta)+(1-\delta)A(\theta)$, where $P_b(\theta)$ is the "black-market price," that is, the equilibrium price at which a quality-θ good is sold by firms choosing to evade taxes. A fine is paid at the rate f on these assets. We assume here that the firm owners are unable to pass the burden of the fine on to labor by reneging on the payment of $W(\theta)$ and instead use this amount to pay the fine. One interpretation is that the firm owner is also the supplier of the labor ("self-employment"). Alternatively, the firm owner pays workers prior to the sale of output, using the proceeds from the sale of his own labor services to other firms.

The expected fine is $\pi f[P_b(\theta)+(1-\delta)A(\theta)]$. Since firms are risk neutral, competition drives their expected profits to zero. Thus, the black-market price must be sufficient to cover the expected fine and the unit input cost, $W(\theta)+\delta A$:

$$P_b(\theta) = \frac{W(\theta) + [\delta + \pi f(1-\delta)]A(\theta)}{1-\pi f}. \tag{4}$$

As previously noted, the government has an incentive to set the fine rate f as high as possible, to minimize the audit costs needed to obtain the desired level of deterrence. But to emphasize the desirability of a black market, even in the presence of low audit costs, the subsequent analysis will sometimes assume that audits are costless.

3. Where is the black market?

The government can create a black market by choosing a sufficiently low audit probability or fine. But at what quality levels will the black market exist, and should the government allow it to exist? This section addresses the first question.

Assuming continuous quality, consumers make their quality choices according to Eq. (2). As for firms, they base their decisions about whether to operate in the legal or black market on a comparison of the tax payments and expected fines. Since all firms are identical, each firm producing a given θ will make the same choice. If good θ is produced in the legal market, then the equilibrium price $P(\theta)$ is the legal price $P_\ell(\theta)$, as determined by Eq. (3), and the expected fine is at least as great as the tax liability:

$$\pi f[P_\ell(\theta + (1-\delta)A(\theta)] \geq t P_\ell(\theta). \tag{5}$$

If Eq. (5) did not hold, then a firm could deviate to the black market and, taking the market price as given, increase its profits. Similarly, if good θ is produced in the black market, then $P(\theta)$ is the black-market price, $P_b(\theta)$, determined by Eq. (4), and

$$\pi f[P_b(\theta) + (1-\delta)A(\theta)] \leq t P_b(\theta). \tag{6}$$

Clearly, $\pi f<1$ is necessary for a black market. At the θ where either Eq. (5) or Eq. (6) are satisfied with equality, denoted θ^*, Eqs. (3) and (4) give $P_\ell(\theta)=P_b(\theta)$, and so both Eqs. (5) and (6) hold with equality. Firms are then indifferent between the two markets.

The legal- and black-market price schedules are drawn in Fig. 1a under the assumption that $\frac{A'(\theta)}{A(\theta)}>\frac{W'(\theta)}{W(\theta)}$ for all θ. As θ rises above θ^*, the left sides of Eqs. (5) and (6) rise faster than the right sides, and firms choose to operate in the legal market, with $P(\theta)=P_\ell(\theta)$. The basic idea is that the
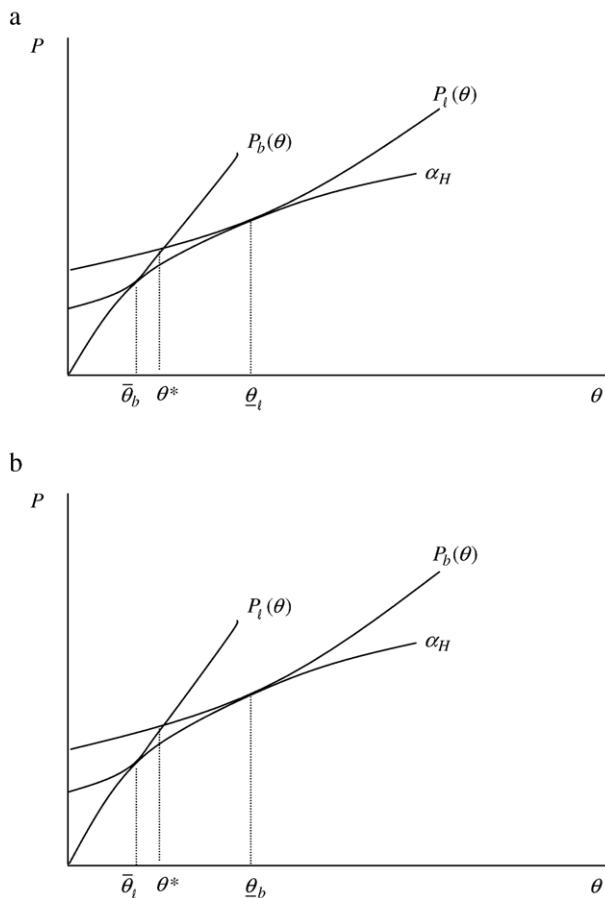
a

b

Fig. 1. a: Black market in low-quality goods. b: Black market in high-quality goods.

tax base, $P_\ell(\theta)$ or $P_b(\theta)$, includes both labor and capital costs, but the expected fine depends not only on revenue from sales of the good, but also on non-depreciated capital, which becomes relatively more important as θ rises. Thus, the black market becomes increasingly unattractive as θ rises. From Eqs. (3) and (4), the excess of the expected fine over the tax liability implies that the black-market price lies above the legal-market price at values of θ above θ^*, as shown.[8] By similar reasoning, the legal-market price is above the black-market price at values of θ below θ^*, and firms selling these goods choose to operate in the black market.

Putting these observations together, we see that the schedule of market prices will be the lower envelope of the black- and legal-market price schedules. This property would also hold if $\frac{A'(\theta)}{A(\theta)} < \frac{W'(\theta)}{W(\theta)}$ for all θ, in contrast to the case depicted in Fig. 1a. Moving to this new case would reverse the positions of $P_b(\theta)$ and $P_\ell(\theta)$, as shown in Fig. 1b, with the $P_\ell(\theta)$ curve now relatively

---

[8] The proof that the two curves have the properties depicted in Fig. 1a is a special case of the proof of Proposition 1A in the Appendix. Briefly, differentiate Eqs. (3) and (4) with respect to quality to find that the legal-market schedule has a greater slope than the black-market schedule wherever they cross. Thus, there can only be one crossing, as shown.

steeper. Then high-quality goods characterize the black market.[9] In our subsequent analysis, we allow either inequality to hold.

No consumer ever buys θ^* or any quality level nearby. The reason is that the price schedule faced by consumers has a kink at θ^*. For this reason, if both legal- and black-market goods are consumed, some consumer is indifferent between consuming good $\underline{\theta}_{\ell}$, the lowest quality in the legal market, and a lower-quality good $\bar{\theta}_b$, which is the highest quality in the black market (for the case depicted in Fig. 1a). Between these qualities, no goods are sold, that is, the set of black-market goods differs by a discrete amount (in quality) from the set of legal goods.

4. Desirable and undesirable black markets

We now prove our main results about the desirability of black markets, beginning with the high-quality case.

Proposition 1. *If black-market goods are high quality, then a black market exists under the government's optimal tax and enforcement system, even if audits are costless.*

Proof. We need only demonstrate the desirability of at least "small" black markets. Consider Fig. 1b. To construct a small black market, lower the expected fine to the level where the black market consists of only a single quality.[10] Then the consumer with the highest α (shown as α_H in Fig. 1b) is indifferent between the highest-quality legal good, $\bar{\theta}_{\ell}$, and the single black-market good, $\underline{\theta}_b$. If we now reduce the expected fine further by a small amount, thereby reducing black-market prices (see Eq. (4)), consumers who are approximately indifferent between black and legal markets switch to the black market. Given the discrete difference between the two markets, however, government revenue changes by a discrete amount. This amount must be positive because firms providing $\bar{\theta}_{\ell}$ have decided not to participate in the black market, since their expected fines would exceed their tax payments, and the expected fine rises with quality. Since the extra revenue can be used to lower taxes and fines, it is clearly possible to make everyone better off by introducing the black market. □

Matters are more complicated if black-market goods are low quality. Start once again with an expected fine that is just low enough to produce a black market consisting of a single quality, $\bar{\theta}_b$. As illustrated by Fig. 1a, the consumer of the good with the lowest quality in the legal market, $\underline{\theta}_{\ell}$, is now not only indifferent between this quality and no consumption, but also quality $\bar{\theta}_b$. If we now slightly reduce the fine, thereby shifting down the $P_b(\theta)$ curve in Fig. 1a, consumers jump from the legal market into the black market, which now discontinuously lowers their tax payments. However, new consumers are induced to jump from no consumption to black-market consumption, implying that they now pay taxes. At first blush, it would appear that in this case, the desirability of a black market would depend on the relative importance these two changes. But the next proposition proves that this is not the case.

Proposition 2. *If black-market goods are low quality and the government's tax and enforcement system is initially optimal, conditional on no black market, then introducing a small black market must lower welfare.*

---

[9] We are assuming here that the inequality is either reversed for all θ or for none. We later discuss a third possibility, where the black market exists in one or more intermediate intervals of θ.

[10] Either the fine or the audit rate may be reduced here. But to the extent that audits are costly, lowering the audit rate generates an additional benefit in the form of lower audit costs.

Proof. Consider an initial equilibrium without a black market, and let α_H denote the taste parameter for the consumer of the good with the lowest legal quality. As depicted in Fig. 1a, this consumer is indifferent between this good ($\underline{\theta}_\ell$) and no consumption. Now lower the expected fine to the point where this consumer is also indifferent to consuming a black-market good ($\bar{\theta}_b$), also illustrated in Fig. 1a. To shorten notation, let $v_\ell \equiv v(\underline{\theta}_\ell)$, $v_b \equiv v(\bar{\theta}_b)$, $P_\ell \equiv P_\ell(\underline{\theta}_\ell)$, and $P_b \equiv P_b(\bar{\theta}_b)$. Given the consumer's indifference,

$$\alpha_H v_\ell - P_\ell = \alpha_H v_b - P_b = 0. \tag{7}$$

Let T_ℓ represent the unit tax rate (tP_ℓ), and let T_b to denote the per unit expected fine ($\pi f(P_b + (1-\delta)A)$). Their values are given by the difference between the prices determined by Eq. (7) and the unit input costs, $C_\ell \equiv W(\underline{\theta}_\ell) + \delta A(\underline{\theta}_\ell)$ and $C_b \equiv W(\bar{\theta}_b) + \delta A(\bar{\theta}_b)$:

$$T_\ell = \alpha_H v_\ell - C_\ell \quad \text{and} \quad T_b = \alpha_H v_b - C_b. \tag{8}$$

To create a black market with positive measure, we may lower T_b or increase T_ℓ. Using the assumption that tax revenue (G) is initially optimized, it can be shown that both changes move welfare in the same direction. Hence, we consider a reduction in T_b, chosen so that P_b falls by a unit. As a result, there is a rise in the taste parameter possessed by the consumer who is indifferent between the two markets (α_H), as can be seen by implicit differentiation of the first equality in Eq. (7): $d\alpha_H = \frac{1}{v_\ell - v_b}$. In addition, the drop in T_b lowers α_L, the value of α for the consumer who is indifferent between black-market consumption and no consumption. By the definition of α_L, we know that $\alpha_L v_b - P_b = 0$ Starting from the initial equilibrium, with $\alpha_L = \alpha_H$, implicit differentiation gives $d\alpha_L = -\frac{1}{v_b}$. With α_H now above α_L, a type-α_H consumer strictly prefers legal consumption over no consumption.

To summarize, the decline in the expected fine causes some consumers to move from no consumption to black-market consumption, as represented by $d\alpha_L$, while causing others to move from legal-market consumption to black-market consumption, represented by $d\alpha_H$. The total change in revenue is a positive multiple of

$$-T_b d\alpha_L + (T_b - T_\ell)d\alpha_H, \tag{9}$$

where the size of the multiple depends on the value of $h(\alpha)$ at $\alpha_L = \alpha_H$.

The welfare effect of this small black market has the same sign as this change in tax revenue, since any consumers who are switching among no consumption, black-market consumption, and legal-market consumption are initially indifferent about the three choices. By substituting Eq. (8) and the expressions for $d\alpha_L$ and $d\alpha_H$ into Eq. (9) and then canceling terms, we find that the sign of Eq. (9) is negative if

$$\frac{C_\ell}{v_\ell} < \frac{C_b}{v_b}. \tag{10}$$

To see that this condition must hold, observe from Eqs. (2) and (3) that $\alpha v'(\theta)(1-t) = C'(\theta)$ at all θ chosen by consumers. But Eqs. (3) and (7) give $\alpha_H v_\ell(1-t) - C_\ell = 0$. Combining these equalities yields, $v'_\ell/v_\ell = C'_\ell/C_\ell$. This is a condition for the minimization of $C(\theta)/v(\theta)$ Consequently, Eq. (10) must hold. □

Some intuition behind Proposition 2 is suggested by Eq. (10). In the absence of a black market, we have found that the average cost of generating utility, which is a multiple of C/v for each consumer, is minimized at the lowest-quality legal good. In this sense, this good is produced most efficiently. Note, in particular, that the tax t has no impact the minimum legal quality consumed in

this model; the tax does cause some consumers to exit the market, but the lowest quality consumed by those who remain is no different than before. In contrast, the introduction of a black market induces some consumers to reduce their quality levels below this efficient level, that is, they move up the U-shaped "average-cost curve" for C/v. The reason for this difference is that tax payments are proportional to cost C in this model, whereas the expected fine payments rise at a faster percentage rate than costs as quality rises, given our assumption that the capital-labor ratio increases with quality. As a result, the black-market price schedule tilts in favor of consumption on the downward-sloping portion of the average-cost curve. Given this source of inefficiency, we are able to prove that a black market cannot be welfare improving.

We have shown that it is possible to predict the welfare implications of black markets, depending on whether they contain low- or high-quality goods. We could construct black markets consisting of *only* intermediate-quality goods, by allowing the capital intensity to rise with quality over some intervals of qualities but fall over other intervals. In this case, it is possible for the price schedules in Fig. 1 to have multiple crossing. Creating such a black market would effectively lower tax burdens on goods in this intermediate interval, which would have ambiguous welfare effects. More can be said by moving to a two-quality model, in which case all black markets are "large" in terms of the share of quality levels that they encompass, though not necessarily in terms of the share of the population participating in them. A new and interesting possibility in this case is that it may be possible to use the black market to replicate an optimal discriminatory tax system (see Davidson et al., 2005 for details).

As another extension, assume now that labor and capital are substitutable in the production of variable-quality goods. In the absence of a black market, this assumption would not matter, because the tax t is imposed on sales, leaving the relative prices of labor and capital unchanged. But since the fine paid by black-market firms is based on the firm's assets, including capital, it increases the cost of capital for tax-evading firms without altering the social cost of production. Firms will therefore shed capital as they enter the black market, resulting in a new factor intensity that is sub-optimal. Despite this additional negative feature of the black-market activity, Propositions 1 and 2 hold in this more general setting. See the Appendix for details.

So far, we have assumed random audits. Alternatively, it seems reasonable to view tax evaders as more easy to detect if they possess relatively large amounts of capital. But this possibility reinforces our finding that labor-intensive firms operate in the black market, and also that that black-market firms choose inefficiently low capital-labor ratios if factor substitutability is possible. The current paper shows that the government does not necessarily need to go to the expense of designing complicated non-random auditing strategies as a means of inducing particular firms to operate in the black market.

5. Differential fines and taxes

We have demonstrated the desirability of a black market in cases where the government does not distinguish between a firm's different types of assets when levying the fine. In our model, these assets consist of revenue from the sale of output (P) and the capital used in production (A). Let us now extend the government's enforcement powers by allowing them to make this distinction. In particular, suppose that the total fine is a linear function of P and A: $f_r P + f_a A$. Then we may modify Eq. (3) to get the following black-market price schedule:

$$P_b(\theta) = \frac{W(\theta) + [\delta + \pi f_a(1-\delta)A(\theta)]}{1-\pi f_r} \qquad (11)$$

For simplicity of exposition, we maintain the assumption of fixed coefficients in production, although the reasoning in the Appendix can be use to extend the results to the case of a variable input mix. We could also replace our single tax rate on sales with separate taxes on sales and capital, but the point we wish to make here is that only the new fine schedule is needed to ensure that a welfare-improving black market can always be created, regardless of whether the capital intensity of production increases or declines with quality.[11] This result can be obtained with or without a more complicated tax system.

In the case of a uniform fine structure ($f_f = f_a$), we already know from the previous analysis that a small black market is desirable if $A(\theta)/W(\theta)$ declines with θ, since then the black market consists of high-quality goods, enabling us to apply Proposition 1. Thus, we assume instead that $A(\theta)/W(\theta)$ rises with θ, in which case any small black market arising under a uniform fine structure consists of low-quality goods, and Proposition 2 implies that such a black market worsens welfare.

If we now allow the fine structure to distinguish between P and A, then in this case, we can move the black market to the high-quality goods and, by so doing, raise welfare. Using Eqs. (3) and (11), a high-quality black market emerges if the fine per unit of output is sufficiently low and

$$\frac{W'(\theta^*) + [\delta + \pi f_a(1-\delta)]A'(\theta^*)}{W(\theta^*) + [\delta + \pi f_a(1-\delta)]A(\theta^*)} < \frac{W'(\theta^*) + \delta A'(\theta^*)}{W(\theta^*) + \delta A(\theta^*)}, \tag{12}$$

where θ^* is the quality level at which the legal and black-market price schedules cross. If $f_a = 0$, then both sides of Eq. (12) are equal. But with $A(\theta)/W(\theta)$ increasing with θ, the inequality in Eq. (12) can be produced by setting $f_a < 0$, no matter how small. Thus, start with a zero fine on capital assets, and set the tax rate equal to the expected fine on revenues; i.e., $t = \pi f_r$. Under these assumptions, the legal and back-market price schedules coincide. We may then introduce a small black market at high quality levels by increasing f_r while lowering f_a slightly below zero. In words, the fine falls as the ratio of capital to revenue rises, holding constant total assets. The negative f_a flattens the black-market price schedule relative to the legal market price schedule, because high-quality goods are capital intensive, implying that they receive a relatively large benefit from the negative fine on capital. By raising f_r enough, we may ensure that the black-market schedule lies below the legal-market schedule at all quality levels except those near the top of the quality range. In contrast, a positive f_a would produce a black market at low qualities, as previously described.

Having produced a small high-quality black market, we may then use the proof of Proposition 1 to again show that this black market raises welfare. This proof consists of showing that the introduction of the black market induces some consumers to switch to higher-quality goods, causing government revenue to increase. That this revenue is now partially obtained from a non-uniform fine structure does not alter the argument.

6. Concluding remarks

Proponents of a Haig–Simons income tax argue on both equity and efficiency grounds that all sources of income should be treated equally for tax purposes. Under this view, black-market activities are clearly inefficient, because they destroy this "neutrality." Modern public finance has

---

[11] We continue to assume that the capital-labor ratio is either monotonically increasing or monotonically decreasing in quality.

taught us, however, that optimal tax systems are usually characterized by unequal treatment of income. But unequal treatment involves higher administrative and collection costs, arising in part from informational asymmetries that limit the government's power to tax. To the extent that a black market can be used to circumvent such informational problems, it can bring the tax system closer to the one that would be optimal if the government could optimally discriminate among different income-generating activities. The current paper has shown that a black market may or may not serve this role, depending on the attributes of the goods supplied there. In our model, goods are distinguished by their "quality," and it is always the case that a small black market containing high-quality goods raises welfare, whereas a black market with low-quality goods worsens welfare. In both cases, however, it is the relatively labor-intensive firms that choose to occupy the black market, given our initial assumption that the fine imposed on tax evaders depends on their total assets at the time they are caught, not on the mix of different assets. But by using a fine that depends on this mix, the government is able to locate the black market at high-quality goods, even if they are capital-intensive, thereby ensuring a welfare improvement.

Given the potential for welfare improvements, these results are not fully consistent with the large welfare losses from a black market identified by Alm (1985), who assumes a labor-intensive back market (see Footnote 3), or with Slemrod and Bakija's argument, described in our introduction. Alm and Rosen also identify circumstances under which a black market may be increase welfare, but their description of welfare-improving black markets seems more closely related to our welfare-worsening black markets at low quality levels.[12]

Our analysis has abstracted from labor-market issues, including both sources of inefficiency and the government's policy response. These considerations provide additional avenues by which black markets might raise welfare. Suppose, for example, that illegal migrants are hired to perform work in the black market. If immigration is restricted, then those individuals who are allowed to immigrate can be expected to earn incomes above those available to similar workers left behind in the source countries. In other words, individuals who are picked to be immigrants receive economic rents (assuming the selection process does not impose costs on immigrants that either eliminate such rents or transfer them to the host country). In contrast, if we introduce a black market with a sizable expected fine for employers caught hiring illegal immigrants, then firms will only hire these immigrants if their wages are low enough to offset the expected fine. In this way, a black market provides a means by which the host country can capture the economic rents generated by restrictions on migration within the legal market. Looked at another way, the expected fine serves the role of a tariff on the importation of foreign labor. By the usual optimal tariff arguments from international trade theory, the importing country can raise welfare by choosing a positive expected fine, provided it has terms-of-trade effects in the form of a change in the foreign wage.[13]

---

[12] As previously described, Rosen's argument centers on the potential for a black market to increase incentives to provide resources to taxed activities, which is also the case for our low-quality black market. Similarly, Alm's argument concerns, "illegal employment for people who otherwise would be unemployed, or some underground activities that arise in an attempt to avoid inefficient government regulations may also increase welfare" (p. 259).

[13] Bond and Chen (1987) develop a similar argument, but rather than model a "black market" with illegal immigrants, they assume a single production sector that responds to possible fines on the hiring of illegal workers by paying different wages to illegal and legal workers. When this wage discrimination is not possible, they find that the government's expenditures on detecting illegal workers are less likely to raise welfare.

Alternatively, suppose that the government imposes labor-related regulations, such as health-and-safety standards for the workplace. If these regulations are a normal good, then we should expect a uniform set of standards to be overly stringent for low-wage workers. To save on the excess costs associated with stringent standards, firms employing low-skill labor intensively would then face incentives to operate in a black market. As a result, black markets that exist to evade regulations might raise welfare. Moreover, the expected fine on firms caught evading the regulations could serve the role of a Pigouvian tax on any negative externalities associated with the evasion of workplace standards (e.g., uninsured medical costs). To push this argument further, however, we would need to investigate whether other considerations might prevent those firms facing overly-stringent standards from self-selecting into the black market. Our previous analysis has taught us that this self-selection process does not always work efficiently (e.g., the case of black markets with low-quality goods). In some cases, workplace regulations might be particularly desirable from a social-welfare viewpoint for firms employing large amounts of capital per worker (e.g., mining companies). Here, at least, our previous model predicts that such firms choose not to evade regulations; the government's ability to seize their assets implies that the penalty from detection is too high.

Black markets may also exist to circumvent minimum-wage laws, thereby increasing employment. But a full welfare analysis of this possibility would again require a determination of which types of firms self-select into the black market. With some firms evading minimum-wage laws, but not others, resources might be misallocated across industries to an extent that largely negates the benefits of increased employment.

Returning to our basic model, a natural question to ask is, which type of black market might be favored by a country's political process. A median-voter model would tend to favor black markets with intermediate qualities, which have ambiguous efficiency effects. On the other hand, if we extended the model to make quality a normal good, then it is not difficult to imagine competition among political pressure groups producing an equilibrium favoring high-income consumers through lax enforcement activities for the taxation of our high-quality goods. As a result, the possibility that the political equilibrium produces efficiency-enhancing tax evasion should not be dismissed.

Finally, the analysis could be extended to recognize the possibility that tax authorities have some discretion over how to allocate their enforcement activities. Along with this discretion comes the possibility of corruption, whereby officials take bribes in return for allowing particular black-market activities to occur. Shleifer and Vishney (1993, p. 612) claim, "Efforts to avoid detection and punishment cause corruption to be more distortionary than taxation...Government officials will then use their powers to induce substitution into the goods on which bribes can be more easily collected without detection." In our model, black-market firms should be willing to pay a bribe that reduces the expected fine on black-market activities by more than the bribe. To the extent that it is easier to collect bribes for goods with high value per unit volume, our analysis suggests that corruption can be less distortionary than taxation.

Acknowledgements

Appendix A

Assume that firms produce output according to a standard neo-classical constant-returns production function, $Q_\theta(W(\theta), A(\theta))$ Each firm chooses its factor mix to minimize costs subject to the constraint that $Q_\theta(W(\theta), A(\theta)) = 1$ We use $W_\ell(\theta)$ and $A_\ell(\theta)$ to denote the factor mix chosen by a quality-θ firm that operates in the legal market, whereas $W_b(\theta)$ and $A_b(\theta)$ play the same role for black-market firms. Since the effective cost of capital is relatively high for tax-evading firms, it follows that $W_\ell(\theta) < W_b(\theta)$ and $A_\ell(\theta) > A_b(\theta)$.

Let $C^*(\theta)$ denote the effective unit input cost for a quality-θ good when the socially optimal input mix is used. Since firms that operate in the legal market operate efficiently, we have $C_\ell(\theta) \equiv W_\ell(\theta) + \delta A_\ell(\theta) = C^*(\theta)$. But since black-market firms use an inefficiently low capital-labor ratio, the cost of producing these goods is artificially high: $C_b(\theta) \equiv W_b(\theta) + \delta A_b(\theta) > C^*(\theta)$.

It is straightforward to show that the nature of the equilibrium is driven by the same sort of condition that arose in Section 3:

Proposition 1A. *Low-quality firms will populate the black market if $\frac{A_b'(\theta)}{A_b(\theta)} > \frac{W_b'(\theta)}{W_b(\theta)}$ for all θ, whereas high-quality firm will operate in the black market if the inequality is reversed for all θ.*

Proof. Our goal is to show that $P_b(\theta)$ is steeper than $P_\ell(\theta)$ at θ^* if $\frac{A_b'(\theta)}{A_b(\theta)} > \frac{W_b'(\theta)}{W_b(\theta)}$ for all θ. We first note that the definition of θ^* gives $P_b(\theta^*) = P_\ell(\theta^*)$. From Eqs. (3) and (4), we have

$$\frac{1-\pi f}{1-t} = \frac{W_b(\theta^*) + [\delta + \pi f(1-\delta)]A_b(\theta^*)}{W_\ell(\theta^*) + \delta A_\ell(\theta^*)} \tag{A.1}$$

Now differentiate Eqs. (3) and (4) with respect to θ and compare $P_b'(\theta)$ and $P_\ell'(\theta)$ to show that $P_b'(\theta) > P_\ell'(\theta)$ at θ^* if

$$\frac{1-\pi f}{1-t} < \frac{W_b'(\theta^*) + [\delta + \pi f(1-\delta)]A_b'(\theta^*)}{W_\ell'(\theta^*) + \delta A_\ell'(\theta^*)} \tag{A.2}$$

Using Eq. (A.1) to substitute for the left side of Eq. (A.2) yields

$$\frac{W_b'(\theta^*) + [\delta + \pi f(1-\delta)]A_b'(\theta^*)}{W_b(\theta^*) + [\delta + \pi f(1-\delta)]A_b(\theta^*)} > \frac{W_\ell'(\theta^*) + \delta A_\ell'(\theta^*)}{W_\ell(\theta^*) + \delta A_\ell(\theta^*)} \tag{A.3}$$

Consider the case in which $\pi f = 0$ (so that $A_b(\theta) = A_\ell(\theta)$ and $W_b(\theta) = W_\ell(\theta)$). Then the right side of Eq. (A.3) equals the left side. Now, suppose that we increase πf.[14] Below, we show that that the left side of Eq. (A.3) is strictly increasing in πf if $\frac{A_b'(\theta^*)}{A_b(\theta^*)} > \frac{W_b'(\theta^*)}{W_b(\theta^*)}$. Since the right side of Eq. (A.3) is independent of πf, this implies that if $\frac{A_b'(\theta^*)}{A_b(\theta^*)} > \frac{W_b'(\theta^*)}{W_b(\theta^*)}$, then Eq. (A.3) will hold for all $\pi f > 0$.

To prove this last claim, note that black-market firms choose $(W_b(\theta), A_b(\theta))$ to minimize expected unit costs, which are given by $W_b(\theta) + \delta A_b(\theta) + \pi f(1-\delta)A_b(\theta)$. Thus, by the envelope theorem we have

$$\frac{d[W_b(\theta) + \delta A_b(\theta) + \pi f(1-\delta)A_b(\theta)]}{d(\pi f)} = (1-\delta)A_b(\theta)$$

---

[14] Of course, with $\pi f = 0$, t must equal 0 as well. Moreover, when we increase πf, we must adjust t so that (A.1) continues to hold.

Differentiating by θ then yields

$$\frac{d^2[W_b(\theta) + \delta A_b(\theta) + \pi f(1-\delta)A_b(\theta)]}{d(\pi f)d\theta} = \frac{d^2[W_b(\theta) + \delta A_b(\theta) + \pi f(1-\delta)A_b(\theta)]}{d\theta d(\pi f)}$$
$$= (1-\delta)A'_b(\theta)$$

The latter equality implies that the derivative of the numerator of the left side of Eq. (A.3) with respect to πf is equal to $(1-\delta)A'_b(\theta)$. Thus, the numerator of the left side of Eq. (A.3) rises by a greater percentage than the denominator (and inequality Eq. (A.3) holds) if

$$\frac{A'_b(\theta^*)}{W'_b(\theta^*) + [\delta + \pi f(1-\delta)]A'_b(\theta^*)} > \frac{A_b(\theta^*)}{W_b(\theta^*) + [\delta + \pi f(1-\delta)]A_b(\theta^*)} \tag{A.4}$$

In Eq. (A.4), the left side is the percentage increase in the numerator of the term on the left side of Eq. (A.3), whereas the right side of Eq. (A.4) is the percentage increase in the denominator of the term on the left side of Eq. (A.3). We can rewrite Eq. (A.4) as

$$\frac{A'_b(\theta^*)}{A_b(\theta^*)} > \frac{W'_b(\theta^*) + [\delta + \pi f(1-\delta)]A'_b(\theta^*)}{W_b(\theta^*) + [\delta + \pi f(1-\delta)]A_b(\theta^*)}$$

or,

$$1 > \frac{\frac{W'_b(\theta^*)}{A'_b(\theta^*)} + \delta + \pi f(1-\delta)}{\frac{W_b(\theta^*)}{A_b(\theta^*)} + \delta + \pi f(1-\delta)} \tag{A.5}$$

But, Eq. (A.5) holds if $\frac{A'_b(\theta^*)}{A_b(\theta^*)} > \frac{W'_b(\theta^*)}{W_b(\theta^*)}$. $\square$

Since the schedule of market prices is again kinked where $P_b(\theta)$ crosses $P_\ell(\theta)$, there is again a gap that separates the quality levels offered in the two markets. With no change in the properties of the equilibrium, the extension of the welfare analysis is fairly straightforward as well.

References

Alm, J., 1985. The welfare cost of the underground economy. Economic Inquiry 23, 243–263.
Andreoni, J., 1992. IRS as loan shark: tax compliance with borrowing constraints. Journal of Public Economics 49 (1), 35–46.
Andreoni, J., Erard, B., Feinstein, J., 1998. Tax compliance. Journal of Economic Literature 36, 818–860.
Becker, G.S., 1968. Crime and punishment: an economic approach. Journal of Political Economy 76, 169–217.
Boadway, R., Keen, M.J., 1995. Evasion and time consistency in the taxation of capital income. International Economic Review 39, 461–476.
Boadway, R., Marchand, M., Pestieau, P., 1994. Towards a theory of the direct–indirect tax mix. Journal of Public Economics 55, 71–78.
Bond, E., Chen, T.J., 1987. The welfare effects of illegal immigration. Journal of International Economics 23 (3–4), 315–328.
Brito, D.L., Hamilton, J., Slutsky, S., Stiglitz, J.E., 1995. Randomization in optimal income tax schedules. Journal of Public Economics 56, 189–224.
Chang, F., 1986. Randomization of commodity taxes: an expenditure minimization approach. Journal of Public Economics 31, 329–346.
Cremer, H., Gahvari, F., 1993. Tax evasion and optimal commodity taxation. Journal of Public Economics 50, 261–275.
Davidson, C., Martin, L.W., Wilson, J.D., 2005. Tax evasion as an optimal tax device. Economics Letters 86 (2), 285–290.

Feldstein, M., 1995. Tax avoidance and the deadweight loss of the income tax. NBER Working Paper, vol. 5055.

Kaplow, L., 1990. Optimal taxation with costly enforcement and evasion. Journal of Public Economics 43, 221–236.

Kesselman, J.R., 1989. Income tax evasion: an intersectoral analysis. Journal of Public Economics 38, 137–182.

Kopczuk, W., 2001. Redistribution when avoidance behavior is heterogeneous. Journal of Public Economics 81, 51–71.

Palda, F., 1998. Evasive ability and the efficiency cost of the underground economy. Canadian Journal of Economics 31 (5), 1118–1138.

Polinsky, A.M., Shavell, S., 2000. The economic theory of public enforcement of law. Journal of Economic Literature 38, 45–76.

Rosen, H.S., 2005. Public Finance, Seventh edition. McGraw-Hill/Irwin, New York.

Schneider, F., Enste, D.H., 2000. Shadow economies: size, causes, and consequences. Journal of Economic Literature 38, 77–114.

Shleifer, A., Vishney, R.W., 1993. Corruption. Quarterly Journal of Economics 108, 599–617.

Slemrod, J., Bakija, J., 2004. Taxing Ourselves: A Citizen's Guide to the Great Debate over Tax Reform, Third edition. MIT Press, Cambridge, MA.

Slemrod, J., Yitzhaki, S., 1995. The costs of taxation and the marginal cost of funds. IMF Working Paper, vol. 95/83.

Slemrod, J., Yitzhaki, S., 2002. Tax avoidance, evasion, and administration. In: Auerbach, A., Feldstein, M. (Eds.), Handbook of Public Economics, vol. 3. North-Holland, Amsterdam, pp. 1423–1470.

Stiglitz, J., 1982a. Utilitarianism and horizontal equity: the case for random taxation. Journal of Public Economics 18, 1–34.

Stiglitz, J., 1982b. Self-selection and Pareto efficient taxation. Journal of Public Economics 17, 213–240.

Usher, D., 1994. Tax evasion and the marginal cost of public funds. The Collected Papers of Dan Usher. Welfare Economics and Public Finance, vol. 2. Ashgate, Brookfield, Vt, pp. 46–69.

Weiss, L., 1976. The desirability of cheating incentives and randomness in the optimal income tax. Journal of Political Economy 84, 1343–1352.

Wilson, J.D., 1989. On the optimal tax base for commodity taxation. American Economic Review 89, 1196–1206.

Yitzhaki, S., 1979. A note on optimal taxation and administrative costs. American Economic Review 69, 475–480.

Yitzhaki, S., 1987. On the excess burden of tax evasion. Public Finance Quarterly 15 (2), 123–137.