Price-setting newsvendor with strategic consumers

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ABSTRACT

We consider a newsvendor who sells a single product over a single season with the objective of determining both the selling price and stock quantity to maximize the expected profit. The customers are strategic and we consider two demand cases: additive and multiplicative. For each case, we derive the newsvendor’s optimal decisions and demonstrate that neglecting the price-sensitivity of demand leads the newsvendor to make sub-optimal decisions. Moreover, we show that under certain conditions, strategic consumer behavior may positively affect the newsvendor’s optimal expected profit in the additive demand case.

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1. Introduction

A fundamental building block in the operations management literature is the classical newsvendor setting wherein a newsvendor orders a quantity of a perishable or seasonal product from an outside supplier with ample capacity to meet a random demand with certain distributions in a single period; any lost sales would incur the corresponding penalty costs, and any excess inventories can be salvaged at a given per-unit salvage value at the end of the period. The newsvendor’s objective is to determine his stock quantity before the demand is realized to maximize his own expected profit.

As it is convenient to analyze, many studies use the classical newsvendor model and its extensions. We refer to [14,20], and [5] for extensive reviews on this topic. In this study, we extend the classical newsvendor model to a case where a newsvendor sells a single product over a single selling season in order to determine both the selling price and stock quantity to maximize expected profit. We assume strategic, forward-looking customers who recognize that the product may become available on the salvage market and consider delaying their purchase until the end of the selling season to maximize their expected surplus by purchasing the product at the salvage value. Note that strategic consumer behavior is widely observed in many industries, e.g., fashion apparel and consumer electronics. A typical clothing retailer must always determine an initial selling price before bringing a certain type of clothing to market. If the clothing is very popular, the price remains the same until the items are sold out. However, if the demand is less than the corresponding stock quantity, the retailer has to offer markdowns close to the end of the selling season. Some customers may delay their purchase when they can anticipate that the item will be available in the future at a lower price to obtain a relatively greater surplus. Accordingly, the retailer must consider potential strategic consumer behavior when making the pricing and stock decisions. The model and the corresponding results in this study aim to provide insights into this decision-making situation.

We consider two demand cases: additive and multiplicative. In the former, demand is defined as the sum of a deterministic demand function, \( d(p) \) that is dependent on the selling price \( p \) and a random variable \( e \) that is independent of the price. In the multiplicative case, demand is defined as the product of \( d(p) \) and \( e \). For both demand cases, we develop the newsvendor’s decisions in the context of strategic consumer behavior. Moreover, we compare the decisions in our setting with those in two special cases. Case M (in which customers are myopic) and Case I (in which demand is independent of the selling price). The results show that neglecting price-sensitivity as a factor of demand leads to sub-optimal decisions. Additionally, we demonstrate that under certain conditions, strategic consumer behavior may positively affect the newsvendor’s optimal expected profit in the additive demand case.

While a significant number of studies address the price-setting newsvendor model, we refer to [19] for a comprehensive review, wherein the newsvendor’s decisions about price and stock quantity are derived for both additive and multiplicative demand cases. In addition, they propose a unified framework for these two demand cases and demonstrate that the optimal prices in both cases can be interpreted as a base price plus a premium. The single-period, single-product price-setting newsvendor model has
recently been extended to many other cases. For instance, [4,28], and [13] focus on the competitive setting; [6] and [21] consider the effectiveness of return policies and the structural properties of return contracts in the price-setting newsvendor model, respectively. Ref. [7] analyze the effects of order and price postponement in a decentralized newsvendor model with multiplicative and price-dependent demand. Refs. [29] and [30] focus on the effects of uncertain demand and uncertain supply, respectively, in price-setting newsvendor models. Ref. [10] consider a price-setting newsvendor model with service and loss constraints. In this stream, demand is always assumed as either additive or multiplicative. Moreover, each of these studies implicitly assume that customers are myopic and they have only one opportunity to purchase the product. In this study, we also use the price-setting newsvendor model and assume that the demand is either additive or multiplicative, though we assume that customers are strategic.

Another related research stream uses the newsvendor model in the context of strategic consumer behavior, though most of these studies, e.g., [23,24,22,2,11,27], and [31] treat the classical newsvendor model as the building block to investigate the impacts of strategic consumer behavior by some important, such as supply chain efficiency, the value of commitment, the value of quick response, and the value of fast fashion. In these studies, although the newsvendor must determine the price (similar to the price-setting newsvendor model), demand is always assumed to be independent of price (a departure from in the price-setting newsvendor model). That is, random demand is interpreted as the total mass of infinitesimal consumers in the market. However, in most industries, demand is price-sensitive. In a novel extension of the price-setting newsvendor model, this study occurs in the context of strategic consumer behavior.

The research stream dedicated to the secondary market (see, e.g., [16,8,1,12,18,9]) is also related to our study. However, our context does not include resellers in the salvage market. Thus, the model framework in our study is considerably different from those in this research stream.

The remainder of this study is organized as follows. In the next section, we describe the basic model, including notations, definitions, and some necessary assumptions. In Section 3, we derive the newsvendor’s optimal decisions in the additive demand case and compare this to the two special cases. The optimal decisions in the multiplicative demand case are developed in Section 4. Finally, we summarize the results in Section 5. All proofs are presented in the Appendix.

2. Basic model

We consider a newsvendor who sells a single product over a single selling season to determine both the selling price p and stock quantity q based on beliefs about customers’ reservation prices, \( \xi \). The selling price is publicly announced such that all market participants can observe it. Next, the random demand \( D(p) \) is realized based on the newsvendor’s selling price, and then

\[
D(p) = \text{additive and multiplicative. Specifically, } D(p) \text{ is defined as}
\]

\[
D(p) = d(p) + e
\]

in the additive case and

\[
D(p) = d(p)\xi
\]

in the multiplicative case, where \( d(p) \) is a deterministic decision function dependent upon the selling price and \( e \) is a random variable that is independent of \( p \), with \( \text{cdf } F(e) \), pdf \( f(e) \), mean \( \mu \), and support \([A, B] \), where \( B > A > 0 \). For technical reasons, we assume that \( f(e) = 0 \) is continuous and \( f(A) > 0 \). Define the failure rate of \( e \)'s distribution as \( \eta(e) = f(e)/(1 - F(e)) \). We assume that \( e \)'s distribution has an increasing failure rate (IFR). This assumption is not too restrictive because it includes many common distributions. According to expressions (1) and (2), we have

\[
G(k) = F(x - d(p))
\]

in the additive case and

\[
G(k) = F\left( \frac{x}{d(p)} \right)
\]

in the multiplicative case. Moreover, similar to [19], we let \( d(p) = a - bp \ (a > b > 0) \) in the additive case and \( d(p) = ap^{-b} \ (a > b > 1) \) in the multiplicative case.

Our context assumes strategic customers who recognize that the product may become available on the salvage market at price \( s \) and may consider delaying their purchase until the end of the selling period to maximize their expected surplus. Let \( r \) denote the customer’s valuation of the product and \( r (r \leq v) \) denote the customer’s reservation price. Naturally, we assume that newsvendors cannot observe customers’ reservation price. Furthermore, the retailer’s selling price is observable, while their stock quantity is not. Given these conditions, all parties form unobservable beliefs: the newsvendor forms beliefs \( \xi \), about customers’ reservation prices, and customers form beliefs \( \xi \), about the probability of availability on the salvage market. Similar to [23], we assume that customers are: (i) homogeneous in the sense that they share the same reservation price \( r \) and beliefs \( \xi \), and (ii) risk neutral in that they do not discount future payoffs. The latter assumption is easily relaxed by introducing a discount factor in developing consumers’ purchasing decisions. However, the analytical results are very difficult to derive once the former assumption is relaxed.¹

¹ We sketch out this problem as follows. Suppose that customers have a heterogeneous valuation distributed according to the continuous function \( H(v) \) with support \([v_o, v_u]\). A customer with valuation \( v \) individually decides whether to purchase the product at full price \( p \) and obtain surplus \( v - p \), or wait for the markdown to obtain surplus \( v - s \). Given the customers’ beliefs, \( \xi \), about availability on the salvage market, this customer chooses to buy the product at price \( p \) if and only if \( v - p > s \xi \). Correspondingly, we can obtain a critical customer valuation \( v^* = \min(v_o, p - s\xi, 1 - \xi) \) such that all customers with \( v > v^* \) would purchase at price \( p \) and all customers with \( v < v^* \) would purchase at salvage price \( s \). Additionally, the newsvendor’s demand can be separated into two parts: \( \pi = \pi^*(D(p)) \) and \( \pi^*(D(p)) \). Note that only the former must be considered in modeling the newsvendor’s profit function.

In this context, the newsvendor forms beliefs about the critical customer valuation, which we label \( \xi \), not about customers’ reservation price. The newsvendor’s expected profit can now be expressed as \( \pi = \text{max} \{q(\xi)\pi^*(D(p)) - c - s \xi + q(\xi)\pi^*(D(p)) - c\} \). For both additive and multiplicative demand functions, it is possible to obtain the newsvendor’s best response inventory level, though it is difficult to obtain the optimal price \( p^* \). It must be noted that in the context of strategic consumer behavior, if the inventory level is a decision variable, heterogeneous customer valuation is considered when only the selling price \( p = \xi \) is exogenously given, as in, for example, [26].
consumers decide when to buy based on their beliefs about the probability of obtaining the product at the salvage price, $\xi_p$. At the beginning of the selling season, sales occur at selling price $p$. Finally, all leftover inventory is sold at salvage price $s$.

Before deriving the newsvendor’s optimal decisions, we must address strategic consumer purchasing decisions. Suppose a customer decides to purchase the product; the unique decision is when to buy, i.e., whether to buy immediately at full price or wait for the salvage price. The surplus derived from an immediate purchase at price $p$ is $v - p$, while a deferred purchase at the end of the selling period can generate a greater surplus $v - s$. However, customers who choose to delay the purchase risk experiencing stock outs. Recall that customers form beliefs about customers’ reservation price, $\xi$, on the basis of price $p$ and beliefs about product availability. Therefore, an individual customer waiting for the salvage price can experience a surplus of $v - s$.

Define $\xi_p = \xi / \mu$, where $\mu$ is the expected leftovers when $z = 0$. For each of the $(p, q)$, which occurs with probability $Pr(D(p^*) < q^*) = G(q^*) / p^*$. 

3. Additive demand case

In the additive demand case, $D(p) = d(p) + c$ and $d(p) = a - bp$. First, we write the newsvendor’s expected profit as

$$\pi(p, q) = p \min(D(p), q) + \varepsilon[\min(q - D(p))] - cq,$$

where $\varepsilon$ is the expectation operator.

Using expression $z = q - d(p)$, $\pi(p, q)$ can be rewritten as

$$\pi(p, z) = (p - c)d(p) + p \min(c, z) + \varepsilon(z - c) - cz = \Psi(p) - \Gamma(p, z),$$

where $\Psi(p) = (p - c)[d(p) + \mu]$, $\Gamma(p, z) = (c - s)\Lambda(z) + (p - c)\Theta(z)$, $\Lambda(z) = \int_{a - p}^{z} f(x)dx$, and $\Theta(z) = \int_{z}^{a} [z - x]f(x)dx$. The term $\Psi(p)$ represents the newsvendor’s profit function when the random term in the demand model is replaced by its constant mean $\mu$. Further, another way, $\Psi(p)$ is the newsvendor’s riskless profit function [19,17]. $\Gamma(p, z)$ is the loss function, which accesses an overage lost $(c - s)$ for each of the $\Lambda(z)$ expected leftovers when $z$ is too high and an undercost $(p + s - c)$ for each of the $\Theta(z)$ expected shortages when $z$ is too low.

Since strategic consumers are homogeneous, all would purchase at the same time, either during the selling season or at the end of the season. Given that $s < c$, all strategic consumers purchase at salvage price $s$ leading to an equilibrium where the newsvendor does not order any inventory. As a result, we need to consider only the equilibrium in which the newsvendor induces customers to buy at full price. In other words, in the rational expectations equilibrium, in addition to the conditions presented in Definition 1, the newsvendor’s optimal selling price, $p^*$, must satisfy

$$p^* \leq v - \xi_p = v - (v - s)G(q^*) .$$

Compared to (3), expression (4) indicates that in equilibrium, the newsvendor’s optimal selling price must be less than or equal to the customers’ reservation price.
When the total demand \( D(p) \) is replaced by its random term \( c \), that is, the newsvendor’s demand is independent of the selling price, we can find that for a given \( z \), the newsvendor’s expected profit strictly increases with respect to \( p \), i.e., \( \frac{\partial \pi(p,z)}{\partial p} |_{a=b=0} = \mu - \Theta(z) = \mu - \int_0^z f(x) dx \geq \mu - \int_0^{b_0} f(x) dx = A > 0 \). Thus, the newsvendor would price the product at \( v = (v-s)F(z) \) and extract all of the consumer surplus. Many relevant studies have arrived at this result, for example, \([23]\) and \([3]\). However, when the demand is price-sensitive, \( \pi(p,z) \) is not necessarily increasing with respect to \( p \) for a given \( z \), thus complicating the newsvendor’s optimal decisions. Taking the first and second partial derivatives of the newsvendor’s expected profit function, \( \pi(p,z) \), with respect to \( p \), gives

\[
\frac{\partial \pi(p,z)}{\partial p} = 2b(p^0 - p) - \Theta(z).
\]

\[
\frac{\partial^2 \pi(p,z)}{\partial p^2} = -2b,
\]

where \( p^0 = (a+bc+\mu)/2b \) is the optimal riskless selling price, that is, the price that maximizes \( \pi(p) \). Since \( \partial^2 \pi(p,z)/\partial p^2 < 0 \), we know that \( \pi(p,z) \) is concave in \( p \) for a given \( z \). Based on the first-order condition of \( \pi(p,z) \) with regard to \( p \), we have

\[
p^*(z) = p^0 - \Theta(z)/2b.
\]

Because \( \partial \Theta(z)/\partial z = 1 + f(z) < 0 \), we know that \( p^*(z) \) is strictly increasing with \( z \). Because \( c^* \)’s support is on \([A,B]\), there would be \( p^* \in [a+bc+\mu]/2b, p^0] \).

Note that the optimal selling price expressed by (5) can maximize the newsvendor’s expected profit but may not be able to induce all customers to buy immediately. To guarantee that the selling price (as a function of \( z \)) can simultaneously maximize the newsvendor’s expected profit and tend towards the rational expectations equilibrium for the game between the newsvendor and customers, the newsvendor would choose

\[
p^*(z) = \min \left\{ v - (v-s)F(z), p^0 - \frac{\Theta(z)}{2b} \right\}.
\]

**Lemma 1.** The parameters \( a, b, c, \mu, v, \) and \( s \) satisfy

\[
a + \frac{bc + \mu}{2b} > s \quad \text{and} \quad a + \frac{bc + A}{2b} < v.
\]

Moreover, there exists a \( z \), namely \( \bar{z} \), such that \( p^*(z) = p^0 - \Theta(z)/2b \) if \( z < \bar{z} \) and \( p^*(z) = v - (v-s)F(z) \) if \( z \geq \bar{z} \), where \( \bar{z} \) is the unique root of

\[
p^0 - \frac{\Theta(z)}{2b} - (v-s)F(z) = 0.
\]

The results presented in Lemma 1 are fundamental and must be developed further. When \( p^*(z) = p^0 - \Theta(z)/2b \), the newsvendor no longer need to form rational expectations \( \xi \), because this guarantees that any price that maximizes the expected profit can induce customers to buy immediately at the full price. Therefore, the newsvendor’s problem is optimizing the expected profit over \( p \) and \( z \). On the other hand, when \( p^*(z) = v - (v-s)F(z) \), the rational expectations equilibrium for the game between the newsvendor and the customers is solved based on the conditions presented in Definition 1. To summarize, the newsvendor’s decisions may be optimal or based on the rational expectations equilibrium. To avoid confusion, we use “optimal decisions” in the following.

**Proposition 1.** Given the newsvendor’s optimal decisions, all strategic consumers buy immediately. If

\[
a + \frac{bc + A}{2b} > s,
\]

then,

(i) In the region \([A,\bar{z}]\), we have \( p^*_1 = p^0 - \Theta(z^*_1)/2b \) and \( z^*_1 \) is determined by either: (1) if the unique root of equation

\[
\left[p^0 - \frac{\Theta(z)}{2b} - s \right] \left[1 - F(z)\right] - (c - s) = 0
\]

is in the interval \([A,\bar{z}]\), sets \( z^*_1 \) as the root; otherwise, (2) when \( z^*_1 \) approximately equals \( \bar{z} \).

(ii) In the region \([\bar{z},B]\), we have

\[
p^*_2 = s + \sqrt{(c-s)(v-s)} \quad \text{and} \quad z^*_2 = F^{-1}\left(1 - \sqrt{\frac{c-s}{v-s}}\right).
\]

(iii) The newsvendor’s decisions are determined as

\[
(p^*, z^*) = \text{arg max}_{p,z} \pi(p,z).
\]

Condition (8) guarantees the existence of \( z^*_2 \). Although (8) is more restrictive than condition (7), it is still general because it is satisfied by many more parameter combinations. Note that if (8) is satisfied, then (7) immediately holds true. In the following, we present a simple example to illustrate how to obtain the newsvendor’s decisions.

**Example 1.** Suppose that demand \( D(p) \) is given by

\[
D(p) = (10 - 2p) + c,
\]

where \( c \) is uniformly distributed over the interval \((0, 1)\); we thus have \( f(x) = 1, f(x) = x, \mu = 0.5, A(z) = 0.5z^2, \Theta(z) = 0.5z^2 - z + 0.5 \). The other parameters are \( v = 6, c = 3, \) and \( s = 2 \). First, the critical value of \( z \) is: \( \bar{z} = 0.4773 \). By using the solution technique presented in Proposition 1, we obtain the results provided in Table 1. In this example, \( (p^*_1, z^*_1) \) generates the maximum optimal expected profit. Thus, the newsvendor would price the product at 4.0986 per unit and stock an inventory of 0.4773 + (10 - 2 \times 4.0986) = 2.3057.

We now consider the effect that strategic customer behavior and price-sensitive demand have on the optimal selling price, stock quantity, and expected profit. Let \( M \) and \( l \) correspond to the case with only myopic customers and the case where demand is the random variable \( c \) that is independent of the selling price, respectively.

**Proposition 2.** \( z^*_1 \) and \( A^*_1 \) denote the rational expectations equilibrium stock factor and the equilibrium expected profit, respectively. In Case I, we have \( z^*_1 > z^*_2 \) and \( A^*_1 > A^*_2 \), where \( A^*_1 = A(p^0, z^*_1) \).

**Proposition 2** states that the optimal/equilibrium stock factor in the general case is less than in Case I. However, this does not mean that the newsvendor can stock less in the general case, since the optimal/equilibrium stock quantity is \( z^* = dp^*(z_1, z^*_1) \) in the general case and \( z^*_1 \) in Case I. Another finding from Proposition 2 is that

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Results for Example 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value interval of ( z )</td>
<td>( z^*_1 )</td>
</tr>
<tr>
<td>0 &lt; ( c &lt; 0.4773 )</td>
<td>( z^*_1 \approx 0.4773 )</td>
</tr>
<tr>
<td>0.4773 ( \leq c &lt; 1 )</td>
<td>( z^*_2 \approx 0.5000 )</td>
</tr>
</tbody>
</table>
neglecting the price-sensitivity of the demand leads the newsvendor to make sub-optimal decisions.

**Proposition 3.** $z^*_M$ and $\pi^*_M$ denote the optimal stock factor and the optimal expected profit in Case $M$, respectively, and suppose that condition (8) holds. If the unique root of Eq. (9) is in the interval $[A, Z]$, we then have $z^*_M \leq z^*$ and $\pi^*_M \geq \pi^*$.

According to Proposition 3, the optimal stock factor in Case $M$ is less than in the general case. Although $z$ (i.e., $z = q - d(p)$) is the same in both cases, it is not possible to directly compare the optimal stock quantities in these two cases because we do not know the relationship between $p^*$ and $p_m^*$. Moreover, we observe that under certain conditions, strategic consumer behavior has a positive impact on the newsvendor’s optimal expected profit. However, if the unique root of Eq. (9) is in the interval $[\bar{z}, \bar{B}]$, the results in Proposition 3, could, but need not be, the opposite. For instance, we consider the parameters provided in Example 1. The newsvendor’s optimal decisions in Case $M$ are $p_m^* = 4.0966$ and $z^*_M = 0.5236 > z^* = 0.4773$; and the optimal expected profit is $\pi^*_M = 2.2681 > \pi^* = 2.2660$.

4. Multiplicative demand case

In this case, $D(p) = d(p)x$ and $d(p) = ap^{-b}$, $a > 0$, $b > 1$. To differentiate this from the additive demand case, we apply superscript $m$ whenever necessary. As in the previous sub-section, the newsvendor’s expected profit can be written as

$$\pi^m(p, z) = \Psi^m(p) - \Gamma^m(p, z),$$

where

$$\Psi^m(p) = \frac{d(p)}{p}(c - s)\pi(z) + (c - d(p))\Phi(z),$$

and $z = q/d(p)$. Although $z$ is defined differently for the multiplicative demand case, the effect is the same and there exists a constant managerial interpretation for $z$, i.e., the stock factor. We refer to [19] for a detailed discussion on this issue. Moreover, the optimal riskless selling price here becomes $p_m^m = bc/(b - 1)$, which maximizes $\Psi^m(p) = (c - d(p))\pi(z)$, by comparing $\Gamma^m(p, z)$ to $\Gamma(z, p)$, the expected leftovers and shortages become $\pi(x)\pi(z)$ and $\pi(p)\pi(z)$, respectively, in the multiplicative demand case.

To derive the newsvendor’s optimal decisions in the multiplicative demand case, we follow the same sequential procedure detailed previously. By taking the first partial derivative of $\pi^m(p, z)$ with respect to $p$, we obtain

$$\frac{\partial \pi^m(p, z)}{\partial p} = \frac{d(p)(b - 1)(\mu - \Theta(z))}{p} \left[ p_m^m + \frac{b}{b - 1} (c - s)\Lambda(z) \right].$$

Because $\mu - \Theta(z) \geq A > 0$, it follows that $\pi^m(p, z)$ is concave in $p$ and attains its maximum at

$$p_m^m(z) = p_m^m + \frac{b}{b - 1} (c - s)\Lambda(z).$$

**Lemma 2.** $p_m^m(z)$ is strictly increasing with $z$, and

$$p_m^m(z) \in \left[ \frac{bc}{b(c - s) + \mu bs}, \frac{b(c - s) + \mu bs}{b - 1} \right].$$

Similar to the additive demand case, the newsvendor would ensure that all customers buy immediately at full price when determining the selling price. Combining (4) with (10), the newsvendor would choose

$$p_m^m(z) = \min \left\{ p_m^m + \frac{b}{b - 1} (c - s)\Lambda(z), v - \frac{v}{C_0} s \right\}.$$  \hspace{1cm} (11)

Note that expression (11) can be rewritten as

$$p_m^m(z) = \min \left\{ p_m^m(z), p_m^m(z) \right\},$$

where $p_m^m(z) = p_m^m + b(c - s)\Lambda(z)/(b - 1)(\mu - \Theta(z))$ and $p_m^m(z) = v - (v - s)/C_0$. This is the optimal price (as a function of $z$) with only myopic consumers, and the rational expectations equilibrium price (as a function of $z$) where demand is independent of price, respectively. In the multiplicative demand case, for a given $z$, both strategic consumer behavior and price-sensitivity of demand leads the newsvendor to a lower price. However, we are unable to compare the detailed optimal/equilibrium selling prices between the different cases since the optimal/equilibrium values of $z$ vary between them.

**Lemma 3.** The parameters $b, c, v$ satisfy

$$v > \frac{bc}{b - 1} = p_m^m.$$

Moreover, there exists a $z$, namely $z^m$, such that $p_m^m = p_m^m + b(c - s)\Lambda(z)/(b - 1)(\mu - \Theta(z))$ if $z < z^m$ and $p_m^m(z) = v - (v - s)/C_0$ if $z \geq z^m$, where $z^m$ is the unique root of

$$p_m^m + \frac{b}{b - 1} (c - s)\Lambda(z) = v - (v - s)/C_0.$$  \hspace{1cm} (12)

Based on Lemma 3, we can propose a similar technique as in the additive demand case to derive the newsvendor’s decisions about the price and stock quantity in the multiplicative demand case. The results are formally provided in the following proposition.

**Proposition 4.** Given the newsvendor’s optimal decisions, all strategic consumers buy immediately. Suppose that $b \geq 2$ and $z^m \leq F^{-1}(1 - \sqrt{C_0}/(v - s))$, then

(i) In the region $[A, z^m]$, we have $p_1^m = p_m^m + b(c - s)\Lambda(z^m)/(b - 1)(\mu - \Theta(z^m))$ and $z^m \approx z^m$.

(ii) In the region $[z^m, B]$, we have

$$p_2^m = s + \sqrt{(c - s)(v - s)}$$

and

$$z_2^m \approx F^{-1}(1 - \sqrt{c - s}/v - s).$$

(iii) The newsvendor’s demands are

$$\pi^m(z) = \arg \max_{(p, z) \in [p_m^m, \infty) \times [z, z^m]} \pi^m(p, z).$$

We provide the following example in Table 2 to better illustrate the technique and how the newsvendor chose the price and stock quantity.

**Example 2.** Suppose that demand $D(p)$ is given by $D(p) = p^{-2}e^p$, where $e$ is uniformly distributed over the interval (10, 15). We thus have $f(x) = 0.2$, $F(x) = 0.2x - 2$, $\mu = 12.5$, $\Lambda(z) = 0.12z^2 - 2z + 10$, and $\Theta(z) = 0.1z^2 - 3z + 22.5$. The other parameters are $v = 12$, $c = 3$, and $s = 2$. When $v = 12$, we have $z^m = 12.9289 < F^{-1}$.

<table>
<thead>
<tr>
<th>Value interval of z</th>
<th>$z^m$</th>
<th>$p_m^m(z^m)$</th>
<th>$\pi^m(p_m^m(z^m), z^m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 &lt; z &lt; 12.9289$</td>
<td>$z^m \approx 12.9289$</td>
<td>$p_m^m(z^m) \approx 6.1421$</td>
<td>$\pi^m(p_m^m(z^m), z^m) \approx 0.9826$</td>
</tr>
<tr>
<td>$12.9289 &lt; z &lt; 15$</td>
<td>$z^m = 13.4189$</td>
<td>$p_m^m(z^m) = 5.1622$</td>
<td>$\pi^m(p_m^m(z^m), z^m) = 0.9501$</td>
</tr>
</tbody>
</table>

**Table 2** Results for Example 2.

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Another extension to this research would apply such a demand reality and thus, be worthy of further study. In addition, we think that a model that relaxes this assumption would be closer to homogeneous valuations, leading to a tractable model. We think results. Moreover, we assume that strategic consumers have equal to the salvage price. It is clear in our context that introducing this study. Existing studies with both strategic and myopic con-

5. Conclusion

In this study, we consider a newsvendor that sells a single product over a single selling season in order to simultaneously decide the selling price and stock quantity to maximize the expected profit. The customers are strategic and forward thinking, in that they recognize that the product may become available on the salvage market and may consider postponing their purchase to obtain the salvage price to maximize their expected surplus. We consider two price-sensitive demand cases: additive and multi-

Proposition 5. \(z^*\) and \(\pi^*\) denote the newsvendor’s equilibrium stock factor and the equilibrium expected profit in Case I, respectively. We thus have \(z^*\) \(\geq z^*\) and \(\pi^*\) \(\leq \pi^*\).

Proof of Proposition 1. According to the discussions above this proposition and expression (6), in equilibrium, all customers would purchase immediately at full price. For \(z \in [0, \pi]\), \(p^* = p^* - \theta(2b) / 2b\), we have

\[
\frac{dx}{dz} = \frac{f(z)}{2b}(p^0 - \theta(2b) / 2b) \quad \text{and} \quad \frac{d^2x}{dz^2} = \frac{f(z)}{2b^2} \left(2b(p^0 - \theta(2b) - 1 - F(z)) - \frac{1 - F(z)}{\eta(z)}\right).
\]

Let

\[
\delta_2(z) = 2b(p^0 - \theta(2b) - \theta(2b) - 1 - F(z)) - \frac{1 - F(z)}{\eta(z)}
\]

and to differentiate it from \(z\), we obtain

\[
\frac{d\delta_2(z)}{dz} = \frac{f(z)\eta(z)[1 - F(z)]}{[\eta(z)]^2} > 0,
\]

that is, \(\delta_2(z)\) is strictly increasing with \(z\). Moreover, the value of \(\delta_2(z)\) at point \(z = 0\) is

\[
\delta_2(A) = a + b + A - 2b > 0,
\]

where the inequality arises due to (8). Therefore, we can conclude that \(\delta_2(z) > 0\) for all \(z \in [0, \pi]\), and consequently \(\pi(p^0, z)\) is con-
cave in \(z\). \(z^*\) is obtained by setting \(d\pi(p^0, z) / dz = 0\).

For \(z \in [0, B]\), taking the first and second derivatives of \(\pi(p, z)\) with respect to \(z\), we obtain

\[
\frac{d\pi(p, z)}{dz} = (p - c) - (p - s)F(z) \quad \text{and} \quad \frac{d^2\pi(p, z)}{dz^2} = -(p - s)f(z) < 0.
\]

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Appendix

Proof of Lemma 1. Let

\[
\delta_1(z) = p^0 - \frac{\theta(2b)}{2b} - v + (v - s)F(z),
\]

By taking the first derivative of \(\delta_1(z)\) with respect to \(z\), we have

\[
\frac{d\delta_1(z)}{dz} = 1 - \frac{F(z)}{2b} + (v - s)f(z) > 0,
\]

that is, \(\delta_1(z)\) is strictly increasing with \(z\). Further, the values of \(\delta_1(z)\) at \(z\)’s endpoints are

\[
\delta_1(0) = a + bc + \frac{A}{2b} - v \quad \text{and} \quad \delta_1(B) = a + bc + \frac{A}{2b} + s - p^0 - s,
\]

respectively. If \(\delta_1(0) \geq 0\), we have \(v - (v - s)F(z) \leq p^0 - \theta(2b) / 2b\) and \(p^0(z) = v - (v - s)F(z) < v \leq (a + bc + A) / 2b\) for all values of \(z\). Because the lower bound of \(p^0 - \theta(2b) / 2b\) is \((a + bc + A) / 2b\), it follows that the condition \((a + bc + A) / 2b \geq v\) would lead the newsvendor to a sub-optimal price (or local-optimal), though it seems impossible for a rational newsvendor to do so. Moreover, if \(\delta_1(B) \leq 0\), we then have \(\delta_1(z) \leq 0\) for all \(z\). In other words, there would always be \(p^0(z) = p^0 - \theta(2b) / 2b\). However, we can easily prove that this does not hold true.

If \(\delta_1(B) \leq 0\), there would be \(p^0 \leq s\). Since \(p^0\) is the upper bound of \(p^0 - \theta(2b) / 2b\) and \(p^0(z) = p^0 - \theta(2b) / 2b\), it follows that \(p^0(z) \leq p^0 \leq s < c\). Clearly, such a price is unacceptable for the newsvendor. Therefore, \((a + bc + \mu / 2b) > s\). Given that \(p^0 > s\), if \((a + bc + A) / 2b < v\), there would exist a unique \(z\), namely \(\pi\), such that \(\delta_1(\pi) = 0\), because \(\delta_1(z)\) is monotonically increasing with \(z\). Moreover, we have \(\delta_1(z) < 0\) for \(z < \pi\) and \(\delta_1(z) \geq 0\) for \(z \geq \pi\). This completes the proof.
therefore, the optimal \( z \) (as a function of \( p \)) is given by
\[
F(z) = \frac{p - c}{p - s}
\]
Simultaneously solving the above equation and \( p(z) = v - (v - s)F(z) \)
gives \( p^*_z = s + \sqrt{(c - s)(v - s)} \) and \( z^*_z = F^{-1}(1 - \sqrt{(c - s)(v - s)}) \).
The remainder of this proposition is obvious.\( \square \)

**Proof of Proposition 2.** According to [23] the newsvendor’s equilibrium decisions in Case I can be expressed as
\[
z^*_I = F^{-1} \left( 1 - \frac{c - s}{\sqrt{v - s}} \right) = z^*_2 \quad \text{and} \quad p^*_I = s + \sqrt{(c - s)(v - s)} = p^*_2,
\]
respectively, and the corresponding equilibrium expected profit is \( \pi^*_I = \pi(p^*_I, z^*_I) \). Since \( z^*_I < z^*_2 \), it immediately follows from part (iii) of Proposition 1 that \( z^*_I \geq z^* \) and \( \pi^*_I \leq \pi^* \).\( \square \)

**Proof of Proposition 3.** In Case M, the optimal \( z \) is determined by Eq. (9), and the optimal selling price is determined by \( p^*_m = p^0 - \Theta(z^*_m) / 2b \). According to the proof of Proposition 1, we know that under condition (8), the root of Eq. (9) is unique. If the unique root is in the interval \([a, z_0]\), we have \( z^*_m = z^*_1, p^*_m = p^*_1 \), and \( \pi^*_m = \pi(p^*_1, z^*_1) \). From part (iii) of Proposition 1, we thus have \( z^*_m \leq z^* \) and \( \pi^*_m \leq \pi^* \), where the former inequality is due to \( z^*_1 < z^*_2 \).\( \square \)

**Proof of Lemma 2.** Differentiating \( p^m(z) \) with respect to \( z \) gives
\[
\frac{dp^m(z)}{dz} = \frac{b}{b - 1} \left( \frac{(c - s)\mu - \Theta(z) - (c - s)(1 - F(z))\Lambda(z)}{[\mu - \Theta(z)]^2} \right)
\]
Since \( \mu - \Theta(z) > 0 \), it is sufficient to prove that \( zF(z) - \Lambda(z) > 0 \) for all \( z \in [a, b] \).
Let
\[
\delta(z) = zF(z) - \Lambda(z).
\]
By taking the first derivative of \( \delta(z) \) with respect to \( z \), we obtain
\[
\frac{d\delta(z)}{dz} = \frac{dzF(z)}{dz} > 0.
\]
Moreover, the value of \( \delta(z) \) at point \( A \) is
\[
\delta(A) = AF(A) - \Theta(A) = 0.
\]
We can thus conclude that \( \delta(z) > 0 \) for all \( z \in [a, b] \).
Since \( p^m(z) \) is strictly increasing with \( z \), the lower bound and the upper bound of \( p^m(z) \) can be obtained by substituting \( z = A \) and \( z = B \) into \( p^m(z) \), respectively.\( \square \)

**Proof of Lemma 3.** Let
\[
\delta(z) = p^0 + \frac{b}{b - 1} \left( \frac{(c - s)\Lambda(z)}{\mu - \Theta(z)} \right) + (v - s)F(z).
\]
The first derivative of \( \delta(z) \) with regard to \( z \) is
\[
\frac{d\delta(z)}{dz} = \frac{b}{b - 1} \left( \frac{(c - s)\mu - \Theta(z) - (c - s)\Lambda(z)(1 - F(z))}{[\mu - \Theta(z)]^2} \right) + (v - s)F(z).
\]
We know from the proof of Lemma 2 that the first term at the right hand side of \( d\delta(z)/dz \) is positive. Since \( (v - s)F(z) > 0 \), it follows that \( d\delta(z)/dz > 0 \), that is, \( \delta(z) \) is strictly increasing with \( z \).
In addition, the values of \( \delta(z) \) at the endpoints of \( z \) are
\[
\delta(A) = \frac{bc}{b - 1} - v \quad \text{and} \quad \delta(B) = \frac{\mu s + b(c - s)}{\mu(b-1)}.
\]
respectively. Clearly, \( \delta(B) > 0 \). According to Lemma 2, the lower bound of \( p^m + b(c - s)\Lambda(z)/(b - 1) - \Theta(z) \) is \( p^m = bc/(b - 1) \).
Since \( p^m \) must satisfy \( p^m < v \), it follows that if \( bc/(b - 1) > v \), the newsvendor would charge a sub-optimal price (or local-optimal), which seems impossible. As a result, we conclude that the parameters \( b, c, \) and \( v \) must satisfy \( bc/(b - 1) < v \). Under this condition, we have \( \delta(A) < 0 \). Since \( \delta(z) \) is strictly increasing with \( z \), there would exist a unique \( z \), namely \( z^m \), such that \( \delta(z^m) = 0 \), \( \delta(z) < 0 \) for \( z < z^m \), and \( \delta(z) > 0 \) for \( z > z^m \).\( \square \)

**Proof of Proposition 4.** According to equation (10), we know that the newsvendor’s decisions inevitably lead to that in the rational expectations equilibrium, and all strategic consumers buy immediately at full price. We now prove that the newsvendor’s decisions can be obtained with three-step technique.
For \( z \in [A, z^m] \), \( p^m(z) = p^m(0) + b(c - s)(z)/(b - 1)(\mu - \Theta(z)) < v - (v - s)F(z) \). Taking the first derivative of \( \pi^m(p^m(z), z) \) with respect to \( z \), we obtain
\[
\frac{d\pi^m(p^m(z), z)}{dz} = \frac{dp^m(z)}{dz} \left[ 1 - F(z) \right] \left[ \frac{p^m(z) - s}{1 - F(z)} \right]
\]
Let
\[
\delta(z) = p^m(z) - s - \frac{c - s}{1 - F(z)}
\]
By taking the first and second derivatives of \( \delta(z) \) with respect to \( z \), we have
\[
\frac{d\delta(z)}{dz} = \frac{dp^m(z)}{dz} \left( \frac{c - s}{1 - F(z)} \right)
\]
and
\[
\frac{d^2\delta(z)}{dz^2} = \frac{d^2p^m(z)}{dz^2} \left( \frac{c - s}{1 - F(z)} \right) + \left( \frac{c - s}{1 - F(z)} \right) \frac{d\delta(z)}{dz}.
\]
Since
\[
\frac{d\delta(z)}{dz} = \frac{b}{b - 1} \left( \frac{(c - s)\mu - \Theta(z) - (c - s)\Lambda(z)(1 - F(z))}{[\mu - \Theta(z)]^2} \right) + (v - s)F(z)
\]

it follows that
\[
\frac{d^2\delta(z)}{dz^2} = \frac{b}{b - 1} \left( \frac{c - s}{1 - F(z)} \right) \left( \frac{c - s}{1 - F(z)} \right) \frac{d\delta(z)}{dz}.
\]
If \( d\delta(z)/dz \neq 0 \), then \( \delta(z) \) is monotone in \( z \); if \( d\delta(z)/dz = 0 \), we then have
\[
\frac{d^2\delta(z)}{dz^2} \bigg|_{d\delta(z)/dz=0} = \frac{b}{b - 1} \left( \frac{c - s}{1 - F(z)} \right) \left( \frac{c - s}{1 - F(z)} \right) \frac{d\delta(z)}{dz}.
\]
Assuming that \( b \geq 2 \) and \( d\delta(z)/dz > 0 \), we know that \( \delta(z) \) is concave in \( z \). In summary, \( \delta(z) \) is either monotone or concave in \( z \).
To find the optimal \( z \) \( \in [A, z^m] \) that maximizes \( \pi^m(p^m(z), z) \), we must still calculate the values of \( \delta(z) \) at the endpoints of \( z \):
\[
\delta(A) = \frac{bc}{b - 1} - c - s > 0 \quad \text{and} \quad \delta(B) = \frac{bc}{b - 1} - c - s = 0.
\]
When \( z^m \leq F^{-1}(1 - \sqrt{(c - s)/(v - s)}) \), \( \delta(z^m) \geq 0 \); when \( z^m > F^{-1}(1 - \sqrt{(c - s)/(v - s)}) \), \( \delta(z^m) < 0 \). We first consider the case with \( z^m \leq F^{-1}(1 - \sqrt{(c - s)/(v - s)}) \), \( \delta(z^m) \geq 0 \). Then the optimal \( z \) is determined by \( \zeta^* \mathrm{P}(\zeta^*) = -c - s - \frac{c - s}{1 - F(z^m)} \).
thus corresponds to a local maximum of $\pi^m(p^m(z), z)$. Where the optimal $z$ is designed by $\delta_0(z) = 0$.

For $z \in [\pi^m, B]$, because $\Gamma^m(p, z) = d(p)\Gamma(p, z)$ and $p^m(z) = v - (v-s)F(z) = p^*(z)$, it follows that $p^m_2 = p^*_2$ and $z^m_2 = z^*_2$.

Further, note that if $\pi^m > F^{-1}(1/\sqrt{(c-s)/(v-s)})$, there does not exist a $z \in [\pi^m, B]$ that tends to the rational expectations equilibrium. Therefore, we must assume that the parameters satisfy $\pi^m \leq F^{-1}(1/\sqrt{(c-s)/(v-s)})$. The remainder of this proposition is obvious and is omitted. □

**Proof of Proposition 5.** The proof is similar as that of Proposition 2 and is omitted. □

**References**


