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Highlights

- We investigate the optimal pricing policies for differentiated brands.
- The retailer's purchase decision criterions are based on two thresholds.
- Power structures have no effect on the retailer's purchase decision criterions.
- Power structures have effects on supply chain members' decisions and performances.

The effect of customer value and power structure on retail supply chain product choice and pricing decisions

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Abstract: Customer value in goods not only affect his purchase decision but also bring about a big challenge for retail supply chain management. A two-stage retail supply chain consisting of one manufacturer and one retailer is investigated in this paper. The manufacturer produces two substitute products which belong to two different generations while the retailer determines his product choice decisions and pricing policies with considering heterogeneous customer demand based on different customer value. The key issues faced by the retailer are which products to purchase, single or both? And how to make pricing policies in different power structures? From three different game theoretical perspectives, we found that the retailer's purchase decision criterions are based on two thresholds, and in each power structure the optimal pricing policies of manufacturer and retailer are obtained. In addition, the impact of power structure has been explored and it shows that different power structures have no effect on the retailer's product choice decision criterions and behaviors, however, they have a great influence on supply chain members' pricing policies and performances. The revenue sharing contract achieves a Pareto improvement and makes a bigger pie, and the power structure determines the pie split between the supply chain members. Additionally, revenue sharing contract will not affect the retailer's purchase decision criterions and behaviors, either.

Keywords: Product choice; pricing; customer value; power structure; game theory; revenue sharing contract.

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1 Introduction

In respond to an increasingly fierce market competition and meet diverse customer demand, the production and sale substitutable products have become a common behaviour of manufacturers and retailers (Draganska and Jain, 2006). For example, Nike Inc. designs and produces shoes with different kinds of styles, colours, qualities, and functions all the year around. And just about one year ago Apple Inc. released iPhone SE, however, the iPhone 7, a new generation mobile phone, has been in a sale at the present time. What's more, the other older versions such as iPhone 6/6s are still on sales.

From the perspective of customers, product diversification has led customers to hold different customer values or customer reservation prices in different products due to their styles, colours, qualities and so on (Holbrook, 1999). For a newer or higher-quality product, the customer may hold higher reservation price than others (Martin, 1996), and customers determine their buying decisions by comparing their reservation price with the actual price of the product (Kaplan *et al.*, 2011). Consumer behaviour is becoming more and more rational. The price factor cannot be considered as the only pricing decision-making reference any more. Therefore, the customer value has become one of the importance factors that influence customer's product choice (Shioda *et al.*, 2011). From the perspective of manufacturers and retailers, there exist phenomena that some stores, such as online stores or the stores in poor regions, may only sell specific types of products, and on the contrary, others may sell all kinds of products. Further, the same-type product in different stores which serve for different customers may labelled with different prices (Xia *et al.*, 2004). Therefore, big challenges of product choice and pricing decisions have arisen to both manufacturers and retailers (Winer, 1986; Petroshius and Monroe, 1987; Akcay *et al.*, 2010; Ferrer *et al.*, 2010).

In addition, the market position between manufacturers and retailers are not equal in different industries. For instance, in some electronics supply chains, Microsoft and Intel act as a leader with more powerful than downstream members. Some retailers, such as, Wal-Mart and Carrefour, however, may be in a relative strong competitive position and act as a leader than their upstream suppliers (Ertek and Griffin, 2002). In many cases, supply chain members may be in balanced market position, in which they are engaged in vertical

Nash competition (Cotterill and Putsis, 2001; Zhao *et al.*, 2014). To our best knowledge, there are very limited studies that combine the product choice and the pricing model based on the customer value with considering power structures. Hence, our research aims to fulfil this gap through addressing the following key questions:

(1) How does the retailer decide which products to purchase based on customer value with power structures?

(2) How to develop pricing policies for the manufacturer and retailer based on customer value when the retailer sells a single product or both products with power structures?

(3) What is the influence of the customer acceptance and the power structure on product choice, pricing decisions and profits?

In order to solve the above problems, a two-stage retail supply chain consisting of one manufacturer and one retailer is investigated. This is an essential supply chain structure, based on which many papers study interesting operational management and marketing problems. The manufacturer produces two substitute products which belong to different generations. The retailer sells directly to end-customers who have heterogeneous customer value in these two generations of products, which is characterized by different product acceptances. We investigate the retailer's product choice decisions and obtain the optimal pricing policies based on heterogeneous customer value in each purchase strategy and power structure. Therefore, using this basic and appropriate two stage supply chain model is fairly enough to solve this problem, and also it is easy to understand by readers. And indeed, we have done many important analysis and obtain some meaningful insights, which can provide useful decision supports to the end retailer. This study contributes to the theory and practice by investigating how customer acceptance and power structure influence retail supply chain management:

(1) Through the analysis of customer surplus (customer reservation price minus the actual retail price) for heterogeneous customer, we expand the demand function used in a dual-channel supply chain to product choice, which brings about a little enrichment in theory.

(2) We obtain pairs of thresholds for the retailer to make product choice decisions. The thresholds, measured by the production cost of the two substitute products, include a low

threshold and a high threshold in each power structure. Therefore, a purchase decision criterions can be established based on the thresholds, by which the retailer can estimate the product acceptance based on some historical data, expertise, or the industrial reports on the similar products and then make product choice decisions. In addition, the retailer's purchase decision criterions will not be influenced with revenue sharing contract.

(3) The Power structure has a great influence on retailer supply chain pricing decisions and profits, however, it interesting to know that it has no effect on the product choice decisions. Namely, no matter which market position the retailer is in, the purchase decision criterions stay unchanged.

This paper is organized as follows. A survey of related literature is presented in Section 2. The model formulation and assumptions are provided Section 3, in which we formulate the demand functions based on heterogeneous customer value and obtain profit functions of manufacturer and retailer. In Section 4, we investigate the product choice decisions and obtain the equilibriums based on customer value in each power structure. In Section 5, we focus on the impact of customer acceptance and power structure on optimal pricing policies and profits. In Section 6 provides an extended model with revenue sharing model to coordinate the retail supply chain and investigate related conclusions. Finally, the research findings and highlight possible future work are concluded in Section 7.

2 Literature review

The literature reviewed here primarily relates to three streams of research: (i) customer value or reservation price; (ii) product choice strategies, and (iii) effect of power structure on decisions and profits.

Many literatures are on customer value or reservation price. Some researchers were focusing on estimating and measuring by using different methods, such as Sweeney and Soutar (2001), Jedidi and Zhang (2002), Wang *et al.* (2007), and Kaplan *et al.* (2011), etc. In the research of Sweeney and Soutar (2001), they use a 19-item measure, called PERVAL, which is an empirical research method and is used to ssess customers' perceptions of the value of a consumer durable good at a brand level. This measure is usually used in a retail purchase situation to determine what consumption values drive purchase attitude and behavior. Jedidi and Zhang (2002) proposed a conjoint-based approach to estimate consumer-level reservation prices. Form the perspective of consumer, they modeled consumer's decision of not only which product to buy ,but also whether to buy at all in a category. Also, Kaplan *et al.* (2011) proposed a two-stage method to elicit consumers' price acceptability range. Others conducted case studies from different perspectives and industries, such as Thompson and Troester (2002) from case of the natural health microculture, Webster and Rennie (2011) in leisure travel, Koller *et al*. (2011) considering green consumption, and Perrea *et al.* (2015) in food product industry, etc. Further, from the perspective of operational management, Shioda *et al.* (2011) assumed that product choices of consumer depend on the reservation prices. They formulated maximum utility model an as a mixed-integer programming problem, and investigated a product line pricing problem. Abbey *et al.* (2015) took customer reservation price into consideration. By using a model of consumers' preferences, they studied the optimal pricing of the new and remanufactured products based on extensive experimentation. Hu *et al.* (2015) considered that consumers are sufficiently heterogeneous in product valuations in a crowdfunding mechanism, they examined the optimal pricing and product decisions and found that the firm should offer a line of products with different levels of product quality and prices. The above literature on customer value or reservation price are not taking supply chain management into consideration. However, in some cases, the customer value may not only affect the retailer who serves consumer directly but also retailer's upstream firms, i.e., distributors and manufacturers.

The second relevant stream of literatures are the researches on product choice. Moorthy (1988) examined two identical firms competing on product quality and price. He assumed that the customer prefers the high quality product to the low quality. The quality-price equilibrium strategies of both a simultaneous-product-choice model and sequential-product-choice model were obtained. Liefeld *et al.* (1996) investigated the Dutch customers' product choice. They found that Dutch customers bought one product not others mainly based on their heterogeneity in tastes and preferences and rely little on extrinsic information cues, and also they took little consideration of country-of-origin as a choice cue. Rath and Zhao (2001) studied two producers' location and pricing policies with consideration customer product choice. They found that the equilibrium prices and locations rely on relative magnitudes of the customer reservation price and the transportation cost. Friese *et al.* (2006) tested the assumption that customers may have explicit and implicit preferences toward a product at the same time. By using Implicit Association Test (IAT), the authors measured consumer preferences regarding generic food products and well-known food brands and found that the customer are more likely to choose the implicitly preferred brand when product choice has to be made in a short time. Mack and Sharples (2009) investigated the important factors which affect people in mobile phone product choice with the methods, such as active information search, structured preference elicitation, ranking and interviews. They found that usability, features, aesthetics and cost are the most important. Peng *et al.* (2012) examined the role of animosity in customers' product choices. They found that animosity has a significant effect on product choice and the trade-offs between price and animosity can be used to adjust the customer product choice behave. Swahn *et al.* (2012) observed 1,623 consumers' choice of apples and found that sensory description labels have an important impact on consumer product choice. Huber *et al.* (2015) aimed to study the impact of different ways of presenting the life insurance price on customer demand. Their finds showed that unlike other products, there are no impacts of ways of presenting prices on purchase intention. Instead, customer experience and price perception have a great effect. However, the above studies on product choice did not consider the impact of customer value on customer buying behavior and firm purchasing decision.

Additionally, many studies have examined the different power structures between supply chain members and they showed that power structure affects decision-making and profit (Gaski and Nevin, 1985; Kim and Kwak, 2007; Kolay and Shaffer, 2013). Ingene and Parry (1995) examined one manufacturer supplying multiple exclusive retailers, and focused on the channel coordination. Choi (1991) studied the pricing decision based on supply chain with two manufacturers and a common retailer based on a linear demand and a nonlinear demand. He discussed three non-cooperative games, namely Manufacturer-Stackelberg, Retailer-Stackelberg, and Nash game between the manufacturers and retailer. Choi (1996) extended this research by examining two manufacturers supply the product to two differentiated common retailers, and the horizontal competition has been into consideration. This paper revealed that horizontal product differentiation will help the retailers but hurt the manufacturers. Raju and Zhang (2005) studied a channel model with the Retailer-Stackelberg and discussed the coordination mechanism for the manufacturer. They found that such channel structure can be coordinated by quantity discounts. Yang *et al.* (2006) analyzed a two-echelon supply chain with one manufacturer and two competitive retailers. The different competitive behaviors of the two retailers were discussed in their study. From the perspectives of supplier-Stackelberg, retailer-Stackelberg, and the Nash game, Cai *et al.* (2009) discussed the effect of the price discount contracts and found that the price discount contracts are benefiting for the supply chain members. Fan *et al.* (2013) analyzed dynamic pricing and production planning problems in a game of one leader and multiple followers with unknown demand parameters. They found that the leader will always outperforms the followers, and each member can improve its revenue by demand learning. Shi *et al.* (2013) examined how power structure and demand models affect supply chain members' performance. They found that the effect of power structure not only depends on the model of expected demand, but also depends on the demand shock. Chen *et al.* (2014) explored the effect power structure on assembly supply chains composed by one assembler and two suppliers. They found when the assembler is the leader, the whole supply chain profit is the highest and so is the assembler's profit. Chen and Wang (2014) investigated the smart phone supply chain that consists of a handset manufacturer and a telecom service operator. Different power structures were considered and the corresponding impacts were discussed. They showed that the smart phone supply chain would choose a bundled channel in the telecom service operator Stackelberg as well as in the manufacturer Stackelberg power structures under certain conditions; while would select a free channel in a vertical Nash power structure. Chen *et al.* (2015) studied the pricing policies for an O2O mixed channel with different power structures. They found power structure has great influence on pricing policies and economic performances. However, the above studies did not consider the customer's valuation on the product as a factor in the operational management decision. Further, being different from the above literatures, our research focuses on the retailer's product purchase choice based on heterogeneous customer value in two substitute products. The impact of customer acceptance and power structure have been examined. This paper aim to fill the gap in literatures and guide business decisions in practice.

This paper contributes to the literature in several perspectives. First, our study considers customers with fully heterogeneous valuations on two generations of products, namely new product and old product. The customers are segmented based on the customer acceptance of old product and the customer surplus in new product and old product respectively. Second, we clearly identify the bounds in which the retailer should do purchase new product, old product or both of them, we provide the corresponding pricing strategies of the manufacturer and the retailer. To the best of our knowledge, there are very limited studies that combine product preference and customer valuation in the pricing model.

3 The model

3.1 Model description and assumption

We consider that a retailer purchases from a manufacturer who produces two generations of products, which have similar attributes, for example iPhone 6 and iPhone 7.These two generations of products are not only produced with different production processes and hardware configurations, but also have differentiated after-sale services. We define that these two generations of products as new product and old product, and the new one are superior to the old one. We consider the problem that the retailer sets the retail prices for new product and old product, while the manufacturer determine the wholesale prices of them. The unit production cost is c_i, unit wholesale price is w_i, and unit retail price is p_i where $i = 1, 2$, and 1 stands for new product and 2 stands for old product. Note that unit retail price is high than that of unit wholesale price and unit production cost, i.e. $p_i > w_i > c_i$. To avoid null results, we assume that $c_1 > c_2$ due to the new product is better than old product. Otherwise, $c_1 \leq c_2$ will lead to negative demand for old production and meaningless lower bound and upper bound (See more detailed explanations in Section 4.1).

For customers, they are heterogeneous in the valuation of new product and old product. We assume that the customer reservation price (v) is uniformly distributed over $[0, 1]$ within the market size (number of customers) from 0 to 1 with density of 1, which catch the individual difference in the product valuation (Chiang *et al.*, 2003). Considering a product that is priced at p, the customer with a net surplus $v - p \geq 0$ will buy it (Chen and Bell, 2012). From Figure 1, all the customers with valuations in the interval $[p, 1]$ will buy the product. Therefore, the demand of the product is $Q = \int_p^1 dv = 1 - p$ for $0 \le p \le 1$.

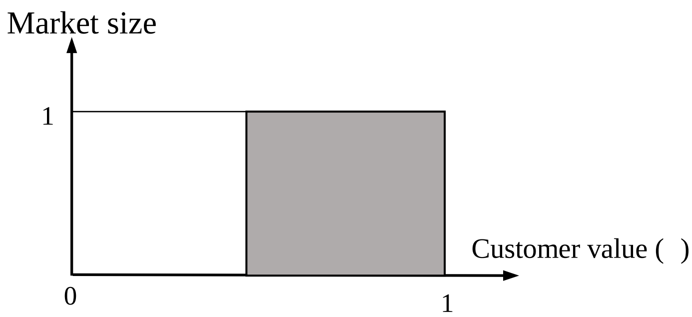

Figure 1. Distribution of customer value

We further assume that the customer perceives new product as a perfect product while the parameter $\theta \in (0,1)$ represents the customer acceptance of old product due to new product's being superior to old products (Martin, 1996). Thus, we use v and θ to capture individual difference in valuing product with different generations: a customer perceives new product and old product to be worth v and θv, respectively.

3.2 Demand and profit functions

A customer with a valuation of v may purchase new product if it has a nonnegative surplus $v - p_1 \ge 0$ and may buy old product if it has a nonnegative surplus $\theta v - p_2 \ge 0$. The customer will choose new product rather than old one if $\theta v - p_2 < v - p_1$. We denote the indifferent values in whether it purchases or not the product as $v_1 = p_1$ and $v_2 = \frac{p_2}{\theta}$, respectively. The indifferent value of purchasing is $v_{21} = \frac{p_1 - p_2}{1-\theta}$. Then two scenarios should be considered:

Scenario 1: if $v_1 > v_2$ or $p_1 > \frac{p_2}{\theta}$, we can derive $\frac{p_1 - p_2}{1-\theta} > p_1$ (or equivalently, $v_{21} > v_1$). Therefore, when $1 > v_{21} > v_1 > v_2$ (or equivalently, $p_2 + 1 - \theta > p_1 > \frac{p_2}{\theta}$), namely $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$, this implies that the customer whose reservation price v is in the range $[v_{21}, 1]$ will purchase new product while purchasing old product if v is in the range $[v_2, v_{21}]$. The customer whose reservation price v is in the range $[0, v_2]$ will buy neither. Since the retailer can select to sell either both new and old product, or new product only, or old product only, if $p_2 + 1 - \theta > p_1 > \frac{p_2}{\theta}$, it is equivalent to the case that the retailer will choose both of them. Therefore, the demands for new product and for old product are $D_1(p_1, p_2) = \int_{v_{21}}^{1} dv = 1 - v_{21} = 1 - \frac{p_1 - p_2}{1-\theta}$ and $D_2(p_1, p_2) = \int_{v_2}^{v_{21}} dv = v_{21} - v_2 = \frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}$ respectively.

When $v_{21} \geq 1 > v_1 > v_2$, it is equivalent to $\theta \geq 1 - p_1 + p_2$. This implies that no customer will purchase new product and the customer whose reservation price v is in the range $[v_2, 1]$ will only purchase old product. Therefore, the demands of new product and old product are $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = \int_{v_2}^{1} dv = 1 - v_2 = 1 - \frac{p_2}{\theta}$.

Scenario 2: if $v_1 \leq v_2$ or $p_1 \leq \frac{p_2}{\theta}$, suggesting $\frac{p_1 - p_2}{1-\theta} \leq p_1$, which is equivalent to $v_{21} \leq v_1$, then we have $v_{21} \leq v_1 \leq v_2 < 1$ or $\theta \leq \frac{p_2}{p_1}$. This implies that no customer will purchase old product and the customer will only purchase new product if v is in the range $[v_1, 1]$. Therefore, demands of new product and old product are $D_1(p_1, p_2) = \int_{v_1}^{1} dv = 1 - v_1 = 1 - p_1$ and $D_2(p_1, p_2) = 0$ respectively.

Therefore, the demand function of new product $D_1(p_1, p_2)$ and old product $D_2(p_1, p_2)$ can be modelled as:

$$D_1(p_1, p_2) = \begin{cases} 1 - p_1 & 0 < \theta \leq \frac{p_2}{p_1} \\ 1 - \frac{p_1 - p_2}{1-\theta} & \frac{p_2}{p_1} < \theta < 1 - p_1 + p_2 \\ 0 & 1 - p_1 + p_2 \leq \theta < 1 \end{cases} \quad (1)$$

$$D_2(p_1, p_2) = \begin{cases} 0 & 0 < \theta \leq \frac{p_2}{p_1} \\ \frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta} & \frac{p_2}{p_1} < \theta < 1 - p_1 + p_2 \\ 1 - \frac{p_2}{\theta} & 1 - p_1 + p_2 \leq \theta < 1 \end{cases} \quad (2)$$

This piecewise demand function gives us an intuitive insight that the product demand depends on the retailer's pricing decisions and the customer acceptance of old product θ. In addition, both the manufacturer and retailer are rational and self-interested, that is, their objective is to maximize their own profit respectively. The model is described in Figure 2.

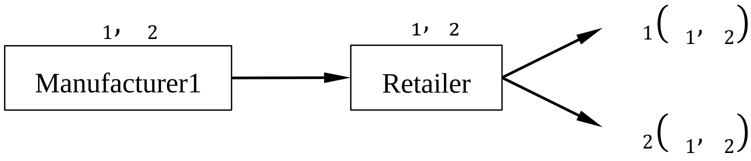

Figure 2. The model framework

We use m and r to represent the manufacturer and the retailer, respectively, then the profit function of manufacturer is:

$$\pi_m(w_1, w_2) = (w_1 - c_1)D_1(p_1, p_2) + (w_2 - c_2)D_2(p_1, p_2) \tag{3}$$

The profit function of retailer is:

$$\pi_r(p_1, p_2) = (p_1 - w_1)D_1(p_1, p_2) + (p_2 - w_2)D_2(p_1, p_2) \tag{4}$$

4 Equilibrium

This section studies the retailer's product choice and discusses under which conditions the retailer sells one generation product or both generations of products in different channel power structures. We first study the case that the retailer sells both generations of products and investigate the retailer's product choice decisions in Section 4.1, and then discuss the case of retailer selling one generation product in Section 4.2.

To examine the supply chain members' competitive dynamics in different market positions, we build a vertical competition model between the manufacturer and retailer as either a Stackelberg game or a Nash game. We use k to represent a model, where $k \in \{MS, VN, RS\}$.

Manufacturer Stackelberg (MS) model

In MS model, the manufacturer is the Stackelberg leader while the retailer is the follower in deciding prices, the decision sequence of the manufacturer and retailer is as follows. In the first-stage game, the manufacturer announces the wholesale price to the retailer, anticipating the retailer's price. In the second-stage game, given the manufacturer's wholesale price, the retailer decides the retail price.

Vertical Nash (VN) model

Under the vertical Nash model, the manufacturer and retailer make their pricing decisions simultaneously. The decision sequence is: the manufacturer decides wholesale price to maximum his profit, anticipating the retailer's margin profit while the retailer decides retail prices to maximum his profit, anticipating the manufacturer's wholesale price.

Here we denote the marginal profits of new product and old product as m_1 and m_2 respectively (Choi, 1991), hence we have $p_1 = m_1 + w_1$ and $p_2 = m_2 + w_2$ that will be used in deriving the optimal policies in proof.

Retailer Stackelberg (RS) model

In RS model, the retailer is the Stackelberg leader while the manufacturer is the follower in deciding prices, the decision sequence of the manufacturer and the retailer: in the first-stage game, the retailer announces the retail price to the manufacturer, anticipating the manufacturer's wholesale price; in the second-stage game, the manufacturer decides the wholesale price, anticipating the retailer's margin profit.

4.1 Retailer sells both generations of products

When $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$, the demand of both generations of products is positive. With (1) to (4), we can prove that there exists unique optimal solutions to the optimal wholesale prices of manufacturer (w_1^k, w_2^k) and to optimal retail prices of retailer (p_1^k, p_2^k) when both products are available, which be summarized in Lemma 1.

Lemma 1: ***For any game model k, there exists a unique optimal solution to the optimal retail prices of retailer (p_1^k, p_2^k) and to the optimal wholesale prices of manufacturer (w_1^k, w_2^k) for the case of retailer selling both generations of products, which are summarized in Table 1.***

Table 1. The optimal retail price, wholesale price, corresponding sales volume, and profit under different power structures when selling both generation of products

Game models	p_1^k	p_2^k	w_1^k	w_2^k	D_1^k	D_2^k	π_m^k	π_r^k
MS	$\frac{3+c_1}{4}$	$\frac{3\theta+c_2}{4}$	$\frac{1+c_1}{2}$	$\frac{\theta+c_2}{2}$	$\frac{1-\theta-c_1+c_2}{4(1-\theta)}$	$\frac{\theta c_1-c_2}{4(1-\theta)\theta}$	$\frac{A}{8}$	$\frac{A}{16}$
VN	$\frac{2+c_1}{3}$	$\frac{2\theta+c_2}{3}$	$\frac{1+2c_1}{3}$	$\frac{\theta+2c_2}{3}$	$\frac{1-\theta-c_1+c_2}{3(1-\theta)}$	$\frac{\theta c_1-c_2}{3(1-\theta)\theta}$	$\frac{A}{9}$	$\frac{A}{9}$
RS	$\frac{3+c_1}{4}$	$\frac{3\theta+c_2}{4}$	$\frac{1+3c_1}{4}$	$\frac{\theta+3c_2}{4}$	$\frac{1-\theta-c_1+c_2}{4(1-\theta)}$	$\frac{\theta c_1-c_2}{4(1-\theta)\theta}$	$\frac{A}{16}$	$\frac{A}{8}$

where $A = \frac{\theta(1-c_1)(1-c_1-\theta+c_2)+(\theta-c_2)(\theta c_1-c_2)}{(1-\theta)\theta}$ and $k \in \{MS, VN, RS\}$. The assumption $c_1 > c_2$ is needed here. Otherwise $D_2^k < 0$. That is, if $c_1 \le c_2$, the demand for old product is meaningless.

In different power structure, we denote the lower bound and upper bound with a superscript k. From Table 1 and the condition $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$ in demand function, the lower bound $\underline{\theta}^k$ and upper bound $\overline{\theta}^k$ in different power structure can be obtained, and which summarized in Proposition 1.

Proposition 1:

1) For any game model k, the retailerôs product choice decision criterions are: the lower bound $\underline{\theta}^{MS} = \underline{\theta}^{VN} = \underline{\theta}^{RS} = \frac{c_2}{c_1}$ and the upper bound $\overline{\theta}^{MS} = \overline{\theta}^{VN} = \overline{\theta}^{RS} = c_2 - c_1 + 1$, where $\frac{c_2}{c_1} < c_2 - c_1 + 1$ for any $0 < c_2 < c_1 < 1$[1].

2) For any game model k, when $0 < \theta \leq \frac{c_2}{c_1}$, the retailer will choose new product only; and if $\frac{c_2}{c_1} < \theta < c_2 - c_1 + 1$, the retailer will choose both generations of products; and if $c_2 - c_1 + 1 \leq \theta < 1$, the retailer will choose old product only.

This proposition provides the retailer's product choice decision criterions and behaviors. On one hand, Part 1) gives the robustness of retailer's product choice decision criterions. The thresholds among MS, VN, and RS power structure are consistent with each other. The lower bound in MS, VN and RS models are the same, i.e., $\frac{c_2}{c_1}$, and the upper bound are also the same, i.e., $c_2 - c_1 + 1$. From this, we can confirm that power structure between supply chain members has no effect on retailer's product choice decision criterions, and the retailer's product choice criterions is steady in any competition situations. This is a very interesting conclusion because from the proof we know that these two bounds highly depend on the optimal retail prices, and in each game model the optimal retail prices are not the same. However, the product choice decision criterions are consistent. Actually the retailer use pricing policy with customer value to segment customer into different types. In addition, the thresholds depend on the cost of new product and old product. If the retailer could acquire manufacturer's cost information (c_1, c_2) and estimate customer acceptance of old product θ based on historical data, expertise, or industrial reports on similar products, he can build up a visual product choice standard and assess retailer's product choice decision behaviors that whether it should choose both products, new product only, or old product only. If the customer acceptance of old product is sufficiently high (or low), no customer will purchase old product (or new product). Therefore, selling old product (or new product) is the optimal choice for the retailer to maximum its profit. When the customer acceptance of old

[1] The assumption $c_1 > c_2$ is needed here. Otherwise, if $c_1 \leq c_2$ the lower bound $\frac{c_2}{c_1} \geq 1$ and the upper bound $c_2 - c_1 + 1 \geq 1$, and the lower bound will be larger than the upper bound, $\frac{c_2}{c_1} \geq c_2 - c_1 + 1$, that is unreasonable. Therefore, $c_1 \leq c_2$ would make no sense to our model.

product is moderate, the retailer's optimal product choice is to sell both of them.

4.2 Retailer sells one generation product

When the customer acceptance of old product is either sufficiently low or sufficiently high, the retailer will sell either new product only or old product only, respectively. To obtain the equilibrium, we start with resolving the last-stage game and moving back to the first-stage game for all three game models. With (1) to (4), we can prove that there exists unique optimal solutions to the optimal wholesale price of manufacturer w_i^k and to optimal retail price of retailer p_i^k when one generation product is available. Therefore, the optimal pricing strategy to the retailer and manufacturer can be summarized in Lemma 2. The subscript $s1$ means retailer only sells new product and $s2$ means retailer only sells old product.

Lemma 2: ***For any game model k, there exists a unique optimal solution to the optimal retail price of retailer p_i^k and to the optimal wholesale price of manufacturer w_i^k for the case of retailer selling one generation product, which are summarized in Table 2.***

Table 2. The optimal retail price, wholesale price, corresponding sales volume, and profit under different power structures when selling one generation product

	$0<\theta\le\frac{c_2}{c_1}$				$c_2-c_1+1\le\theta<1$		
	MS	VN	RS		MS	VN	RS
p_{s1}^k	$\frac{3+c_1}{4}$	$\frac{2+c_1}{3}$	$\frac{3+c_1}{4}$	p_{s2}^k	$\frac{3\theta+c_2}{4}$	$\frac{2\theta+c_2}{3}$	$\frac{3\theta+c_2}{4}$
w_{s1}^k	$\frac{1+c_1}{2}$	$\frac{1+2c_1}{3}$	$\frac{1+3c_1}{4}$	w_{s2}^k	$\frac{\theta+c_2}{2}$	$\frac{\theta+2c_2}{3}$	$\frac{\theta+3c_2}{4}$
D_{s1}^k	$\frac{1-c_1}{4}$	$\frac{1-c_1}{3}$	$\frac{1-c_1}{4}$	D_{s2}^k	$\frac{\theta-c_2}{4\theta}$	$\frac{\theta-c_2}{3\theta}$	$\frac{\theta-c_2}{4\theta}$
π_{ms1}^k	$\frac{(1-c_1)^2}{8}$	$\frac{(1-c_1)^2}{9}$	$\frac{(1-c_1)^2}{16}$	π_{ms2}^k	$\frac{(\theta-c_2)^2}{8\theta}$	$\frac{(\theta-c_2)^2}{9\theta}$	$\frac{(\theta-c_2)^2}{16\theta}$
π_{rs1}^k	$\frac{(1-c_1)^2}{16}$	$\frac{(1-c_1)^2}{9}$	$\frac{(1-c_1)^2}{8}$	π_{rs2}^k	$\frac{(\theta-c_2)^2}{16\theta}$	$\frac{(\theta-c_2)^2}{9\theta}$	$\frac{(\theta-c_2)^2}{8\theta}$

where $c_2<\theta<1$ and $k\in\{MS,VN,RS\}$.

5 Discussions

In this part, we discuss the effect of the customer acceptance of old product as well as power structure on the decisions and profits of the supply chain.

5.1 Impact of the customer acceptance of old products θ

Proposition 1 shows that when $\theta \in (\frac{c_2}{c_1}, c_2 - c_1 + 1)$, the retailer sells both products. From Table 1, we can obtain the following proposition.

Proposition 2: ***For any game model k, when $\theta \in (\frac{c_2}{c_1}, c_2 - c_1 + 1)$, then $w_1^k > w_2^k$ and $p_1^k > p_2^k$; if $\theta \in (\frac{c_2}{c_1}, \theta_0]$, then $D_1^k \geq D_2^k$, and if $\theta \in (\theta_0, c_2 - c_1 + 1)$, then $D_1^k < D_2^k$, where $\theta_0 = \frac{1-2c_1+c_2+\sqrt{(1-2c_1+c_2)^2+4c_2}}{2}$.***

This proposition indicates that in MS, VN and RS power structures, when the retailer sells both new and old product, the optimal wholesale prices and optimal retail prices for new product are higher than that of for old product. Since the customer perceives new product as higher value, the retailer thus can charge a higher retail price, which leaves a room for new product to charge a higher wholesale price as well. On the other hand, due to the low customer acceptance of old product, to attract more lower-value customers, the retailer should set a lower retail price which leads to a lower wholesale price. For the demand, it is easy to understand that low acceptance of old product will trigger much more demand of new product, so it is better for the retailer to order new product more under low customer acceptance of old product, and vice versa.

Proposition 3: ***When $\theta \in \left(\frac{c_2}{c_1}, c_2 - c_1 + 1\right)$, we have $\frac{\partial w_2^{MS}}{\partial \theta} > \frac{\partial w_2^{VN}}{\partial \theta} > \frac{\partial w_2^{RS}}{\partial \theta} > 0$, $\frac{\partial p_2^{MS}}{\partial \theta} = \frac{\partial p_2^{RS}}{\partial \theta} > \frac{\partial p_2^{VN}}{\partial \theta} > 0$, $\frac{\partial \pi_m^{MS}}{\partial \theta} > \frac{\partial \pi_m^{VN}}{\partial \theta} > \frac{\partial \pi_m^{RS}}{\partial \theta} > 0$ and $\frac{\partial \pi_r^{RS}}{\partial \theta} > \frac{\partial \pi_r^{VN}}{\partial \theta} > \frac{\partial \pi_r^{MS}}{\partial \theta} > 0$.***

This proposition does a sensitivity analysis of the customer acceptance of old product on the optimal wholesale prices, retail prices and maximum profits. No surprising that as the customer acceptance of old product increases, the optimal wholesale prices and retail prices of old product, the maximum profits of manufacturer, retailer and entire supply chain will improve as well. However, they are changing in different degrees. For the manufacturer, the changes of optimal wholesale prices and maximum profits in MS market power is larger than that in VN and RS market power. Namely, the manufacturer is more sensitive with the customer acceptance of old product when he is a Stackelberg leader. For the retailer, the optimal retail prices of old product change less when the supply chain members involved in more intense competition than when he is dominate or dominated by the manufacturer. However, for retailer's maximum profits, the more powerful the retailer is, the more sensitive it is with the customer acceptance of old product.

5.2 Impact of power structure

5.2.1 Retailer sells both generations of products

Define that when the retailer sells both generations of products, the profit of the entire supply chain is $\pi_s = \pi_m + \pi_r$. Furthermore, we denote the proportion of new product purchased or sold and of old product purchased or sold as $\alpha_1 = \frac{D_1}{D_1+D_2}$ and $\alpha_2 = \frac{D_2}{D_1+D_2}$, respectively. With Table 1, the following proposition indicates the impact of power structure for the case when the retailer sells both products.

Proposition 4: ***When the retailer sells both products (if*** $\boldsymbol{\frac{c_2}{c_1} < \theta < c_2 - c_1 + 1}$***), the following properties hold:***

(1) $\boldsymbol{w_1^{MS} > w_1^{VN} > w_1^{RS}}$ ***and*** $\boldsymbol{w_2^{MS} > w_2^{VN} > w_2^{RS}}$.

(2) $\boldsymbol{p_1^{MS} = p_1^{RS} > p_1^{VN}}$ ***and*** $\boldsymbol{p_2^{MS} = p_2^{RS} > p_2^{VN}}$.

(3) $\boldsymbol{D_1^{MS} = D_1^{RS} < D_1^{VN}}$ ***and*** $\boldsymbol{D_2^{MS} = D_2^{RS} < D_2^{VN}}$***;*** $\boldsymbol{\alpha_1^{MS} = \alpha_1^{VN} = \alpha_1^{RS}}$ ***and*** $\boldsymbol{\alpha_2^{MS} = \alpha_2^{VN} = \alpha_2^{RS}}$.

(4) $\boldsymbol{\pi_m^{MS} > \pi_m^{VN} > \pi_m^{RS}}$, $\boldsymbol{\pi_r^{RS} > \pi_r^{VN} > \pi_r^{MS}}$ ***and*** $\boldsymbol{\pi_s^{VN} > \pi_s^{MS} = \pi_s^{RS}}$.

This proposition illustrates the impact of power structure when the retailer sells both new and old product. Part (1), (2) and (3) are similar with some conclusions in Proposition 4: the more power the manufacturer has, the higher wholesale prices of new and old products he sets. The retail prices and product demand of the new and old products are the same respectively when the retailer is more dominant or dominated by the manufacturer. And intense competition in balanced power structure results in lower retail prices, which is benefit for customers and in turn triggers much product demand of both new and old products. It is interesting to see that from part (3), no matter which power structure between the supply chain members, and no matter how many products the retailer purchases or sells, the proportion of new product or old product in different market power stays unchanged respectively. Part (4) shows that when the manufacturer or retailer is a Stackelberg leader (MS or RS model), he will gain more profits compared with that when he is Stackelberg follower (RS or MS model). The profit of the entire supply chain is as same to either the manufacturer is Stackelberg leader or the retailer is the Stackelberg leader in the supply chain, and however, balanced power between the manufacturer and retailer is profitable for the entire supply chain.

5.2.2 Retailer sells one generation product

Define that when the retailer sells new product only if $0 < \theta \leq \frac{c_2}{c_1}$, the profit of the entire supply chain is $\pi_{s1} = \pi_{ms1} + \pi_{rs1}$; when the retailer sells old product only if $c_2 - c_1 + 1 \leq \theta < 1$, the profit of the entire supply chain is $\pi_{s2} = \pi_{ms2} + \pi_{rs2}$. With Table 2, the following proposition indicates the impact of power structure for the case when the retailer sells one generation product only.

Proposition 5:

(1) When the retailer sells new product only (if $\mathbf{0 < \theta \leq \frac{c_2}{c_1}}$***), we have*** $\boldsymbol{p_{s1}^{MS} = p_{s1}^{RS} > p_{s1}^{VN}}$, $\boldsymbol{w_{s1}^{MS} > w_{s1}^{VN} > w_{s1}^{RS}}$, $\boldsymbol{D_{s1}^{VN} > D_{s1}^{MS} = D_{s1}^{RS}}$, $\boldsymbol{\pi_{ms1}^{MS} > \pi_{ms1}^{VN} > \pi_{ms1}^{RS}}$, $\boldsymbol{\pi_{rs1}^{RS} > \pi_{rs1}^{VN} > \pi_{rs1}^{MS}}$ ***and*** $\boldsymbol{\pi_{s1}^{VN} > \pi_{s1}^{MS} = \pi_{s1}^{RS}}$.

(2) When the retailer sells old product only (if $\mathbf{c_2 - c_1 + 1 \leq \theta < 1}$***), we have*** $\boldsymbol{p_{s2}^{MS} = p_{s2}^{RS} > p_{s2}^{VN}}$, $\boldsymbol{w_{s2}^{MS} > w_{s2}^{VN} > w_{s2}^{RS}}$, $\boldsymbol{D_{s2}^{VN} > D_{s2}^{MS} = D_{s2}^{RS}}$, $\boldsymbol{\pi_{ms2}^{MS} > \pi_{ms2}^{VN} > \pi_{ms2}^{RS}}$, $\boldsymbol{\pi_{rs2}^{RS} > \pi_{rs2}^{VN} > \pi_{rs2}^{MS}}$ ***and*** $\boldsymbol{\pi_{s2}^{VN} > \pi_{s2}^{MS} = \pi_{s2}^{RS}}$.

This proposition illustrates the impact of power structure when the retailer sells one generation product only. We know that if the customer acceptance of old product is lower ($0 < \theta \leq \frac{c_2}{c_1}$) or higher ($c_2 - c_1 + 1 \leq \theta < 1$), results in Proposition 5 are in line with some studies in the literature (for example, Choi, 1991; Chen and Wang, 2014; Chen *et al.*, 2015). Part (1) or (2) indicates that when the retailer sells new product only or old product only, the imbalanced power between the retailer and manufacturer (MS and RS power structure) has no influence on retailer's optimal prices, therefore, the retailer may have more flexibility compared with manufacturer in different power structures. However, the balanced power (VN power structure) will lead to lower retail prices, which can be explained by the fact that in VN power structure more intense competition between supply chain members will drive the prices down, and that will benefit customers. As to the lower retail prices in VN power structure, it drives much more product demand than that of other imbalanced power structures. Powerful manufacturer will set high wholesale prices, which give him high margin profits, therefore, the manufacturer who is more dominant will gain more profits than when he is dominated by the retailer. Similarly, when the retailer is a leader, he will also gain more profits than that when he is a follower. In other words, either the manufacturer or the retailer will gain more profits when one of them is more powerful in the supply chain. The entire supply chain as well as the customer, however, will benefit from higher profits and lower prices when there is no channel member is dominant.

6 Extended model with revenue sharing contract

6.1 Integrated retail supply chain

In this section, we discuss the optimal retail prices of integrated retail supply chain which is used as a benchmark. Here we just analyze the complex case when the retailer sells both generations of products, that is $\frac{c_2}{c_1} < \theta < c_2 - c_1 + 1$. Therefore, the profit function of integrated retail supply chain, denoted as $\pi^I(p_1, p_2)$, is

$$\pi^I(p_1, p_2) = (p_1 - c_1)(1 - \frac{p_1 - p_2}{1-\theta}) + (p_2 - c_2)(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}) \qquad (5)$$

It is easy to obtain the optimal retail prices of integrated retail supply chain, as well as corresponding sales volume and maximum profit. We summarize the above in Lemma 3.

Lemma 3: $\boldsymbol{p_1^I = \frac{1+c_1}{2},\ p_2^I = \frac{\theta+c_2}{2},\ D_1^I = \frac{1-\theta-c_1+c_2}{2(1-\theta)},\ D_2^I = \frac{\theta c_1 - c_2}{2(1-\theta)\theta}}$ ***and*** $\boldsymbol{\pi^I(p_1^I, p_2^I) = \frac{A}{4}}$, ***where*** $\boldsymbol{A = \frac{\theta(1-c_1)(1-c_1-\theta+c_2)+(\theta-c_2)(\theta c_1 - c_2)}{(1-\theta)\theta}}$.

Lemma 3 indicates that the integrated retail supply chain system will gain a maximal profit $\pi^I(p_1^I, p_2^I) = \frac{A}{4}$ with optimal retail prices $p_1^I = \frac{1+c_1}{2}$ and $p_2^I = \frac{\theta+c_2}{2}$. Compared with decentralized retail supply chain under different power structure, integrated system has a lower retail price, high sales volume and maximum profit than that of decentralized one. Both the manufacturer and retailer in decentralized supply chain aim to capture the most profit which will cause double marginalization. Therefore, a price contract with a specified quantity cannot coordinate the supply chain effectively. If and only if the manufacturer sets its wholesale price equal to the production cost, the retailer can get a profit of $\frac{A}{4}$, but the manufacturer will get nothing. Therefore, without other contract to guarantee positive profit, manufacturer will never decrease wholesale price to production cost.

6.2 Revenue sharing contract

The previous study is based on a price contract with a specified quantity, which is actually a price contract with a specified quantity. However, in this section we extend our work based on other coordination contract, for example, revenue sharing contract. This contract is an important and typical contract, which was early used in the video cassette rental industry and gain great success (Cachon and Lariviere, 2005) and now is widely studied and applied in many areas (Gerchak and Wang, 2004; Li *et al*., 2009; Kong *et al*., 2013; Tang *et al*., 2014; Zhang *et al*., 2015). Here we work on the extended model with revenue sharing contract to study the complex case when the retailer sells both generations of products only, because the case when the retailer sells one generation product is relative simple. We assume that the manufacturer sets wholesale price w_{r1} and w_{r2} (in this section, all the variables with a new subscript r mean the revenue sharing contract), the retailer gives the manufacturer a percentage of his revenue. Let ϕ be the fraction of retailer's revenue that retailer himself keeps, so $1-\phi$ is the fraction the manufacturer earns where $0<\phi<1$. Therefore, for the case when the retailer sells both generations of products, the profit function of retailer is:

$$\pi_r(p_{r1},p_{r2}) = (\phi p_{r1}-w_{r1})\left(1-\frac{p_{r1}-p_{r2}}{1-\theta}\right)+(\phi p_{r2}-w_{r2})\left(\frac{p_{r1}-p_{r2}}{1-\theta}-\frac{p_{r2}}{\theta}\right) \quad (6)$$

The profit function of retailer is:

$$\pi_m(w_{r1},w_{r2}) = [w_{r1}-c_1+(1-\phi)p_{r1}]\left(1-\frac{p_{r1}-p_{r2}}{1-\theta}\right)+[w_{r2}-c_2+(1-\phi)p_{r2}]\left(\frac{p_{r1}-p_{r2}}{1-\theta}-\frac{p_{r2}}{\theta}\right) \quad (7)$$

Regarding the supply chain coordination with the revenue sharing contract, the following proposition can be obtained.

Proposition 6: ***The retail supply chain can be coordinated with revenue sharing contract with the condition satisfies*** $\boldsymbol{w_{r1}=\phi c_1}$, $\boldsymbol{w_{r2}=\phi c_2}$***. With this contract, the retailerôs profit is*** $\boldsymbol{\frac{A}{4}\phi}$ ***and the manufacturerôs profit is*** $\boldsymbol{\frac{A}{4}(1-\phi)}$***, where*** $\boldsymbol{\phi}$ ***satisfies*** $\boldsymbol{\frac{1}{4}<\phi<\frac{1}{2}}$ ***in MS power structure,*** $\boldsymbol{\phi=\frac{1}{2}}$ ***in VN power structure, and*** $\boldsymbol{\frac{1}{2}<\phi<\frac{3}{4}}$ ***in RS power structure, respectively.***

This proposition indicates that the revenue sharing contract can coordinate the retail supply chain and achieve the Pareto improvement under the given conditions for different power structures. With the revenue sharing contract we have designed, the manufacturer should set the wholesale prices which is less than unit production cost namely, $w_{r1} = \phi c_1$ and $w_{r2} = \phi c_2$. From this perspective, the manufacturer cannot gain any profit from the product sales directly, however, the retailer obtains profit from the product sales, and it share a fraction of revenue to the manufacturer to compensate the manufacturer's sacrifice. After making "a bigger pie" through collaboration by revenue sharing contract (Please note that in MS model $\frac{A}{8} + \frac{A}{16} < \frac{A}{4}$, in VN model $\frac{A}{9} + \frac{A}{9} < \frac{A}{4}$, and in RS model $\frac{A}{16} + \frac{A}{8} < \frac{A}{4}$), the pie can be split between manufacturer and retailer, $\frac{A}{4}\phi$ for the retailer while $\frac{A}{4}(1-\phi)$ for the manufacturer. However, the allocation is not arbitrary, since the particular profit allocation ratio ϕ chosen probably depends on the firms' relative bargaining power. In MS power structure, the manufacturer has more bargaining power than the retailer, so the manufacturer will ask for more than half of the revenue (because $\frac{1}{4} < \phi < \frac{1}{2}$, then $\frac{1}{2} < 1 - \phi < \frac{3}{4}$). Similarly, in the RS model, the retailer has more power to own more than half of his revenue ($\frac{1}{2} < \phi < \frac{3}{4}$). And in the VN power structure, the retailer and the manufacturer have balanced power, theoretically they will divide all the profits equally ($\phi = \frac{1}{2}$). Therefore, we can conclude that the supply chain coordination by revenue sharing contract helps make the pie bigger than that without coordination, and the power structure determines how to split the pie between supply chain members, the supply chain member who has more power will share more pie. The following figure gives us an obvious description of mechanism design of coordination parameter ϕ in different power structure.

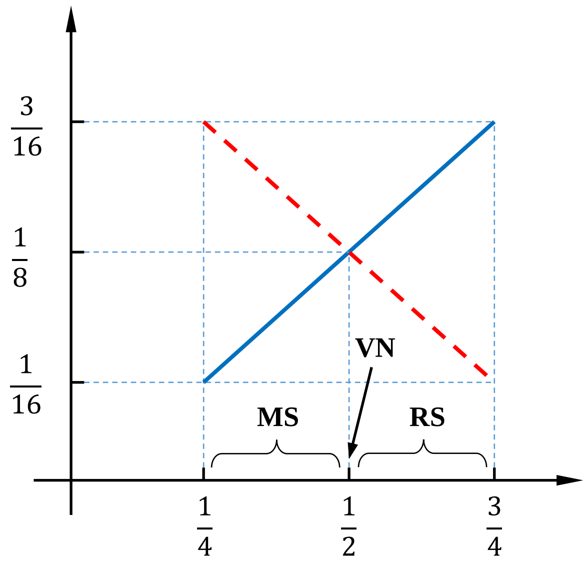

Figure 3. Profit allocation ratio in different power structure

From the above propositions we know that this revenue can coordinate the retail supply chain, so the retailer can sell the products as the optimal price $p_{r1} = p_1^I = \frac{1+c_1}{2}$ and $p_{r2} = p_2^I = \frac{\theta+c_2}{2}$. However, will this coordinating contract influence the product choice decision? The following proposition gives us an answer.

Proposition 7: ***With the revenue sharing contract, the retailerôs product choice decision criterions are: the lower bound*** $\underline{\boldsymbol{\theta}}^r = \frac{c_2}{c_1}$ ***and the upper bound*** $\overline{\boldsymbol{\theta}}^r = c_2 - c_1 + 1$***, where*** $\frac{c_2}{c_1} < c_2 - c_1 + 1$ ***for any*** $0 < c_2 < c_1 < 1$.

This proposition gives us an insight of the retailer's product choice decision with revenue sharing contract. We find that there exist a lower bound $\underline{\theta}^r = \frac{c_2}{c_1}$ and an upper bound $\overline{\theta}^r = c_2 - c_1 + 1$, which provide retailer the product choice decision criterions. According to Proposition 1, we know that the bounds are dependent on the optimal pricing policies. However, it is interesting to see that different optimal pricing policies with wholesale price contract and revenue sharing contract result to the same product choice decision criterions. Therefore, we believe that the revenue sharing contract does well in coordinating the retail supply chain and achieving a Pareto improvement, but does not affect retailer's product choice. This is an important conclusion for a retailer who faces end-user directly, the retailer does not need to worry about product choice problem with different contract.

7 Conclusions

In this paper, we studies two-stage retail supply chain consisting of one manufacturer and one retailer. The manufacturer produces two generations of products which are classified into new and old. However, customers are heterogeneous in the valuation of these two products due to the different production processes, hardware configurations, and differentiated after-sale services, or result from two different generation products. Based on the customer's net surplus, we segment the markets and derive demand functions for the new and old products. To reduce inventory risk while effectively meet customer demand, the retailer should decide which products (single or both) to purchase and how to pricing. In addition, market power has been considered in our models. This study provides several interesting observations.

Observation 1. Retailer's product choice decision criterions and behaviors can be quantized by identifying a low threshold and a high threshold that are related to the production costs (c_1,c_2), and estimating the customer acceptance of old product (θ) in each power structure. What's more, the product choice decision criterions and behaviors will not affected by power structure, and they are steady in any competition situations.

Observation 2. Different power structures have a great influence on the retail supply chain's decisions and profits. No matter the retailer sells one generation product or both generations of products, in VN power structure the manufacturer and retailer engage in intense competition, the retailer will set relatively low retail prices than that in MS and RS power structure. The manufacture will set relatively high wholesale prices when he is a leader. Both manufacturer and retailer will capture greater profits when he is more dominate than other. The entire supply chain will be in the best performance when the manufacturer and retailer have balanced power. On the other hand, the sensitivity of supply chain members' and the entire supply chain's profits to the customer acceptance of old products in different power structures is consistent with the relationship of profits in different power structures. That is to say, more power will get firm greater profits, however, it may also bring massive loss as the change of consumer behavior.

Observation 3. A revenue sharing contract can coordinate the retail supply chain and achieve the Pareto improvement under the given conditions. The profit allocation is decided by the bargaining power between them. That is, the revenue sharing contract helps the entire supply chain make a bigger pie, and the power structure helps to split the pie, namely, the one with more power will be allocated more profit. Further, we find that under revenue sharing contract the lower bound and higher bound are consistent with that under wholesale price contract. That is, no matter wholesale price contract or revenue sharing contract between the manufacturer and retailer, the retailer's purchase decision criterions will not be influenced.

This study provides a general analytical framework for pricing and product choice behavior based on customer value theory in a two-stage retail supply chain with one manufacturer and one retailer. For the industrial applications of our model, our research can offer insightful managerial implications. For the retailer, he would like to get the cost information of products, meanwhile estimate the customer acceptance of old product based on some historical data, expertise, or the industrial reports on the similar products, to quantize the product choice decisions. For the manufacturer, though the retailer's product choice decisions depend on the cost information, the manufacturer can manipulate it. Because our model do not consider manufacturer's stock-holding cost, if one generation products encounter poor sales, the manufacturer can adjust wholesale prices purposely to change retailer's purchase decision. However, that will be at the cost of profits. As other models used in the literatures, our model is built based on some assumptions too. For example, our model assumes that a retailer sells two substitutable products belong to different generations purchased from one manufacturer. One meaningful extension of this work is to consider two or multiple retailers who sell substitutable products form two or multiple manufacturers, in which the chain to chain competition can be studied. Another extension is to consider stochastic demand based on customer value theory in the future to explore the effect of demand uncertainty on pricing decision and product choice.

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Appendix

Proof of Lemma 1

From demand function (1) and (2), when $\frac{p_2}{p_1} < \theta < 1 - p_1 + p_2$, the profit function of manufacturer is:

$$\pi_m(w_1, w_2) = (w_1 - c_1)(1 - \frac{p_1 - p_2}{1-\theta}) + (w_2 - c_2)(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}) \qquad \text{(A1)}$$

the profit function of retailer is:

$$\pi_r(p_1, p_2) = (p_1 - w_1)(1 - \frac{p_1 - p_2}{1-\theta}) + (p_2 - w_2)(\frac{p_1 - p_2}{1-\theta} - \frac{p_2}{\theta}) \qquad \text{(A2)}$$

MS model:

Form (A1), we get $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{-1+\theta+2p_1-2p_2-w_1+w_2}{-1+\theta}$, $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = -\frac{2\theta p_1-2p_2-\theta w_1+w_2}{(-1+\theta)\theta}$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} = -\frac{2}{1-\theta} < 0$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} = -\frac{2}{1-\theta} - \frac{2}{\theta}$, and $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{2}{1-\theta}$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} \end{vmatrix} = \frac{4}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_1, p_2)$ is joint concave in p_1 and p_2.

Let $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = 0$, we obtain

$$\frac{-1+\theta+2p_1-2p_2-w_1+w_2}{-1+\theta}=0 \tag{a-1}$$

Let $\frac{\partial\pi_r(p_1,p_2)}{\partial p_2}=0$, we obtain

$$-\frac{2\theta p_1-2p_2-\theta w_1+w_2}{(-1+\theta)\theta}=0 \tag{a-2}$$

Then, we can derive $p_1=\frac{1+w_1}{2}$ and $p_2=\frac{\theta+w_2}{2}$. Substituting p_1 and p_2 into (3), we can get:

$$\pi_m(w_1,w_2)=(-c_1+w_1)\left[1-\frac{\frac{1}{2}(1+w_1)+\frac{1}{2}(-\theta-w_2)}{1-\theta}\right]+(-c_2+w_2)\left[\frac{\frac{1}{2}(1+w_1)+\frac{1}{2}(-\theta-w_2)}{1-\theta}-\frac{\theta+w_2}{2\theta}\right] \tag{A3}$$

From (A3), we get $\frac{\partial\pi_m(w_1,w_2)}{\partial w_1}=\frac{-1+\theta+2w_1-2w_2-c_1+c_2}{2(-1+\theta)}$, $\frac{\partial\pi_m(w_1,w_2)}{\partial w_2}=\frac{c_2-c_1\theta+2w_1\theta-2w_2}{2(1-\theta)\theta}$, $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1^2}=-\frac{1}{1-\theta}<0$, $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2^2}=-\frac{1}{1-\theta}-\frac{1}{\theta}$ and $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1\partial w_2}=\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2\partial w_1}=\frac{1}{1-\theta}>0$. Then $\begin{vmatrix}\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1^2} & \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1\partial w_2}\\ \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2\partial w_1} & \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2^2}\end{vmatrix}=\frac{1}{(1-\theta)\theta}>0$. Therefore, $\pi_m(w_1,w_2)$ is joint concave in w_1 and w_2. Let $\frac{\partial\pi_m(w_1,w_2)}{\partial w_1}=0$ and $\frac{\partial\pi_m(w_1,w_2)}{\partial w_2}=0$, we obtain $\frac{-1+\theta+2w_1-2w_2-c_1+c_2}{2(-1+\theta)}=0$ and $\frac{c_2-c_1\theta+2w_1\theta-2w_2}{2(1-\theta)\theta}=0$. Then, we can derive $w_1^{MS}=\frac{c_1+1}{2}$ and $w_2^{MS}=\frac{\theta+c_2}{2}$. Replacing w_1 and w_2 with w_1^{MS} and w_2^{MS} into p_1 and p_2, then we have $p_1^{MS}=\frac{3+c_1}{4}$ and $p_2^{MS}=\frac{3\theta+c_2}{4}$.

VN model:

We denote the marginal profits of new product and old product as $m_1=p_1-w_1$ and $m_2=p_2-w_2$, respectively. Then the manufacturers' profit functions become

$$\pi_m(w_1)=(w_1-c_1)\left[1-\frac{(m_1+w_1)-(m_2+w_2)}{1-\theta}\right]+(w_2-c_2)\left[\frac{(m_1+w_1)-(m_2+w_2)}{1-\theta}-\frac{(m_2+w_2)}{\theta}\right] \tag{A4}$$

Form (A4), we get $\frac{\partial\pi_m(w_1,w_2)}{\partial w_1}=\frac{1-\theta-w_1+w_2-p_1+p_2+c_1-c_2}{1-\theta}$, $\frac{\partial\pi_m(w_1,w_2)}{\partial w_2}=\frac{\theta c_1-c_2-\theta p_1+p_2-\theta w_1+w_2}{(-1+\theta)\theta}$, $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1^2}=-\frac{1}{1-\theta}<0$, $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2^2}=-\frac{1}{1-\theta}-\frac{1}{\theta}$ and $\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1\partial w_2}=\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2\partial w_1}=\frac{1}{1-\theta}>0$. Then $\begin{vmatrix}\frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1^2} & \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_1\partial w_2}\\ \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2\partial w_1} & \frac{\partial^2\pi_m(w_1,w_2)}{\partial w_2^2}\end{vmatrix}=\frac{1}{(1-\theta)\theta}>0$.

Therefore, $\pi_m(w_1, w_2)$ is joint concave in w_1 and w_2.

Let $\frac{\partial \pi_m(w_1,w_2)}{\partial w_1} = 0$, we obtain

$$\frac{1-\theta-w_1+w_2-p_1+p_2+c_1-c_2}{1-\theta} = 0 \tag{a-3}$$

Let $\frac{\partial \pi_m(w_1,w_2)}{\partial w_2} = 0$, we obtain

$$\frac{\theta c_1-c_2-\theta p_1+p_2-\theta w_1+w_2}{(-1+\theta)\theta} = 0 \tag{a-4}$$

From (a-1) to (a-4), we get $w_1^{VN} = \frac{1+2c_1}{3}$, $w_2^{VN} = \frac{\theta+2c_2}{3}$, $p_1^{VN} = \frac{2+c_1}{3}$ and $p_2^{VN} = \frac{2\theta+c_2}{3}$.

RS model:

From (a-3) and (a-4), we get $w_1 = 1 + c_1 - p_1$ and $w_2 = \theta + c_2 - p_2$. Substitute w_1 and w_2 to (4), we get

$$\pi_r(p_1,p_2) = [p_1 - (1 + c_1 - p_1)]\left(1 - \frac{p_1-p_2}{1-\theta}\right) + [(p_2 - (\theta + c_2 - p_2))]\left(\frac{p_1-p_2}{1-\theta} - \frac{p_2}{\theta}\right) \tag{A4}$$

From (A4), $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{-3+3\theta-c_1+c_2+4p_1-4p_2}{-1+\theta}$, $\frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = \frac{\theta c_1-c_2-4\theta p_1+4p_2}{(-1+\theta)\theta}$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} = -\frac{4}{1-\theta} < 0$, $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} = -\frac{4}{1-\theta} - \frac{4}{\theta}$, and $\frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{4}{1-\theta}$. Then $\begin{vmatrix} \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1^2} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_r(p_1,p_2)}{\partial p_2^2} \end{vmatrix} = \frac{16}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_1,p_2)$ is joint concave in p_1 and p_2.

Let $\frac{\partial \pi_r(p_1,p_2)}{\partial p_1} = \frac{\partial \pi_r(p_1,p_2)}{\partial p_2} = 0$, we can derive $p_1^{RS} = \frac{3+c_1}{4}$ and $p_2^{RS} = \frac{3\theta+c_2}{4}$. Replacing p_1 and p_2 with p_1^{RS} and P_2^{RS} into w_1 and w_2, then we have $w_1^{RS} = \frac{1+3c_1}{4}$ and $w_2^{RS} = \frac{\theta+3c_2}{4}$. This completes the proof.

Proof of Proposition 1

1) From Table 1, for the MS model, when selling both generations of products, the optimal prices are $p_1^{MS} = \frac{3+c_1}{4}$ and $p_2^{MS} = \frac{3\theta+c_2}{4}$. The optimal solutions p_1^{MS} and p_2^{MS} must satisfy $\frac{p_2^{MS}}{p_1^{MS}} < \theta < 1 - p_1^{MS} + p_2^{MS}$, namely $\left(\frac{3\theta+c_2}{4}\right)/\left(\frac{3+c_1}{4}\right) < \theta$ and $\theta < 1 - \frac{3+c_1}{4} + \frac{3\theta+c_2}{4}$. Through simplifying, the first inequality implies $\frac{c_2}{c_1} < \theta$ while the second inequality implies $\theta < 1 - c_1 + c_2$. Therefore, in MS game model the lower bound is $\underline{\theta}^{MS} = \frac{c_2}{c_1}$ and the upper

bound is $\overline{\theta}^{MS} = 1 - c_1 + c_2$. When the real acceptance of old product θ is sufficiently low, satisfying $\theta \leq \underline{\theta}^{MS} = \frac{c_2}{c_1}$, the demand of old product is zero and only new product is sold; and when $\theta \geq 1 - c_1 + c_2$, the demand of new product is zero and only old product is sold. Similarly, we get the lower bound $\underline{\theta}^{VN} = \frac{c_2}{c_1}$ and upper bound $\overline{\theta}^{VN} = 1 - c_1 + c_2$ in VN game model, and the lower bound $\underline{\theta}^{RS} = \frac{c_2}{c_1}$ and the upper bound $\overline{\theta}^{RS} = 1 - c_1 + c_2$ in RS game model. From the above derivation, obviously we have $\underline{\theta}^{MS} = \underline{\theta}^{VN} = \underline{\theta}^{RS} = \frac{c_2}{c_1}$ and $\overline{\theta}^{MS} = \overline{\theta}^{VN} = \overline{\theta}^{RS} = 1 - c_1 + c_2$.

2) From demand function and the proof of part 1), it is easy to know that when θ is lower than the lower bound $\frac{c_2}{c_1}$, the demand for old product is zero; and when θ is higher than the upper bound $1 - c_1 + c_2$, the demand for new product is zero; and when θ is between $\frac{c_2}{c_1}$ and $1 - c_1 + c_2$, the demand for both products are positive. This completes the proof.

Proof of Lemma 2

MS model:

From demand function (1) and (2), when $0 < \theta \leq \frac{c_2}{c_1}$, $\pi_r(p_1, p_2) = \pi_r(p_1) = (p_1 - w_1)(1 - p_1)$. $\frac{d\pi_r(p_1)}{dp_1} = 1 - 2p_1 + w_1$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = -2 < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1. Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we get $p_1 = \frac{1+w_1}{2}$. Substituting p_1 into $\pi_m(w_1, w_2) = \pi_m(w_1) = (w_1 - c_1)(1 - p_1) = (w_1 - c_1)(1 - \frac{1+w_1}{2})$, we get $\frac{d\pi_m(w_1)}{dw_1} = \frac{1+c_1-2w_1}{2}$, $\frac{d^2\pi_m(w_1)}{\partial w_1^2} = -1 < 0$. Therefore, $\pi_m(w_1)$ is concave in w_1. Let $\frac{d\pi_m(w_1)}{dw_1} = 0$, we get $w_{s1}^{MS} = \frac{1+c_1}{2}$. Replacing w_1 with w_{s1}^{MS} into p_1, then we have $p_{s1}^{MS} = \frac{3+c_1}{4}$.

When $c_2 - c_1 + 1 \leq \theta < 1$, $\pi_r(p_1, p_2) = \pi_r(p_2) = (p_2 - w_2)(1 - \frac{p_2}{\theta})$. $\frac{d\pi_r(p_2)}{dp_2} = 1 - \frac{2p_2}{\theta} + \frac{w_2}{\theta}$, $\frac{d^2\pi_r(p_2)}{dp_2^2} = -\frac{2}{\theta} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$, we get $p_2 = \frac{\theta+w_2}{2}$. Substituting p_2 into $\pi_m(w_1, w_2) = \pi_m(w_2) = (w_2 - c_2)\left(1 - \frac{p_2}{\theta}\right) = (w_2 - c_2)(1 - \frac{\theta+w_2}{2\theta})$, we get $\frac{d\pi_m(w_2)}{dw_2} = \frac{\theta+c_2-2w_2}{2}$, $\frac{d^2\pi_m(w_2)}{\partial w_2^2} = -1 < 0$. Therefore, $\pi_m(w_2)$

is concave in w_2. Let $\frac{d\pi_m(w_2)}{dw_2} = 0$, we get $w_{s2}^{MS} = \frac{\theta+c_2}{2}$. Replacing w_2 with w_{s2}^{MS} into p_2, then we have $p_{s2}^{MS} = \frac{3\theta+c_2}{4}$.

VN model:

From demand function (1) and (2), when $0 < \theta \le \frac{c_2}{c_1}$, remember that the marginal profits of new product is $m_1 = p_1 - w_1$, then $\pi_m(w_1, w_2) = \pi_m(w_1) = (w_1 - c_1)(1 - m_1 - w_1)$, we get $\frac{d\pi_m(w_1)}{dw_1} = 1 - p_1 - w_1 + c_1$, $\frac{d^2\pi_m(w_1)}{\partial w_1^2} = -1 < 0$. Therefore, $\pi_m(w_1)$ is concave in w_1. Combine $\frac{d\pi_m(w_1)}{dw_1} = 0$ and $\frac{d\pi_r(p_1)}{dp_1} = 1 - 2p_1 + w_1 = 0$ (from MS model when $0 < \theta \le \frac{c_2}{c_1}$), we get $w_{s1}^{VN} = \frac{1+2c_1}{3}$ and $p_{s1}^{VN} = \frac{2+c_1}{3}$.

When $c_2 - c_1 + 1 \le \theta < 1$, remember that the marginal profits of old product is $m_2 = p_2 - w_2$, then $\pi_m(w_1, w_2) = \pi_m(w_2) = (w_2 - c_2)(1 - \frac{m_2+w_2}{\theta})$, we get $\frac{d\pi_m(w_2)}{dw_2} = \frac{\theta - p_2 - w_2 + c_2}{\theta}$, $\frac{d^2\pi_m(w_2)}{\partial w_2^2} = -\frac{1}{\theta} < 0$. Therefore, $\pi_m(w_2)$ is concave in w_2. Combine $\frac{d\pi_m(w_2)}{dw_2} = 0$ and $\frac{d\pi_r(p_2)}{dp_2} = 1 - \frac{2p_2}{\theta} + \frac{w_2}{\theta} = 0$ (from MS model when $c_2 - c_1 + 1 \le \theta < 1$), we get $w_{s2}^{VN} = \frac{\theta+2c_2}{3}$ and $p_{s2}^{VN} = \frac{2\theta+c_2}{3}$.

RS model:

From demand function (1) and (2), when $0 < \theta \le \frac{c_2}{c_1}$ from $\frac{d\pi_m(w_1)}{dw_1} = 1 - p_1 - w_1 + c_1 = 0$ (from VN model when $0 < \theta \le \frac{c_2}{c_1}$), we get $w_1 = 1 - p_1 + c_1$. Substitute w_1 into $\pi_r(p_1, p_2) = \pi_r(p_1) = (p_1 - w_1)(1 - p_1) = (p_1 - 1 + p_1 - c_1)(1 - p_1)$, we get $\frac{d\pi_r(p_1)}{dp_1} = 3 - 4p_1 + c_1$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = -4 < 0$. Therefore, $\pi_r(p_1)$ is concave in p_1. Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we get $p_{s1}^{RS} = \frac{3+c_1}{4}$. Replacing p_1 with p_{s1}^{RS} into w_1, then we have $w_{s1}^{RS} = \frac{1+3c_1}{4}$.

When $c_2 - c_1 + 1 \le \theta < 1$, from $\frac{d\pi_m(w_2)}{dw_2} = \frac{\theta - p_2 - w_2 + c_2}{\theta} = 0$ (from VN model when $c_2 - c_1 + 1 \le \theta < 1$), we get $w_2 = \theta - p_2 + c_2$. Substitute w_2 into $\pi_r(p_1, p_2) = \pi_r(p_2) = (p_2 - w_2)\left(1 - \frac{p_2}{\theta}\right) = (p_2 - \theta + p_2 - c_2)\left(1 - \frac{p_2}{\theta}\right)$, we get $\frac{d\pi_r(p_2)}{dp_2} = \frac{3\theta - 4p_2 + c_2}{\theta}$

and $\frac{d^2\pi_r(p_2)}{dp_2^2} = -\frac{4}{\theta} < 0$. Therefore, $\pi_r(p_2)$ is concave in p_2. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$, we get $p_{s2}^{RS} = \frac{3\theta+c_2}{4}$. Replacing p_2 with p_{s2}^{RS} into w_2, then we have $w_{s2}^{RS} = \frac{\theta+3c_2}{4}$. This completes the proof.

Proof of Proposition 2

From Table 1, $p_1^{MS} - p_2^{MS} = \frac{3-3\theta+c_1-c_2}{4} > 0$, $w_1^{MS} - w_2^{MS} = \frac{1-\theta+c_1-c_2}{2} > 0$, $p_1^{VN} - p_2^{VN} = \frac{2-2\theta+c_1-c_2}{3} > 0$, $w_1^{VN} - w_2^{VN} = \frac{1-\theta+2c_1-2c_2}{3} > 0$, $p_1^{RS} - p_2^{RS} = \frac{3-3\theta+c_1-c_2}{4} > 0$ and $w_1^{RS} - w_2^{RS} = \frac{1-\theta+3c_1-3c_2}{4} > 0$. $D_1^{MS} - D_2^{MS} = \frac{\theta^2-(1-2c_1+c_2)\theta-c_2}{4(-1+\theta)\theta}$, $D_1^{VN} - D_2^{VN} = \frac{\theta^2-(1-2c_1+c_2)\theta-c_2}{3(-1+\theta)\theta}$ and $D_1^{RS} - D_2^{RS} = \frac{\theta^2-(1-2c_1+c_2)\theta-c_2}{4(-1+\theta)\theta}$. Define $f(\theta) = \theta^2 - (1 - 2c_1 + c_2)\theta - c_2$, assuming $f(\theta) = 0$, because $\Delta = (1 - 2c_1 + c_2)^2 + 4c_2 > 0$, there exit two real roots $\theta_0 = \frac{1-2c_1+c_2+\sqrt{(1-2c_1+c_2)^2+4c_2}}{2}$ and $\theta_0' = \frac{1-2c_1+c_2-\sqrt{(1-2c_1+c_2)^2+4c_2}}{2}$. Because $\theta_0\theta_0' = -c_2$, then $\theta_0 > 0$ and $\theta_0' < 0$, reject θ_0'.

Next, we aim to prove $\theta_0 \in (\frac{c_2}{c_1}, c_2 - c_1 + 1)$. Firstly, $c_2 - c_1 + 1 - \theta_0 = \frac{1+c_2-\sqrt{(1-2c_1+c_2)^2+4c_2}}{2}$, then we have $1 + c_2 - \sqrt{(1 - 2c_1 + c_2)^2 + 4c_2} > 0 \Leftrightarrow 1 + c_2 > \sqrt{(1 - 2c_1 + c_2)^2 + 4c_2} \Leftrightarrow (1 + c_2)^2 > (1 - 2c_1 + c_2)^2 + 4c_2 \Leftrightarrow 4(1 + c_1)(c_1 - c_2) > 0$ for any $0 < c_2 < c_1 < 1$. That is, $\theta_0 < c_2 - c_1 + 1$. Secondly, $\theta_0 - \frac{c_2}{c_1} = \frac{\sqrt{4c_2+(1-2c_1+c_2)^2}-(-1+2c_1-c_2+\frac{2c_2}{c_1})}{2}$. If $-1 + 2c_1 - c_2 + \frac{2c_2}{c_1} \le 0$, we get $\sqrt{4c_2 + (1 - 2c_1 + c_2)^2} - \left(-1 + 2c_1 - c_2 + \frac{2c_2}{c_1}\right) > 0$ for any $0 < c_2 < c_1 < 1$; and if $-1 + 2c_1 - c_2 + \frac{2c_2}{c_1} > 0$, we get $\sqrt{4c_2 + (1 - 2c_1 + c_2)^2} > -1 + 2c_1 - c_2 + \frac{2c_2}{c_1} \Leftrightarrow 4c_2 + (1 - 2c_1 + c_2)^2 > \left(-1 + 2c_1 - c_2 + \frac{2c_2}{c_1}\right)^2 \Leftrightarrow \frac{4(1-c_1)c_2(c_1-c_2)}{c_1^2} > 0$ for any $0 < c_2 < c_1 < 1$. Therefore, $\theta_0 > \frac{c_2}{c_1}$.

Therefore, when $\theta \in (\frac{c_2}{c_1}, \theta_0]$, $f(\theta) = \theta^2 - (1 - 2c_1 + c_2)\theta - c_2 \le 0$, and then $D_1^{MS} \ge D_2^{MS}$, $D_1^{VN} \ge D_2^{VN}$ and $D_1^{RS} \ge D_2^{RS}$, and if $\theta \in (\theta_0, c_2 - c_1 + 1)$, $f(\theta) = \theta^2 - (1 - 2c_1 + c_2)\theta - c_2 > 0$, and then $D_1^{MS} < D_2^{MS}$, $D_1^{VN} < D_2^{VN}$ and $D_1^{RS} < D_2^{RS}$. This completes the proof.

Proof of Proposition 3

From Table 1, $\frac{\partial w_2^{MS}}{\partial \theta} = \frac{1}{2}$, $\frac{\partial w_2^{VN}}{\partial \theta} = \frac{1}{3}$ and $\frac{\partial w_2^{RS}}{\partial \theta} = \frac{1}{4}$. That is, $\frac{\partial w_2^{MS}}{\partial \theta} > \frac{\partial w_2^{VN}}{\partial \theta} > \frac{\partial w_2^{RS}}{\partial \theta} > 0$. $\frac{\partial p_2^{VN}}{\partial \theta} = \frac{2}{3}$ and $\frac{\partial p_2^{MS}}{\partial \theta} = \frac{\partial p_2^{RS}}{\partial \theta} = \frac{3}{4}$. That is, $\frac{\partial p_2^{MS}}{\partial \theta} = \frac{\partial p_2^{RS}}{\partial \theta} > \frac{\partial p_2^{VN}}{\partial \theta} > 0$. $\frac{\partial \pi_m^{MS}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{8(1-\theta)^2\theta^2} > 0$, $\frac{\partial \pi_m^{VN}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{9(1-\theta)^2\theta^2} > 0$ and $\frac{\partial \pi_m^{RS}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{16(1-\theta)^2\theta^2} > 0$. That is, $\frac{\partial \pi_m^{MS}}{\partial \theta} > \frac{\partial \pi_m^{VN}}{\partial \theta} > \frac{\partial \pi_m^{RS}}{\partial \theta} > 0$. $\frac{\partial \pi_r^{MS}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{16(1-\theta)^2\theta^2} > 0$, $\frac{\partial \pi_r^{VN}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{9(1-\theta)^2\theta^2} > 0$ and $\frac{\partial \pi_r^{RS}}{\partial \theta} = \frac{(\theta c_1 - c_2)(\theta c_1 + c_2 - 2\theta c_2)}{8(1-\theta)^2\theta^2} > 0$. That is, $\frac{\partial \pi_r^{RS}}{\partial \theta} > \frac{\partial \pi_r^{VN}}{\partial \theta} > \frac{\partial \pi_r^{MS}}{\partial \theta} > 0$. This completes the proof.

Proof of Proposition 4

1) From Table 1, $w_1^{MS} - w_1^{VN} = \frac{1-c_1}{6} > 0$, $w_1^{MS} - w_1^{RS} = \frac{1-c_1}{4} > 0$ and $w_1^{VN} - w_1^{RS} = \frac{1-c_1}{12} > 0$; $w_2^{MS} - w_2^{VN} = \frac{\theta - c_2}{6} > 0$, $w_2^{MS} - w_2^{RS} = \frac{\theta - c_2}{4} > 0$ and $w_2^{VN} - w_2^{RS} = \frac{\theta - c_2}{12} > 0$. That is, $w_1^{MS} > w_1^{VN} > w_1^{RS}$ and $w_2^{MS} > w_2^{VN} > w_2^{RS}$.

2) From Table 1, $p_1^{MS} - p_1^{RS} = 0$ and $p_1^{MS} - p_1^{VN} = \frac{1-c_1}{12} > 0$; $p_2^{MS} - p_2^{RS} = 0$ and $p_2^{MS} - p_2^{VN} = \frac{\theta - c_2}{12} > 0$. That is, $p_1^{MS} = p_1^{RS} > p_1^{VN}$ and $p_2^{MS} = p_2^{RS} > p_2^{VN}$.

3) From Table 1, $D_1^{MS} - D_1^{RS} = 0$ and $D_1^{MS} - D_1^{VN} = -\frac{1-\theta-c_1+c_2}{12(1-\theta)} < 0$; $D_2^{MS} - D_2^{RS} = 0$ and $D_2^{MS} - D_2^{VN} = -\frac{1-\theta-c_1+c_2}{12(1-\theta)} < 0$; $\alpha_1^{MS} - \alpha_1^{VN} = 0$ and $\alpha_1^{MS} - \alpha_1^{RS} = 0$; $\alpha_2^{MS} - \alpha_2^{VN} = 0$ and $\alpha_2^{MS} - \alpha_2^{RS} = 0$. That is, $D_1^{MS} = D_1^{RS} < D_1^{VN}$, $D_2^{MS} = D_2^{RS} < D_2^{VN}$, $\alpha_1^{MS} = \alpha_1^{VN} = \alpha_1^{RS}$ and $\alpha_2^{MS} = \alpha_2^{VN} = \alpha_2^{RS}$.

4) From Table 1, $\pi_m^{MS} - \pi_m^{VN} = \frac{A}{72} > 0$, $\pi_m^{MS} - \pi_m^{RS} = \frac{A}{16} > 0$ and $\pi_m^{VN} - \pi_m^{RS} = \frac{7A}{144} > 0$; $\pi_r^{MS} - \pi_r^{VN} = -\frac{7A}{144} < 0$, $\pi_r^{MS} - \pi_r^{RS} = -\frac{A}{16} < 0$ and $\pi_r^{VN} - \pi_r^{RS} = -\frac{A}{72} > 0$; $\pi_s^{MS} - \pi_s^{VN} = -\frac{5A}{144} < 0$ and $\pi_s^{MS} - \pi_s^{RS} = 0$. That is, $\pi_m^{MS} > \pi_m^{VN} > \pi_m^{RS}$, $\pi_r^{RS} > \pi_r^{VN} > \pi_r^{MS}$ and $\pi_s^{VN} > \pi_s^{MS} = \pi_s^{RS}$. This completes the proof.

Proof of Proposition 5

1) From Table 2, $p_{s1}^{MS} - p_{s1}^{RS} = 0$, $p_{s1}^{MS} - p_{s1}^{VN} = \frac{1-c_1}{12}$, $w_{s1}^{MS} - w_{s1}^{VN} = \frac{1-c_1}{6} > 0$, $w_{s1}^{MS} - w_{s1}^{RS} = \frac{1-c_1}{4} > 0$, $w_{s1}^{VN} - w_{s1}^{RS} = \frac{1-c_1}{12} > 0$, $D_{s1}^{MS} - D_{s1}^{RS} = 0$, $D_{s1}^{MS} - D_{s1}^{VN} = -\frac{1-c_1}{12} < 0$, $\pi_{ms1}^{MS} - \pi_{ms1}^{VN} = \frac{(1-c_1)^2}{72} > 0$, $\pi_{ms1}^{MS} - \pi_{ms1}^{RS} = \frac{(1-c_1)^2}{16} > 0$, $\pi_{ms1}^{VN} - \pi_{ms1}^{RS} = \frac{7(1-c_1)^2}{144} > 0$, $\pi_{rs1}^{MS} - \pi_{rs1}^{VN} = -\frac{7(1-c_1)^2}{144} < 0$, $\pi_{rs1}^{MS} - \pi_{rs1}^{RS} = -\frac{(1-c_1)^2}{16} < 0$, $\pi_{ms1}^{VN} - \pi_{ms1}^{RS} = -\frac{(1-c_1)^2}{72} < 0$, $\pi_{s1}^{MS} - \pi_{s1}^{RS} = 0$ and $\pi_{s1}^{MS} - \pi_{s1}^{VN} = -\frac{5(1-c_1)^2}{144} < 0$.

2) From Table 2, $p_{s2}^{MS} - p_{s2}^{RS} = 0$, $p_{s2}^{MS} - p_{s2}^{VN} = \frac{\theta-c_2}{12}$, $w_{s2}^{MS} - w_{s2}^{VN} = \frac{\theta-c_2}{6} > 0$, $w_{s2}^{MS} - w_{s2}^{RS} = \frac{\theta-c_2}{4} > 0$, $w_{s2}^{VN} - w_{s2}^{RS} = \frac{\theta-c_2}{12} > 0$, $D_{s2}^{MS} - D_{s2}^{RS} = 0$, $D_{s2}^{MS} - D_{s2}^{VN} = -\frac{\theta-c_2}{12} < 0$, $\pi_{ms2}^{MS} - \pi_{ms2}^{VN} = \frac{(\theta-c_2)^2}{72\theta} > 0$, $\pi_{ms2}^{MS} - \pi_{ms2}^{RS} = \frac{(\theta-c_2)^2}{16\theta} > 0$, $\pi_{ms2}^{VN} - \pi_{ms2}^{RS} = \frac{7(\theta-c_2)^2}{144\theta} > 0$, $\pi_{rs2}^{MS} - \pi_{rs2}^{VN} = -\frac{7(\theta-c_2)^2}{144\theta} < 0$, $\pi_{rs2}^{MS} - \pi_{rs2}^{RS} = -\frac{(\theta-c_2)^2}{16\theta} < 0$, $\pi_{ms2}^{VN} - \pi_{ms2}^{RS} = -\frac{(\theta-c_2)^2}{72\theta} < 0$, $\pi_{s2}^{MS} - \pi_{s2}^{RS} = 0$ and $\pi_{s2}^{MS} - \pi_{s2}^{VN} = -\frac{5(\theta-c_2)^2}{144\theta} < 0$. This completes the proof.

Proof of Lemma 3

From (5), $\frac{\partial \pi^I(p_1,p_2)}{\partial p_1} = \frac{-1+\theta-c_1+c_2+2p_1-2p_2}{-1+\theta}$, $\frac{\partial \pi^I(p_1,p_2)}{\partial p_2} = \frac{\theta c_1-c_2-2\theta p_1+2p_2}{(-1+\theta)\theta}$, $\frac{\partial^2 \pi^I(p_1,p_2)}{\partial p_1^2} = -\frac{2}{1-\theta} < 0$, $\frac{\partial^2 \pi^I(p_1,p_2)}{\partial p_2^2} = \frac{2}{(-1+\theta)\theta}$ and $\frac{\partial^2 \pi^I(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi^I(p_1,p_2)}{\partial p_2 \partial p_1} = \frac{2}{1-\theta}$. We have $\begin{vmatrix} -\frac{2}{1-\theta} & \frac{2}{1-\theta} \\ \frac{2}{1-\theta} & \frac{2}{(-1+\theta)\theta} \end{vmatrix} = \frac{4}{(1-\theta)\theta} > 0$. Therefore, $\pi^I(p_1,p_2)$ is jointly concave in p_1 and p_2. Let $\frac{\partial \pi^I(p_1,p_2)}{\partial p_1} = \frac{\partial \pi^I(p_1,p_2)}{\partial p_2} = 0$, we get $p_1^I = \frac{1+c_1}{2}$ and $p_2^I = \frac{\theta+c_2}{2}$. Then we can get $D_1^I = \frac{1-\theta-c_1+c_2}{2(1-\theta)}$, $D_2^I = \frac{\theta c_1-c_2}{2(1-\theta)\theta}$ and $\pi^I(p_1^I,p_2^I) = \frac{A}{4}$, where $A = \frac{\theta(1-c_1)(1-c_1-\theta+c_2)+(\theta-c_2)(\theta c_1-c_2)}{(1-\theta)\theta}$. This completes the proof.

Proof of Proposition 6

From (6), we get $\frac{\partial \pi_r(p_{r1},p_{r2})}{\partial p_{r1}} = \frac{-\phi+\theta\phi+2\phi p_{r1}-2\phi p_{r2}-w_{r1}+w_{r2}}{-1+\theta}$,

$\frac{\partial \pi_r(p_{r1},p_{r2})}{\partial p_{r2}} = \frac{2\theta\phi p_{r1} - 2\phi p_{r2} - \theta w_{r1} + w_{r2}}{(1-\theta)\theta}$, $\frac{\partial^2 \pi_r(p_{r1},p_{r2})}{\partial p_{r1}^2} = -\frac{2\phi}{1-\theta} < 0$, $\frac{\partial^2 \pi_r(p_{r1},p_{r2})}{\partial p_{r2}^2} = -\frac{2\phi}{(1-\theta)\theta}$ and $\frac{\partial^2 \pi_r(p_{r1},p_{r2})}{\partial p_{r1}\partial p_{r2}} = \frac{\partial^2 \pi_r(p_{r1},p_{r2})}{\partial p_{r2}\partial p_{r1}} = \frac{2\phi}{1-\theta}$. We have $\begin{vmatrix} -\frac{2\phi}{1-\theta} & \frac{2\phi}{1-\theta} \\ \frac{2\phi}{1-\theta} & \frac{2\phi}{(-1+\theta)\theta} \end{vmatrix} = \frac{4\phi^2}{(1-\theta)\theta} > 0$. Therefore, $\pi_r(p_{r1}, p_{r2})$ is jointly concave in p_{r1} and p_{r2}. Let $\frac{\partial \pi_r(p_{r1},p_{r2})}{\partial p_{r1}} = \frac{\partial \pi_r(p_{r1},p_{r2})}{\partial p_{r2}} = 0$, we get $\frac{-\phi + \theta\phi + 2\phi p_{r1} - 2\phi p_{r2} - w_{r1} + w_{r2}}{-1+\theta} = 0$ and $\frac{2\theta\phi p_{r1} - 2\phi p_{r2} - \theta w_{r1} + w_{r2}}{(1-\theta)\theta} = 0$. In order to coordinate the supply chain, replace $p_{r1} = p_1^I$ and $p_{r2} = p_2^I$ to aforementioned equations, we get $w_{r1} = \phi c_1$ and $w_{r2} = \phi c_2$. Therefore, we can get $\pi_r(p_1^I, p_2^I) = (\phi p_1^I - \phi c_1)\left(1 - \frac{p_1^I - p_2^I}{1-\theta}\right) + (\phi p_2^I - \phi c_2)\left(\frac{p_1^I - p_2^I}{1-\theta} - \frac{p_2^I}{\theta}\right) = \phi \pi^I(p_1^I, p_2^I) = \frac{A}{4}\phi$, and $\pi_m(w_{r1}, w_{r2}) = [\phi c_1 - c_1 + (1-\phi)p_1^I]\left(1 - \frac{p_1^I - p_2^I}{1-\theta}\right) + [\phi c_2 - c_2 + (1-\phi)p_2^I]\left(\frac{p_1^I - p_2^I}{1-\theta} - \frac{p_2^I}{\theta}\right) =$
$(1-\phi)\pi^I(p_1^I, p_2^I) = \frac{A}{4}(1-\phi)$. In MS model, the manufacturer is the leader. The retail supply chain can be coordinated and it have to satisfy that $\pi_r(p_1^I, p_2^I) > \pi_r^{MS}(p_1^{MS}, p_2^{MS})$ and $\pi_m(w_{r1}, w_{r2}) > \pi_m^{MS}(w_1^{MS}, w_1^{MS})$, that is $\frac{A}{4}\phi > \frac{A}{16}$ and $\frac{A}{4}(1-\phi) > \frac{A}{8}$, then $\frac{1}{4} < \phi < \frac{1}{2}$. In VN model, the retailer and the manufacturer have balanced power, so they will divide all the profits equally, that is $\phi = \frac{1}{2}$. In RS model, the retailer is the leader. The retail supply chain can be coordinated and it have to satisfy that $\pi_r(p_1^I, p_2^I) > \pi_r^{RS}(p_1^{RS}, p_2^{RS})$ and $\pi_m(w_{r1}, w_{r2}) > \pi_m^{RS}(w_1^{RS}, w_1^{RS})$, that is $\frac{A}{4}\phi > \frac{A}{8}$ and $\frac{A}{4}(1-\phi) > \frac{A}{16}$, then $\frac{1}{2} < \phi < \frac{3}{4}$. This completes the proof.

Proof of Proposition 7

From Lemma 3, we get $p_1^I = \frac{1+c_1}{2}$ and $p_2^I = \frac{\theta + c_2}{2}$. According to Proposition 1, we know that $\frac{p_2}{p_1} < \theta$ provides us the lower bound and $\theta < 1 - p_1 + p_2$ provides us the upper bound. We have $\left(\frac{\theta+c_2}{2}\right)/\left(\frac{1+c_1}{2}\right) < \theta$ and $\theta < 1 - \frac{1+c_1}{2} + \frac{\theta+c_2}{2}$. Though simplifying, the first inequality implies $\frac{c_2}{c_1} < \theta$ while the second inequality implies $\theta < 1 - c_1 + c_2$. Therefore, with revenue sharing contract, we get lower bound $\underline{\theta}^r = \frac{c_2}{c_1}$, and the upper bound $\overline{\theta}^r = c_2 - c_1 + 1$. This completes the proof.