Accepted Manuscript

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PII: S0377-2217(17)30571-4
DOI: 10.1016/j.ejor.2017.06.032
Reference: EOR 14514

To appear in: *European Journal of Operational Research*

Received date: 15 January 2016
Revised date: 15 May 2017
Accepted date: 11 June 2017

Please cite this article as: Xue Cui, Takashi Shibata, Investment strategies, reversibility, and asymmetric information, *European Journal of Operational Research* (2017), doi: 10.1016/j.ejor.2017.06.032

Highlights

- Information asymmetry delays investment timing.
- Under information asymmetry, quantity is increasing in degree of reversibility.
- Loss is increasing in degree of reversibility, but decreasing in volatility.
- Higher volatility increases owner's value but decreases manager's value.
- Higher reversibility causes smaller manager's bonus and larger manager's value.

Investment strategies, reversibility, and asymmetric information

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First Version: January 15, 2016
Second Version: November 7, 2016
Third Version: February 6, 2017
This Version: June 15, 2017

Abstract: In this paper, we examine the reversibility effects on a firm's investment trigger (timing) and quantity strategies in the presence of asymmetric information between the firm owner and the manager. We obtain five main results under conditions of asymmetric information. First, information asymmetry increases (delays) investment trigger (timing). Second, under information asymmetry, investment quantity increases in degree of reversibility, while under information symmetry it is constant. Third, social loss arising from information asymmetry increases in degree of manager's informational rent and degree of reversibility, but decreases in volatility. Fourth, an increase in volatility increases the owner's value, while it decreases the manager's value. Fifth, an increase in volatility increases the ex post manager's value, while it decreases the ex ante manager's value. An increase in degree of reversibility decreases the ex post manager's value, while it increases the ex ante manager's value.

Keywords: Investment analysis; real options; investment quantity; private information.

JEL classification: G31; G34; D81.

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1 Introduction

In this paper, we investigate how changes in the reversibility of investment affect a firm's investment timing and quantity strategies when information asymmetry exists.

This paper is based on many previous studies regarding the investment decision problem. The seminal work by McDonald and Siegel (1986) provides a standard framework for examining the timing of an investment where the investment cost is fully irreversible. Abel and Eberly (1999) incorporate reversibility of investment and examine the optimal investment of a firm. The reversibility of investment means that, when the profitability of capital becomes unfavorable, a firm can sell capital at a lower price than the initial investment cost. Thus, under the assumption of reversibility, the firm owns an abandonment option. Following Abel and Eberly (1999), Wong (2010) examines the effects of reversibility on investment timing and quantity (intensity) strategies. Wong (2010) shows that higher reversibility accelerates investment but has no impact on quantity. The frameworks of Abel and Eberly (1999) and Wong (2010) are made under the full (symmetric) information assumption.

However, in most modern corporations, many investment decisions are made under conditions of asymmetric information. For example, firm owners would like to delegate management to managers, taking advantage of managers' professional skills. In this situation, the presence of asymmetric information is inevitable. Managers may own private information that owners cannot observe. Grenadier and Wang (2005) examine investment timing in the presence of a manager's private information and show that the investment timing under asymmetric information is more delayed than under full information.[1] Cui and Shibata (2017) extend the work of Grenadier and Wang (2005) by incorporating a quantity decision and show that the investment quantity under asymmetric information is higher than under full information.[2] To the best of our knowledge, most studies on the asymmetric information model assume that the investment cost is fully irreversible (i.e., there is no consideration of the reversibility of investment).

To combine the reversibility of investment and asymmetric information, Cui and Shibata

[1]See Shibata (2009) and Shibata and Nishihara (2011) for an extension of the Grenadier and Wang (2005) model.

[2]Besides these, many studies veiw the effects of information asymmetry on investment strategies from different perspectives. For example, Leung and Kwok (2012) examine the impact of information asymmetry on patent-investment strategies. Belleflamme and Peitz (2014) examine the effects of information asymmetry on investment in product quality.

(2016) extend Wong (2010) by incorporating asymmetric information, and alternatively extend Cui and Shibata (2017) to account for the reversibility of investment. Cui and Shibata (2016) find numerically that, under asymmetric information, higher reversibility accelerates the investment and increases the quantity. The former result about investment timing is the same as under full information (i.e., Wong (2010)). By contrast, interestingly, the latter result about quantity is contrary to that under full information (i.e., the quantity under asymmetric information is no longer independent of the degree of reversibility of investment). However, Cui and Shibata (2016) provide no economic interpretation for the mechanism of the interesting result. They also provide no examination of the firm's and manager's values under asymmetric information. Thus, we undertake this study to extend Cui and Shibata (2016) in at least three ways.

The first extension is to provide an economic interpretation for the reversibility effect on the investment trigger (timing) and quantity under asymmetric information. To be more precise, under full information, the optimal quantity is decided by solving only one equation. The optimal investment trigger (timing) is decided by using the optimal quantity. On the contrary, under asymmetric information, the optimal investment trigger and quantity are determined by solving two simultaneous equations. Thus, solutions under asymmetric information become more complex than under full information. In this study, we explore the influence of reversibility effects on investment trigger and quantity under asymmetric information.

The second extension is to analyze reversibility effects on the manager's bonus (ex post manager's value) and ex ante manager's value. In the full information situation, there is no delegation of management. We do not recognize the reversibility effects on the manager's values. However, in the model with information asymmetry, the firm (owner) must provide the manager a bonus incentive to induce the manager to reveal private information. Otherwise, the manager has incentive to divert values for his private interests by giving false reports to the owner. Thus, the manager's bonus is quite an important element in the asymmetric-information model. In this study, we examine how the manager's bonus (ex post manager's value) and ex ante manager's value vary with the degree of reversibility of investment.

The third extension is to investigate the reversibility effects on the social loss arising from information asymmetry. Cui and Shibata (2016) recognize that asymmetric information causes distortions of investment trigger and quantity strategies, making the investment strategies under asymmetric information deviate from those under full information. The deviation of investment strategies causes a social loss, which is defined as the difference between the firm's (owner's) value under full information and the sum of the firm's (owner's) and manager's values under asymmetric information. However, Cui and Shibata (2016) do not state how the degree of reversibility affects social loss. From our intuition, because higher reversibility increases the owner's value, we conjecture that higher reversibility should reduce the loss. This conjecture, however, lacks examination and confirmation in earlier studies. In this paper, by examining the distortion of asymmetric information on the investment strategies, we show the reversibility effects on social loss.

We obtain five results. First, information asymmetry increases (delays) investment trigger (timing). Second, under information asymmetry, investment quantity is increasing in degree of reversibility, while under information symmetry it is constant. Third, social loss arising from information asymmetry is increasing in degree of manager's informational rent and degree of reversibility, but it is decreasing in volatility. Fourth, an increase in volatility increases the owner's value, while it decreases the manager's value. Fifth, an increase in volatility increases the ex post manager's value, while it decreases the ex ante manager's value. An increase in degree of reversibility decreases the ex post manager's value (bonus), while it increases the ex ante manager's value. Among these five results, the first and second results correspond to the numerical findings of Cui and Shibata (2016). In this study, we show these results analytically and explain their economic mechanism. The last three results are new findings of this study.

The remainder of the paper is organized as follows. Section 2 describes the model setup and formulates the firm's optimization problem under asymmetric information. We also provide the solution to the full (symmetric) information model and consider it as a benchmark. Section 3 provides the solution to the asymmetric information model and discusses the solution properties. Section 4 analyzes the model implications by presenting numerical results. Section 5 concludes.

2 The Model

This section describes the model in four ways. First, we describe the model setup. Second, we provide the value function after investment. Third, we formulate the investment problem under asymmetric information by providing the value function before investment. Finally, as a benchmark, we review the investment problem under full information.

2.1 Setup

Consider a risk neutral firm that is endowed with an option to invest in a production facility. To initiate the facility, the firm simultaneously chooses the quantity and the timing of investment. We assume that once the investment is made, the facility starts to produce $q > 1$ units of a single commodity per unit of time. The firm sells the commodity in a perfectly competitive market at a per-unit price, X_t, at time t. The commodity price is stochastic and evolves over time according to the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}Z_t, \quad X_0 = x > 0, \tag{1}$$

where Z_t is a standard Brownian motion, and $\mu > 0$ and $\sigma > 0$ are the constant growth rate and volatility of the commodity price, respectively. We assume that the initial price $X_0 = x$ is too low to make an immediate investment optimal. Let $r > 0$ be the constant interest rate. For convergence, we assume $r > \mu$.[3]

The cost expenditure to undertake the investment is

$$I(q; F) := C(q) + F > 0. \tag{2}$$

We assume that $C(q)$ is a strictly increasing and convex function of q, i.e., $C\ (q) > 0$ and $C\ \ (q) > 0$ for any $q > 1$. F denotes the fixed cost.[4]

We assume that F could take on two possible values: F_1 or F_2, with $F_2 > F_1 > 0$. We denote $\Delta F := F_2 - F_1 > 0$. One could interpret F_1 as a "low-fixed cost" expenditure and F_2 as a "high-fixed cost" expenditure. The probability of drawing $F = F_i$ ($i \quad \{1, 2\}$) is *exogenous*, and $P(F_i) = p_i \quad (0, 1)$ with $p_1 + p_2 = 1$.

Besides the option to invest, we assume that the firm possesses an option to abandon the operation of facility at any time after investment, when the commodity price becomes unfavorable. The abandonment, once made, is irreversible. The salvage at the time of abandonment is $sI(q; F)$, where $s \quad [0, 1]$ gauges the degree of reversibility. Thus, based on the above assumptions, we define a reversible investment as follows.

[3]This assumption is needed to ensure a finite firm value.

[4]We assume that the elasticity of cost function, $qI\ (q; F)/I(q; F)$, is increasing with q, i.e., $(qC\ (q)/I(q; F))\ > 0$. This assumption corresponds to the second-order condition to ensure that there exists a unique solution q. See Cui and Shibata (2017) for details.

Definition 1 *Suppose an investment with the investment cost* $I(q;F)$. *For* s $(0,1]$*, the investment is reversible, where a higher value of* s *implies a higher degree of reversibility of investment. The limiting cases* $s=0$ *and* $s=1$ *imply a fully irreversible and a fully reversible investment, respectively.*

We denote by $q_i = q(F_i)$ the investment quantity for $F = F_i$. In addition, we denote by $\overline{x}_i = \overline{x}(F_i)$ and $\underline{x}_i = \underline{x}(F_i)$ the investment (indicated by "overline") and abandonment (indicated by "underline") triggers for $F = F_i$, respectively. Correspondingly, let $\overline{\tau}_i = \inf\{t \geq 0; X_t = \overline{x}_i\}$ and $\underline{\tau}_i = \inf\{t \geq \overline{\tau}_i; X_t = \underline{x}_i\}$ represent the (random) first passage time when X_t reaches $\overline{x}_i$ from below and then reaches $\underline{x}_i$ from above, respectively.

[Insert Figure 1 here]

In summary, we have three control variables, q_i, $\overline{x}_i$ and $\underline{x}_i$ for a given $F = F_i$. The first two are determined to maximize the firm value *before investment*, while the last one is determined to maximize the firm value *after investment*. We use Figure 1 to explain the scenario of our model. When X_t, starting at x, increases and arrives at $\overline{x}_i$, the firm undertakes the investment and decides q_i endogenously. Afterwards, if X_t, starting at $\overline{x}_i$, decreases and arrives at $\underline{x}_i$, the firm exercises the abandonment. Following Shibata and Nishihara (2012), a smaller (larger) investment trigger $\overline{x}_i$ implies an earlier (later) investment, and a smaller (larger) abandonment trigger $\underline{x}_i$ implies a later (earlier) abandonment.

2.2 Value function after investment

Given q_i and $\overline{x}_i$, we denote by $V(q_i, \overline{x}_i)$ the value function of the firm at the time of investment $\overline{\tau}_i$. The value $V(q_i, \overline{x}_i)$ is defined by

$$V(q_i, \overline{x}_i) := \sup_{\underline{\tau}_i} \mathsf{E}^{\overline{x}_i} \left[\int_{\overline{\tau}_i}^{\underline{\tau}_i} \mathrm{e}^{-r(t-\overline{\tau}_i)} q_i X_t \mathrm{d}t + \mathrm{e}^{-r(\underline{\tau}_i - \overline{\tau}_i)} s I(q_i; F_i) \right], \tag{3}$$

where $\mathsf{E}^{\overline{x}_i}[\cdot]$ denotes the expectation operator conditional on $\overline{x}_i$. The first term on the right-hand side of Equation (3) is the present value of the stream of cash flows. The second term is the present value of salvage, $sI(q_i; F_i)$, upon abandonment. Using the arguments of Dixit and Pindyck (1994) (pp. 315-316), the value $V(q_i, \overline{x}_i)$ is rewritten as

$$V(q_i, \overline{x}_i) = \max_{\underline{x}_i} v q_i \overline{x}_i + (sI(q_i; F_i) - v q_i \underline{x}_i) \left(\frac{\overline{x}_i}{\underline{x}_i} \right)^{\gamma}, \tag{4}$$

where $\overline{x}_i > \underline{x}_i$ for any q_i, $v := (r-\mu)^{-1}$ and $\gamma := 1/2 - \mu/\sigma^2 - ((1/2-\mu/\sigma^2)^2 + 2r/\sigma^2)^{1/2} < 0$. The term $(\overline{x}_i/\underline{x}_i)^\gamma = \mathsf{E}^{\overline{x}_i}[\mathrm{e}^{-r(\underline{\tau}_i - \overline{\tau}_i)}]$ accounts for both the present value and probability of one dollar received at the instant when X_t, starting off at $\overline{x}_i$, reaches $\underline{x}_i$ from above. The optimal abandonment trigger, $\underline{x}_i(q_i)$, is decided to maximize the right-hand side of Equation (4):

$$\underline{x}_i(q_i) = \frac{\gamma}{\gamma - 1}\frac{sI(q_i;F_i)}{vq_i} \geq 0, \tag{5}$$

for a fixed q_i. Note that $\underline{x}_i(q_i)$ is a function of q_i.

2.3 Investment problem under asymmetric information

In this subsection, we formulate the investment problem under asymmetric information.

Consider that the owner delegates the investment decision to a manager. Throughout the analysis, we assume that both the owner and the manager are risk neutral and aim to maximize their expected pay-offs.

We assume that the cash flows $\{q_iX_t, t > 0\}$ are observed by both the owner and the manager. However, the fixed cost F is privately observed only by the manager.[5] That is, the manager observes the realized value of F, while the owner cannot observe it. Thus, we assume that there exists asymmetric information between the owner and the manager. In such a case, the owner must induce the manager to reveal private information truthfully. Otherwise, the owner suffers some losses because the manager could divert value to himself/herself by misreporting the realized value of F. Suppose, for example, when the manager observes $F = F_1$ as the realized value, he/she could divert the difference ΔF to himself/herself by reporting $F = F_2$ to the owner. This means that the owner suffers the loss of ΔF at the time of investment. To prevent the diversion, the owner must encourage the manager to report the realized value of F by giving a bonus-incentive.

Suppose that *at time zero*, the owner signs a contract with the manager regarding the delegation of investment decision. The contract commits the owner to give a bonus-incentive to the manager *at the time of investment*. Once the contract is signed, *no renegotiation* is allowed. While the commitment may cause *ex post* inefficiency at the time of investment, it increases the *ex ante* owner's value. To motivate the manager to reveal private information

[5] In the asymmetric information structure, it is quite common to assume that a portion of investment value is privately observed by one party (here, the manager) and not observed by the other party (here, the owner). Laffont and Martimort (2002) give an excellent overview of situations with asymmetric information.

truthfully, we assume that the owner provides a bonus-incentive $w_i = w(\widetilde{F}_i)$ to the manager ***at the time of investment***.[6] Thus, the contract under asymmetric information is described as a triple:[7]

$$(q(\widetilde{F}_i),\ \overline{x}(\widetilde{F}_i),\ w(\widetilde{F}_i)), \quad i \quad \{1, 2\}.$$

Note that the contract is designed contingent on manager's report value $\widetilde{F}_i$. Because the revelation principle (Laffont and Martimort (2002), pp.48-51) ensures that, in equilibrium, the manager in state F_1 exercises at the trigger $\overline{x}(\widetilde{F}_1)$ and the manager in state F_2 exercises at the trigger $\overline{x}(\widetilde{F}_2)$, we make no distinction between the manager's reported $\widetilde{F}_i$ and true F_i. Thus, we drop the suffix "$\sim$" on the reported $\widetilde{F}_i$ and simply write F_i.

In summary, at time 0, both the owner and the manager make a contract. Neither has recognized the realized value of F. After making the contract, the manager observes the realized value of F, while the owner cannot observe it. Both the manager and the owner observe the realized value of X_t. Because making the contract induces the manager to reveal the realized value of F, at the time of investment, the manager reports the true value of F. Note that, prior to the exercised investment, the owner does not observe any information regarding whether the realized value of F is F_1 or F_2.[8] This setting is the same as in Grenadier and Wang (2005).

Given F_i, the owner's value at time 0 is defined by

$$\sup_{q_i, \overline{\tau}_i, w_i} \mathsf{E}^x \left[\mathrm{e}^{-r\overline{\tau}_i} \{V(q_i, \overline{x}_i) - I(q_i; F_i) - w_i\} \right], \tag{6}$$

where $\mathsf{E}^x[\cdot]$ denotes the expectation operator conditional on x. Using the arguments of Dixit and Pindyck (1994) (pp. 315-316), the value is rewritten as

$$\max_{q_i, \overline{x}_i, w_i} \{V(q_i, \overline{x}_i) - I(q_i; F_i) - w_i\} \left(\frac{x}{\overline{x}_i} \right)^{\beta}, \tag{7}$$

where $x < \overline{x}_i$ and $\beta := 1/2 - \mu/\sigma^2 + ((1/2 - \mu/\sigma^2)^2 + 2r/\sigma^2)^{1/2} > 1$. The term $(x/\overline{x}_i)^{\beta} =$

[6] In the equilibrium, we can verify $w_i < \Delta F$ (i.e., the cost of giving an incentive is smaller than the loss of the firm due to the wrong information provided by the manager). This means that it is better for the owner to induce the manager to tell the truth than to follow the manager's report.

[7] We need not examine the possibility of a pooling equilibrium in which only one investment trigger/quantity/bonus-incentive triple is offered. This is because the pooling equilibrium is always dominated by a separating equilibrium with two investment trigger/quantity/bonus-incentive triples.

[8] Prior to the time of investment, the owner does not have any updating information about the realized value of F. Thus, the equilibrium is defined by the Subgame Perfect Nash Equilibrium, not the Bayesian Nash Equilibrium.

$\mathsf{E}^x[\mathrm{e}^{-r\overline{\tau}_i}]$ accounts for both the present value and probability of one dollar received at the instant when X_t, starting off at x, reaches $\overline{x}_i$ from below.

Under asymmetric information, the owner's optimization is formulated as

$$\max_{q_1,q_2,\overline{x}_1,\overline{x}_2,w_1,w_2} \sum_{i=1,2} p_i \left\{V(q_i,\overline{x}_i) - I(q_i;F_i) - w_i\right\} \left(\frac{x}{\overline{x}_i}\right)^{\beta}, \tag{8}$$

subject to

$$w_1 \left(\frac{x}{\overline{x}_1}\right)^{\beta} \geq (w_2 + \quad F) \left(\frac{x}{\overline{x}_2}\right)^{\beta}, \tag{9}$$

$$w_2 \left(\frac{x}{\overline{x}_2}\right)^{\beta} \geq (w_1 - \quad F) \left(\frac{x}{\overline{x}_1}\right)^{\beta}, \tag{10}$$

$$p_1 w_1 \left(\frac{x}{\overline{x}_1}\right)^{\beta} + p_2 w_2 \left(\frac{x}{\overline{x}_2}\right)^{\beta} \geq 0, \tag{11}$$

$$w_i \geq 0, \quad i \quad \{1,2\}. \tag{12}$$

The objective function (8) is the *ex ante* owner's value. Constraints (9) and (10) are *ex post* incentive-compatibility constraints for the manager in states F_1 and F_2, respectively. Taking Constraint (9) as an example, for the manager who observes F_1, the manager's payoff is $w_1 (x/\overline{x}_1)^{\beta}$ if he/she truly reports F_1, and it is $(w_2 + \quad F)(x/\overline{x}_2)^{\beta}$ if he/she instead reports F_2. If Constraint (9) is satisfied, the manager who observes F_1 has no incentive to report F_2. Similarly, for the manager who observes F_2, Constraint (10) follows. Constraint (11) is participation constraint, where the left-hand side of Constraint (11) represents the *ex ante* manager's value, denoted by $M(x)$. Constraints (12) are *ex post* limited-liability constraints. They are imposed to ensure that the manager could accept the contract.

Our model (asymmetric-information, endogenous quantity, and reversible-investment) includes three previous models: Grenadier and Wang (2005), Wong (2010), and Cui and Shibata (2017). First, when $s = 0$ (irreversible investment) and $q_i = 1$ (exogenous quantity), our model becomes the model of Grenadier and Wang (2005). Second, if $p_1 = 1$ (symmetric information), Constraints (9) and (10) are not required and $w_1 = 0$. Our model is the same as that in Wong (2010). Third, when $s = 0$ (irreversible investment), our model corresponds to Cui and Shibata (2017). We summarize the relationship between our model and previous models in Table 5.

[Insert Table 1 about here]

Before solving the asymmetric information problem, we first briefly review the full (symmetric) information problem and provide the solution as a benchmark.

2.4 Investment problem under symmetric (full) information

In this subsection, as a benchmark, we consider the investment problem when the owner observes the true value of F. This problem is equivalent to the problem in which there is no delegation of the investment decision because the manager has no informational advantage. We then have $w_i = 0$ for any $i \quad \{1, 2\}$. Thus, the contract under symmetric (full) information is described as a couple:

$$(q_i,\ \overline{x}_i), \quad i \quad \{1, 2\}.$$

Under full information, the owner's optimization problem is formulated as

$$\max_{q_1, q_2, \overline{x}_1, \overline{x}_2} p_1 H(q_1, \overline{x}_1; F_1) + p_2 H(q_2, \overline{x}_2; F_2), \tag{13}$$

where

$$H(q_i, \overline{x}_i; F_i) := \{V(q_i, \overline{x}_i) - I(q_i; F_i)\}\, \overline{x}_i^{\,-\beta}. \tag{14}$$

Note that we formulate the symmetric information problem in (13) by dividing the objective function by x^β. In addition, note that, given F_i, the optimization problem is the same as that in Wong (2010). We use the superscript " " to represent the optimum under *symmetric (full) information*.

The optimal contract $(q_i, \overline{x}_i)$ is obtained as follows (see the proof in the Appendix). For any i ($i \quad \{1, 2\}$), q_i is determined by solving the following equation:

$$C\ (q_i) = \frac{\beta}{\beta - 1} \frac{I(q_i; F_i)}{q_i}, \tag{15}$$

and $\overline{x}_i$ is determined by solving the following equation:

$$\overline{x}_i = \frac{\beta}{\beta - 1} \frac{1}{v q_i} \left[I(q_i; F_i) - \frac{\beta - \gamma}{\beta} \left(\frac{\overline{x}_i}{\underline{x}_i(q_i)} \right)^\gamma \left(s I(q_i; F_i) - v q_i\, \underline{x}_i(q_i) \right) \right]. \tag{16}$$

Note that we first obtain q_i by solving Equation (15), then $\overline{x}_i$ is the implicit solution of Equation (16) after substituting q_i. The important property of the symmetric information solution is that q_i does not depend on s. This result is one of the most important results under symmetric information. See Wong (2010) in detail. In addition, we obtain $q_2 > q_1$ by using the assumption of $(q_i C\ (q_i)/I(q_i; F_i))\ > 0$.

Based on q_i and $\overline{x}_i$ ($i \quad \{1, 2\}$), the *ex ante* owner's optimal value under full information, $O\ (x)$, is given by

$$O\ (x) = x^\beta \left(p_1 H(q_1, \overline{x}_1; F_1) + p_2 H(q_2, \overline{x}_2; F_2) \right) > 0. \tag{17}$$

We use the solution and value as a benchmark.

3 Model Solution

In this section, we begin by providing the solution to the investment problem under asymmetric information. We then discuss the solution properties. Finally, we formulate the optimal values and social loss arising from asymmetric information.

3.1 Optimal Contract

In this subsection, we provide the optimal contract for the asymmetric information problem that was described in the previous section.

We show that only two of five constraints (9) – (12) are binding at the equilibrium in three steps. First, Constraint (11) is automatically satisfied because Constraint (12) implies Constraint (11). Second, unlike a manager who observes F_1, a manager who observes F_2 has no incentive to pretend the manager who observe F_1. This is because the manager who observes F_2 suffers a loss $-\Delta F < 0$ from pretending the manager who observe F_1. Thus, Constraint (10) is satisfied automatically, and $w_2 = 0$ at the optimum. Third, suppose that Constraint (9) holds as a strict inequality. Then, by decreasing w_1, the owner's value is increased. Thus, Constraint (9) is binding, and $w_1 = (\overline{x}_1 / \overline{x}_2)^{\beta} \Delta F$ at the optimum.

Consequently, the owner's optimization problem (8) is simplified as

$$\max_{q_1, q_2, \overline{x}_1, \overline{x}_2} p_1 H(q_1, \overline{x}_1; F_1) + p_2 H(q_2, \overline{x}_2; F_2 + \phi \Delta F), \tag{18}$$

where $\phi = p_1/p_2 \geq 0$ and $I(q_2; F_2 + \phi\Delta F) = F_2 + \phi\Delta F + C(q_2)$. Note that we formulate the simplified problem (18) by dividing the objective function (8) by x^{β}. We use the superscript " " to represent the optimum under *asymmetric information*. Then, the optimal contract is obtained as follows (see the proof in the Appendix).

Proposition 1 *Suppose the asymmetric information problem. Then, q_2 and $\overline{x}_2$ are obtained by solving the two simultaneous equations:*

$$C(q_2) = \frac{\beta}{\beta - 1} \frac{1}{q_2} \left[I(q_2; F_2) + \phi \Delta F \left(1 - s \left(\frac{\overline{x}_2}{\underline{x}_2(q_2)} \right)^{\gamma} \right)^{-1} \right], \tag{19}$$

and

$$\overline{x}_2 = \frac{\beta}{\beta - 1} \frac{1}{v q_2} \left[I(q_2; F_2) + \phi \Delta F - \frac{\beta - \gamma}{\beta} \left(\frac{\overline{x}_2}{\underline{x}_2(q_2)} \right)^{\gamma} \left(s I(q_2; F_2) - v q_2 \, \underline{x}_2(q_2) \right) \right]. \tag{20}$$

The optimal contract is

$$\begin{aligned} (\overline{x}_1\ ,\ \ q_1\ ,\ \ w_1\ ) &= (\ \ \overline{x}_1,\ \ \ q_1,\ \ \ (\overline{x}_1/\overline{x}_2\ )^{\beta}\Delta F\ \ \ ), \\ (\overline{x}_2\ ,\ \ q_2\ ,\ \ w_2\ ) &= (\ \ \overline{x}_2\ ,\ \ q_2\ ,\ \ \ \ \ \ \ \ 0\ \ \ \ \ \ \ \ ). \end{aligned} \tag{21}$$

In Proposition 1, there is an important result. We have $q_1\ \ = q_1$ and $\overline{x}_1\ \ = \overline{x}_1$, while $q_2\ \ = q_2$ and $\overline{x}_2\ \ = \overline{x}_2$. The presence of asymmetric information causes distortion on quantity q_2 and investment trigger $\overline{x}_2$, which is captured by the term $\phi\Delta F > 0$ in Equations (19) and (20). Note that if $\phi\Delta F = 0$, Equations (19) and (20) become the same as Equations (15) and (16), respectively.

3.2 Properties

In this subsection, we discuss the properties of an optimal contract.

By comparing the contracts under symmetric and asymmetric information, we obtain the following four results (see the proof in the Appendix).

Proposition 2 *We have the following properties:*

$$q_2\ \ \geq q_2, \quad \overline{x}_2\ \ \geq \overline{x}_2, \quad 0 \leq w_1\ \ \leq \Delta F, \quad \underline{x}_2\ \ \geq \underline{x}_2.$$

The first property of $q_2\ \ > q_2$ is that the quantity is *larger under asymmetric information* than under full information. The second property of $\overline{x}_2\ \ > \overline{x}_2$ is that the investment is exercised *later under asymmetric information* than under full information. In other words, the distance of $\overline{x}_2\ \ - \overline{x}_1\ \ > 0$ is larger than that of $\overline{x}_2 - \overline{x}_1 > 0$. The third property of $0 < w_1\ \ < \Delta F$ is that the owner gives the manager in F_1 the bonus-incentive w_1 as a portion of the informational rent $\Delta F > 0$ to induce the manager to reveal private information. Here, $\Delta F > 0$ can be regarded as the *informational rent* for the manager who observes F_1. These three results are similar to Grenadier and Wang (2005), Shibata and Nishihara (2011), and Cui and Shibata (2017). The fourth property of $\underline{x}_2\ \ \geq \underline{x}_2$ corresponds to the first property, because $\underline{x}_i(q_i)$ is an increasing function of q_i.

Recall that one of the most important results under symmetric (full) information is that q_i is independent of s (see Wong (2010) for details). However, under asymmetric information, q_2 is *no longer* independent of the degree of reversibility s. This suggests the following result (see the proof in the Appendix).

Proposition 3 *Suppose the asymmetric information problem. The optimal quantity q_2 is no longer independent of the degree of reversibility s. More precisely, q_2 is increasing with s.*

The mathematical reason behind Proposition 3 is as follows. Under full information, q_2 is decided by solving only Equation (15). In contrast, under asymmetric information, because of the distortion term $\phi\Delta F > 0$, q_2 and x_2 are determined by solving the two simultaneous Equations (19) and (20). These differences are caused by the existence of $\phi\Delta F > 0$ under asymmetric information. Intuitively, to induce the manager to reveal private information, giving the bonus-incentive to the manager leads to the existence of $\phi\Delta F > 0$ in Equation (18), which changes the problem from Equation (13) to Equation (18). Thus, the quantity under information asymmetry depends on the degree of reversibility.

3.3 Optimal values and social loss

In this subsection, we provide the *ex ante* owner's and manager's optimal values under asymmetric information. Then we define the measure of inefficiency arising from information asymmetry.

The *ex ante* owner's optimal value under asymmetric information, $O(x)$, is given by

$$O(x) = x^{\beta}\Big(p_1 H(q_1, \overline{x}_1; F_1) + p_2 H(q_2, \overline{x}_2; F_2 + \phi\Delta F)\Big) > 0, \tag{22}$$

and the *ex ante* manager's optimal value, $M(x)$, is given by[9]

$$M(x) = p_1\left(\frac{x}{\overline{x}_2}\right)^{\beta}\Delta F > 0. \tag{23}$$

We define the measure of inefficiency arising from information asymmetry by the social loss $L(x) := O(x) - (O(x) + M(x))$. This measure is exactly the same as in Grenadier and Wang (2005). The social loss $L(x)$ is given by

$$L(x) = p_2\Big(H(q_2, \overline{x}_2; F_2) - H(q_2, \overline{x}_2; F_2)\Big)x^{\beta} \geq 0. \tag{24}$$

Here, we have $L(x) \geq 0$ because $(q_2, \overline{x}_2) = \arg\max_{q_2, \overline{x}_2} H(q_2, \overline{x}_2; F_2)$. Note that $L(x)$ is driven by the distance of $(q_2, \overline{x}_2)$ from $(q_2, \overline{x}_2)$. In addition, $L(x)$ does not include the distortion term $\phi\Delta F$ *explicitly*, but includes it *implicitly* through q_2 and $\overline{x}_2$. In addition, to better explain the property of $L(x)$, we define the difference of a revenue-cost ratio of investment as

$$R = p_2\left(\frac{V(q_2, \overline{x}_2)}{I(q_2; F_2)} - \frac{V(q_2, \overline{x}_2)}{I(q_2; F_2)}\right). \tag{25}$$

[9] Substituting $\overline{x}_1 = \overline{x}_1$, $\overline{x}_2 = \overline{x}_2$, $w_1 = w_1$, and $w_2 = w_2$ into $M(x)$ in Constraint (11) gives $M(x)$.

4 Model implications

In this section, we consider more important implications of our model.

To examine the properties of solutions, we consider some numerical examples. In order to do so, the cost function of investment quantity, $C(q_i)$, is assumed to be[10]

$$C(q_i) = q_i^3, \quad i \quad \{1, 2\}. \tag{26}$$

In our numerical examples, we set the parameters to satisfy the second-order conditions (see the Appendix for the second-order conditions). The basic parameters are assumed to be $r = 0.09$, $\mu = 0.025$, $\sigma = 0.3$, $p_1 = 0.5$, $F_1 = 100$, $F_2 = 200$, $s = 0.5$, and $X_0 = x = 5$.

Section 4.1 provides the economic scenario of our model by using the numerical solutions. Sections 4.2 to 4.5 consider the comparative statics of the solutions with respect to four specific parameters: ΔF (degree of asymmetric information), s (degree of reversibility), σ (volatility), and p_2 (probability of $F = F_2$).

4.1 Model's economic scenario and its related empirical studies

In this subsection, we describe the economic scenario of our model by using numerical solutions. In addition, we provide related empirical studies, which correspond to the theoretical results.

[Insert Table 2 about here]

Table 2 shows the numerical solutions. At time 0, the stock price under asymmetric information is $O\ \ (x) = 194.6596$ for $x = 5$, while the stock price under symmetric information is $O\ (x) = 198.6704$. The stock price is equal to the owner's (equity) value. Thus, information asymmetry decreases the stock price. This result is consistent with previous empirical studies. See Schaller (1993), Bharath et. al. (2008), and Tang (2009) for details.

Suppose that the true (realized) value of F is F_1 after making the contract between the investor (owner) and the manager. Then, note that the manager observes that the true value of F is F_1, while the investor (market or owner) cannot observe whether it is F_1 or F_2. Our model scenario is, if X_t, starting at $x = 5$, increases and arrives at $\overline{x}_1\ \ = \overline{x}_1 = 12.5412$ from below, the manager exercises the investment in project quantity $q_1\ \ = q_1 = 8.1321$, and

[10]Here, to make the difference between q_2 and q_2 larger, we assume $C(q_i) = q_i^3$, not $C(q_i) = q_i^2$.

receives the bonus-incentive $w_1 = 29.3438 < \Delta F = 100$, where $\Delta F = 100$ is the manager's informational rent. By observing that the manager exercises the investment at $\overline{x}_1$, the market recognizes that the true value of F is F_1. Then the stock price jumps upward from 890.7340 to 912.0309.[11] The manager's investment behavior signals the true value of F as F_1 to the market.[12] After investment, if X_t decreases and reaches $\underline{x}_1 = \underline{x}_1 = 1.4013$ from above, the manager abandons the investment project.

Suppose that the true (realized) value of F is F_2 after making the contract. Note that the manager observes that the true value of F is F_2, while the investor (market or owner) cannot observe whether it is F_1 or F_2. Our model scenario is that if X_t, starting at $x = 5$, increases and arrives at $\overline{x}_1 = \overline{x}_1 = 12.5412$ from below, the manager does not exercise the investment. By observing that the manager does not exercise the investment at $\overline{x}_1$, the market recognizes that the true value of F is F_2. Then the stock price jumps downward from 890.7340 to 869.4370.[13] Recall that no renegotiation is allowed after making the contract.[14] When X_t increases and reaches $\overline{x}_2 = 26.3172$ ($> \overline{x}_2 = 19.9044$), the manager exercises the investment in project quantity $q_2 = 11.7718$ ($> q_2 = 10.2451$), and does not receive the bonus-incentive ($w_2 = 0$). Thus, asymmetric information delays the investment. Furthermore, once X_t decreases and reaches $\underline{x}_2 = 2.7675 = \underline{x}_2 = 2.2145$ after investment, the manager abandons the investment project.

In summary, at the equilibrium, the manager in state F_1 who has an informational advantage, exercises the investment at $\overline{x}_1 = \overline{x}_1$ in project quantity $q_1 = q_1$ (no distortions in the investment strategies) and receives the bonus-incentive $w_1 < \Delta F$. By contrast, the manager in state F_2 who has no informational advantage, exercises the investment at $\overline{x}_2 = \overline{x}_2$ in project

[11] Prior to the point at which X_t reaches $\overline{x}_1$, the market does not know the true value of F. The market believes that $F = F_1$ with probability p_1 and $F = F_2$ with probability p_2. The stock prices just before investment at $\overline{x}_1$ is

$$O(\overline{x}_1) = p_1\{V(\overline{x}_1, q_1) - I(q_1; F_1) - w_1\} + p_2\left(\frac{\overline{x}_1}{\overline{x}_2}\right)^{\beta}\{V(\overline{x}_2, q_2) - I(q_2; F_2)\}. \tag{27}$$

The stock price just after investment at $\overline{x}_1$ is $V(\overline{x}_1, q_1) - I(q_1; F_1) - w_1$.

[12] See Tetlock (2010) for empirical studies of price impact by resolving information asymmetry.

[13] When the manager does not exercise the investment at $\overline{x}_1$, the stock price jumps downward to $(\overline{x}_1/\overline{x}_2)^{\beta}\{V(\overline{x}_2, q_2) - I(q_2; F_2)\}$.

[14] In our model, while commitment may cause *ex post (after making the contract)* inefficiency, it causes *ex ante (at the time of making the contract)* efficiency. Such a contract is the same as in previous studies such as Grenadier and Wang (2005), Shibata (2009), and Shibata and Nishihara (2011).

quantity $q_2 = q_2$ (distortions in the investment strategies) and does not receive the bonus-incentive ($w_2 = 0$). Under asymmetric information, to induce the manager to reveal private information, the owner distorts $\overline{x}_2 = \overline{x}_2$ and $q_2 = q_2$, while the owner does not distort $\overline{x}_1 = \overline{x}_1$ and $q_1 = q_1$. These properties are similar to those in theoretical studies by Leland and Pyle (1977), Myers and Majluf (1984), Bezalel and Kalay (1983), Brennan and Kraus (1987), and Grenadier and Wang (2005). In addition, the result that information asymmetry delays the investment is consistent with empirical findings by Schaller (1993), Leahy and Whited (1996), Folta, et al. (2006), Bharath et. al. (2008), Tang (2009), and Glover and Levine (2015).

4.2 Effects of asymmetric information

In this subsection, we examine the effects of ΔF (degree of asymmetric information or degree of manager's informational rent). We assume $F_1 = 100$. Here, ΔF is changed from 0 to 100, where $\Delta F = 0$ corresponds to the symmetric information (no informational rent) case.

[Insert Figure 2 about here]

The top-left and top-middle panels of Figure 2 depict q_2 (investment quantity) and $\overline{x}_2$ (investment trigger). Both q_2 and $\overline{x}_2$ are increasing in ΔF with $\lim_{\Delta F \to 0} q_2 = q_2$ and $\lim_{\Delta F \to 0} \overline{x}_2 = \overline{x}_2$. Thus, an increase in ΔF increases the difference of $q_2 - q_2$ and $\overline{x}_2 - \overline{x}_2$, respectively. This is because q_2 and $\overline{x}_2$ do not depend on ΔF. An increase in degree of asymmetric information delays investment timing, but increases investment quantity. For a larger degree of asymmetric information, because the firm suffers from losses (i.e., the firm's value is decreased) due to delayed investment, the firm makes a larger investment quantity to compensate for the losses. These results are similar to those in Shibata and Nishihara (2011) and Cui and Shibata (2017). In addition, these results fit well with empirical findings by Schaller (1993), Bloom et al. (2007), Bharath et. al. (2008), Tang (2009), and Panousi and Papanikolaou (2012). The top-right panel shows $\underline{x}_2$ (abandonment trigger). We see that $\underline{x}_2$ is increasing with ΔF. Such a positive relationship is obtained in a straightforward manner because of $\underline{x}_2 = \underline{x}_2(q_2)$, where $\underline{x}_i(q_i)$ is an increasing function of q_i.[15] The middle-left panel depicts $\overline{x}_1/\overline{x}_2$ (ratio). We see that $\overline{x}_1/\overline{x}_2$ is decreasing with ΔF. This is because $\overline{x}_1$ is constant with ΔF, while $\overline{x}_2$ is increasing with ΔF.

[15] See the proof of the fourth property in Proposition 2 for details.

The middle-middle and middle-right panels illustrates w_1 (manager's bonus) and $M(x)$ (manager's value). Both w_1 and $M(x)$ are increasing with ΔF. The first result is obtained by the following reason. Recall that $w_1 := (\overline{x}_1/\overline{x}_2)^{\beta}\Delta F$, and an increase in ΔF decreases $\overline{x}_1/\overline{x}_2$. The magnitude of the increase in ΔF is larger than that of the decrease in $(\overline{x}_1/\overline{x}_2)^{\beta}$. This leads to the first result that w_1 is increasing with ΔF. The second result is obtained by the following reason. Recall that $M(x) = (1 - p_2)(x/\overline{x}_2)^{\beta}\Delta F$, and an increase in ΔF decreases $x/\overline{x}_2$. Similarly, the magnitude of the increase in ΔF is larger than that of the decrease in $(x/\overline{x}_2)^{\beta}$. This leads to the second result that $M(x)$ is increasing with ΔF. Note that w_1 and $M(x)$ are ***ex post*** and ***ex ante*** values, respectively. Thus, the manager's ***ex ante*** and ***ex post*** values are increasing with the manager's informational rent. The bottom-left panel shows $O(x)$ (owner's value). We see that $O(x)$ is decreasing with ΔF. Thus, the owner's value is decreasing with the manager's informational rent. This result is consistent with empirical studies by Bharath et. al. (2008), Tang (2009), and Glover and Levine (2015).

The bottom-middle and bottom-right panels demonstrate $L(x)$ and R with ΔF. Recall that q_2 and $\overline{x}_2$ are increasing with ΔF, while q_2 and $\overline{x}_2$ are constant. This implies that an increase in ΔF increases the differences of $q_2 - q_2$ and $\overline{x}_2 - \overline{x}_2$, respectively, which increases R. Thus, an increase in ΔF increases $L(x)$. An increase in R corresponds to an increase in $L(x)$.

4.3 Effects of reversibility

This subsection investigates the effects of s (degree of reversibility). Here, s is changed from 0 to 1.

[Insert Figure 3 about here]

The top-left panel of Figure 3 depicts q_2 (investment quantity) with s. Importantly, q_2 is increasing with s, while q_2 is constant, as shown in the symmetric-information model of Wong (2010). Under asymmetric information, q_2 is ***no longer*** independent of s. We show that the relationship between q_2 and s is quite different from that between q_2 and s. We confirm the result in Proposition 3.

We provide the economic implications about three properties of q_2. First, the intuition of $q_2 \geq q_2$ is as follows. Recall that, under full (symmetric) information, q_2 is obtained by solving

Equation (15). Differentiating Equation (15) with q_2 and F_2 gives

$$\frac{\mathrm{d}q_2}{\mathrm{d}F_2} = \frac{q_2 C\ (q_2)/(I(q_2;F_2))^2}{(q_2 C\ (q_2)/I(q_2;F_2))} \geq 0,$$

where we have used the assumption of $(q_2 C\ (q_2)/I(q_2;F_2)) \geq 0$. Thus, an increase in F_2 increases q_2. Recall that, under asymmetric information, q_2 and $\overline{x}_2$ are simultaneously obtained by solving Equations (19) and (20). If $\phi\Delta F = 0$, Equation (19) is equivalent to Equation (15). In other words, the difference between symmetric and asymmetric information is whether the distortion term of $\phi\Delta F$ is zero or strictly positive. The distortion term of $\phi\Delta F > 0$ is regarded as an additional cost due to asymmetric information. Thus, an additional cost $\phi\Delta F > 0$ causes the increase in q_2 , i.e., $q_2 \geq q_2$. Second, we consider the economic interpretation about the property that q_2 is ***no longer*** independent of s. This result implies that asymmetric information distorts the independence between q_2 and s that is obtained under full information. The property of the result is similar to that in Modigliani and Miller (1958) theorem, where financial frictions distort the independence between investment and capital structure that is obtained in a frictionless market. Third, we consider the intuition for q_2 being increasing with respect to s. In Equation (19), an increase in s increases $(\overline{x}_2 / \underline{x}_2(q_2))^\gamma$ which leads to the increase in $(1 - s(\overline{x}_2 / \underline{x}_2(q_2))^\gamma)^{-1}$.[16] The effect of the increase in $(1 - s(\overline{x}_2 / \underline{x}_2(q_2))^\gamma)^{-1}$ is regarded as the effect of the increase in F_2. Thus, an increase in s increases q_2 .

The top-middle panel shows $\overline{x}_2$ (investment trigger). We see that $\overline{x}_2$ is decreasing with s. This result is the same as under full information. Thus, q_2 is increasing with s, while $\overline{x}_2$ is decreasing with s. Interestingly, q_2 and $\overline{x}_2$ have a different effect with s. These effects of s are contrary to those of ΔF, where q_2 and $\overline{x}_2$ have an identical effect with ΔF.

The top-right panel shows that $\underline{x}_2$ (abandonment trigger) is increasing with s. The middle-left panel depicts $\overline{x}_1/\overline{x}_2$ (ratio) with s. We see that $\overline{x}_1/\overline{x}_2$ is decreasing with s, while $\overline{x}_1/\overline{x}_2$ is constant with s. These results imply that an increase in s increases the difference of $\overline{x}_2 - \overline{x}_1$, while it keeps the difference of $\overline{x}_2 - \overline{x}_1$ constant.

The middle-middle and middle-right panels show w_1 (manager's bonus-incentive) and $M\ (x)$ (manager's value). More interestingly, w_1 is decreasing with s, while $M\ (x)$ is increasing with s, where $w_1 = (\overline{x}_1/\overline{x}_2)^\beta \Delta F$ and $M\ (x) = (1 - p_2)(x/\overline{x}_2)^\beta \Delta F$. The reason for the first result is explained by the middle-left panel, where $\overline{x}_1/\overline{x}_2$ is decreasing with s. By

[16] The reason is that $\overline{x}_2$ and $\underline{x}_2(q_2)$ are decreasing and increasing with s, respectively. See the top-middle and top-right panels of Figure 3.

contrast, the reason for the second result is easily shown because $x/\overline{x}_2$ is increasing with s. These two results suggest the following observation.

Observation 1 *Suppose the asymmetric information problem. An increase in degree of reversibility (s) decreases the manager's ex post value (bonus-incentive), while it increases the manger's ex ante value.*

The middle-left panel depicts $O(x)$ (owner's value). We see that $O(x)$ is increasing with s. The intuitive reason for this is that the salvage value $sI(q_2 ; F_2)$ is increasing with s.

The bottom-middle and bottom-right panels depict $L(x)$ and R. In the bottom-middle panel, $L(x)$ is increasing with s. This result is interesting because it is contrary to our intuition. From our intuition, we conjecture that an increase in s reduces the social loss because it increases the owner's value (see the bottom-left panel). However, this counter-intuitive result is explained by using the bottom-right panel. In the bottom-right panel, an increase in s increases R. This means that an increase in s increases the differences of $q_2 - q_2$ and $\overline{x}_2 - \overline{x}_2$. Thus, such an increase in differences leads to a larger social loss. Thus, we have the following observation.

Observation 2 *Suppose the asymmetric information problem. An increase in degree of reversibility increases the social loss.*

We examine the economic interpretation about three results obtained by increasing $s \quad [0, 1]$. Here, note that larger s is regarded as larger salvage value (i.e., larger collateral value). First, an increase in s increases the owner's value $O(x)$ and decreases the investment trigger $\overline{x}_2$. These results imply that larger collateral value increases the stock price and accelerates corporate investment. Second, an increase in s decreases the manager's bonus $w_1 = (\overline{x}_1/\overline{x}_2)^{\beta}\Delta F$, because the ratio $\overline{x}_1/\overline{x}_2$ is decreasing with s. Because the decrease in w_1 is smaller than the decrease in $\overline{x}_2$ if s is increasing, an increase in s increases the manager's value $M(x) = p_1(x/\overline{x}_2)^{\beta}w_1$. Thus, larger collateral value decreases the manager's bonus, while it increases the manager's value. Third, we see that $O(x) - O(x)$ and $M(x)$ are increasing with s, and that the increase in $O(x) - O(x)$ is larger than the increase in $M(x)$. This result leads to the fact that the social value $L(x) = O(x) - O(x) - M(x)$ is increasing with s in Observation 2. Note that, according to the stylized fact, larger firm is regarded to have larger collateral. Thus, the result implies that larger firm has a larger social loss (i.e., larger agency cost). The result is most closely related to the stylized facts and empirical studies such as Kadapakkam (1998) and Ang et al. (2000), where larger (smaller) firm approximately has outsider (insider) manager, lower (higher) equity share of owner-manager, less (greater) monitoring by banks, and larger (smaller) cash flow-investment sensitivity, and higher (lower) agency cost.

4.4 Effects of volatility

This subsection examines the effects of σ (volatility). Here, σ is changed from 0.1 to 0.3.

[Insert Figure 4 about here]

The top-left and top-middle panels of Figure 4 show q_2 (investment quantity) and $\overline{x}_2$ (investment trigger), respectively. In the top-left panel, q_2 is increasing with σ. In the top-middle panel, $\overline{x}_2$ is increasing with σ. These results imply that an increase in volatility increases the quantity and investment trigger. The economic interpretation of these results is as follows. Higher volatility delays the investment via an increase in the value to exercise the investment as in Dixit and Pindyck (1994). Then, an increase in the value increases the investment quantity. These results are consistent with Shibata and Nishihara (2011) and Cui and Shibata (2017).

The top-right panel demonstrates $\underline{x}_2$. We see that $\underline{x}_2$ has a U-shaped relationship with σ. This is because there are two opposite effects. Recall that $\underline{x}_2 = (\gamma/(\gamma-1)v)sI(q_2;F_2)/q_2$. First, an increase in σ decreases $\gamma/(\gamma-1)$. Second, an increase in σ increases q_2 which increases $I(q_2;F_2)/q_2$. The first effect dominates the second effect for a smaller σ, while the second effect dominates the first effect for a larger σ. As a result, $\underline{x}_2$ has a U-shaped relationship with σ. This result is similar to that of Shibata and Nishihara (2010). The middle-left panel depicts $\overline{x}_1/\overline{x}_2$ (ratio). We see that $\overline{x}_1/\overline{x}_2$ is decreasing with σ, while $\overline{x}_1/\overline{x}_2$ is constant with σ. These results imply that an increase in σ increases the difference of $\overline{x}_2 - \overline{x}_1$, while it keeps the difference of $\overline{x}_2 - \overline{x}_1$ constant.

The middle-middle and middle-right panels illustrate w_1 (manager's bonus) and $M(x)$ (manager's value). Interestingly, w_1 is increasing with σ, while $M(x)$ is decreasing with σ, where $w_1 = (\overline{x}_1/\overline{x}_2)^{\beta}\Delta F$ and $M(x) = (1-p_2)(x/\overline{x}_2)^{\beta}\Delta F$. The first result is obtained as follows. As shown in the middle-left panel, $(\overline{x}_1/\overline{x}_2)$ is decreasing with σ. In addition, β is decreasing with σ (see Dixit and Pindyck (1994)). The latter effect dominates the former effect, which leads to the fact that w_1 is increasing with σ. On the other hand, the second result is obtained as follows. Recall that $\overline{x}_2$ is increasing with σ. In addition, β is decreasing with σ. The former effect dominates the latter effect, which leads to the fact that $M(x)$ is decreasing with σ. The middle-left panel shows $O(x)$ (owner's value). We see that $O(x)$ is increasing with σ. This result is the same as in Dixit and Pindyck (1994). We summarize the results as follows.

Observation 3 *Suppose the asymmetric information problem. An increase in volatility (σ) increases the manager's ex post value (bonus), while it decreases the manger's ex ante value. In addition, an increase in volatility increases the owner's ex ante value. Thus, an increase in volatility shifts the (ex ante) value from the manager to the owner.*

Note that wealth is transferred from the manager to the owner by increasing the volatility. This wealth transfer is known as *asset substitution*.[17]

The bottom-middle and bottom-right panels demonstrate $L(x)$ and R with σ. In the bottom-middle panel, $L(x)$ is decreasing with σ. This result is explained using the bottom-right panel. In the bottom-right panel, an increase in σ decreases R. It then leads to a smaller social loss. This suggests the following observation.

Observation 4 *Suppose the asymmetric information problem. An increase in volatility (σ) decreases a social loss.*

4.5 Effects of probability

In this subsection, we investigate the effects of p_2 (probability of drawing $F = F_2$). Here, the parameter p_2 is changed from 0 to 1. Note that $p_2 = 0$ and $p_2 = 1$ correspond to the symmetric (full) information situations in states F_1 and F_2, respectively.

[Insert Figure 5 about here]

The top-left and top-middle panels show q_2 (quantity) and $\overline{x}_2$ (investment trigger). We see that q_2 and $\overline{x}_2$ are decreasing with p_2, while q_2 and $\overline{x}_2$ are constant with p_2. This is because an increase in p_2 reduces the distortion term $\phi \Delta F$ ($\phi = p_1/p_2 = 1/p_2 - 1$), which decreases the differences of $q_2 - q_2$ and $\overline{x}_2 - \overline{x}_2$. As p_2 goes closer to 1, q_2 and $\overline{x}_2$ go closer to q_2 and $\overline{x}_2$,

[17]See Myers (1977) and Bezalel and Kalay (1983) for the asset substitution.

respectively. The top-right panel illustrates that $\underline{x}_2$ (abandonment trigger) is decreasing with p_2. The middle-left panel depicts $\overline{x}_1/\overline{x}_2$ (trigger ratio). We see that $\overline{x}_1/\overline{x}_2$ is increasing with p_2.

The middle-middle and middle-right panels illustrate w_1 (bonus-incentive) and $M \ (x)$ (manager's value). We see that w_1 is increasing with p_2, while $M \ (x)$ has an inverse U-shaped relationship with p_2, where $w_1 = (\overline{x}_1/\overline{x}_2)^{\beta}\Delta F$ and $M \ (x) = (1 - p_2)(x/\overline{x}_2)^{\beta}\Delta F$. The reason for the first result is explained by the middle-left panel, where $\overline{x}_1/\overline{x}_2$ is increasing with p_2. The reason for the second result is explained by the fact that $\overline{x}_2$ is decreasing with p_2, which implies that $(x/\overline{x}_2)^{\beta}$ is increasing with p_2. In addition, for the extreme cases of $p_2 = 0$ and $p_2 = 1$, we have $M \ (x) = 0$ and the maximum value at $p_2 = 0.44$. Because, for $p_2 \quad [0, 0.44)$, the magnitude of the decrease in $(1 - p_2)$ is smaller than that of the increase in $(x/\overline{x}_2)^{\beta}$, $M \ (x)$ is increasing with p_2. Because, for $p_2 \quad [0.44, 1]$, the magnitude of the decrease in $(1 - p_2)$ is larger than that of the increase in $(x/\overline{x}_2)^{\beta}$, $M \ (x)$ is decreasing with p_2.

The bottom-left panel shows $O \ (x)$ (owner's value). We see that $O \ (x)$ is decreasing with p_2. The bottom-middle panel demonstrates $L(x)$. Here, $L(x)$ has an inverse U-shaped relationship with p_2 and $L(x) = 0$ for the extreme values of $p_2 = 0$ and $p_2 = 1$. The bottom-right panel illustrates R. We see that R has an inverse U-shaped curve with p_2. These results are obtained by the fact that differences of $O \ (x) - O \ (x)$ and $M \ (x) - 0$ are increasing with p_2 for a smaller p_2, while they are decreasing with p_2 for a larger p_2. We make the following observation.

Observation 5 *Suppose the asymmetric information problem. The social loss has an inverse U-shaped relationship with the probability of drawing a high fixed-cost.*

5 Concluding remarks

In this paper, we study the reversibility effects on a firm's investment timing and quantity strategies, especially in the presence of manager's private information. We obtain five results. First, information asymmetry increases (delays) investment trigger (timing). Second, under information asymmetry, investment quantity is increasing in degree of reversibility, while under information symmetry it is constant. Third, social loss arising from information asymmetry is increasing in the degree of manager's informational rent and degree of reversibility, but it is decreasing in volatility. Fourth, an increase in volatility increases the owner's value, while it decreases the manager's value. Fifth, an increase in volatility increases the ex post manager's value, while it decreases the ex ante manager's value. An increase in degree of reversibility decreases the ex post manager's value, while it increases the ex ante manager's value.

Acknowledgments

We would like to thank the participants at the conferences of the CEF 2015 (Taiwan), the EURO 2015 (Glasgow), and the RIMS 2015 (Kyoto) for their helpful comments. This work was supported by the Asian Human Resources Fund of the Tokyo Metropolitan Government and JSPS KAKENHI (Grant numbers: 26242028, 16KK0083, and 17H02547).

Appendix

Derivations of Equations (15) and (16)

The derivations of Equations (15) and (16) are similar to those of Equations (19) and (20) in Proposition 1. See the proof of Proposition 1 for the derivations of Equations (19) and (20). More precisely, in the proof of Proposition 1, we have already derived Equations (A.8) and (A.9). We assume $\Delta F = 0$ (i.e., our model turns out to be the full-information model). By substituting $\phi\Delta F = 0$ into Equations (A.8) and (A.9), we obtain Equation (16) and

$$\left(1 - s\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\right)\left(C\,(q_2) - \frac{\beta}{\beta - 1}\frac{I(q_2;F_2)}{q_2}\right) = 0,$$

respectively. Because we have $(1 - s(\overline{x}_2/\underline{x}_2(q_2))^{\gamma}) > 0$ for any s $[0, 1]$, we obtain Equation (15).

Proof of Proposition 1

Because the optimum under asymmetric information is limited, for notational simplicity we drop the superscript "**" and simply write q_2 and $\overline{x}_2$. Recall that under asymmetric information, the optimization problem for $F = F_2$ is

$$\max_{q_2,\overline{x}_2} H(q_2, \overline{x}_2; F_2 + \phi\Delta F), \tag{A.1}$$

where

$$H(q_2, \overline{x}_2; F_2 + \phi\Delta F) = \{V(q_2, \overline{x}_2) - I(q_2; F_2 + \phi\Delta F)\}\, \overline{x}_2^{-\beta}. \tag{A.2}$$

Differentiating H with q_2 gives

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}q_2} &= \frac{\partial H}{\partial q_2} + \frac{\partial H}{\partial \underline{x}_2}\frac{\partial \underline{x}_2}{\partial q_2} && \text{(A.3)}\\ &= \frac{\partial H}{\partial q_2} && \text{(A.4)}\\ &= \overline{x}_2^{-\beta}\left(v\overline{x}_2 + \left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}(sC\,(q_2) - v\underline{x}_2(q_2)) - C\,(q_2)\right), && \text{(A.5)}\end{aligned}$$

where we have applied the envelope theorem from Equation (A.3) to Equation (A.4). The condition of $\mathrm{d}H/\mathrm{d}q_2 = 0$ is

$$v\overline{x}_2 + \left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}(sC\,(q_2) - v\underline{x}_2(q_2)) - C\,(q_2) = 0. \tag{A.6}$$

Differentiating H with $\overline{x}_2$ gives

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}\overline{x}_2} &= \overline{x}_2^{-\beta}\Bigg(\frac{-\beta}{\overline{x}_2}\{V(q_2,\overline{x}_2) - I(q_2;F_2+\phi\Delta F)\} + vq_2 \\ &\quad + \frac{\gamma}{\overline{x}_2}\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}(sI(q_2;F_2) - vq_2\underline{x}_2(q_2))\Bigg).\end{aligned} \tag{A.7}$$

The condition of $\mathrm{d}H/\mathrm{d}\overline{x}_2 = 0$ is

$$(\beta-1)v\overline{x}_2 + (\beta-\gamma)\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\left(s\frac{I(q_2;F_2)}{q_2} - v\underline{x}_2(q_2)\right) - \beta\frac{I(q_2;F_2+\phi\Delta F)}{q_2} = 0. \tag{A.8}$$

Substituting Equation (A.6) into Equation (A.8) yields

$$\left(1 - s\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\right)\left(C\ (q_2) - \frac{\beta}{\beta-1}\frac{I(q_2;F_2)}{q_2}\right) = \frac{\beta}{\beta-1}\frac{\phi\Delta F}{q_2}. \tag{A.9}$$

Rearranging Equations (A.9) and (A.8) gives Equations (19) and (20), respectively.

Derivations of second-order conditions

We provide the second-order conditions as sufficient conditions for the local optimum to the problem. Note that the second-order conditions for the problem $H(\overline{x}_2, q_2; F_2 + \phi\Delta F)$ are the same as those for the problem $H(\overline{x}_2, q_2; F_2)$.

The second-order conditions for the problem are given as

$$P_{qq} < 0, \text{ and } |P| > 0, \tag{A.10}$$

where the Hessian matrix P is

$$P = \begin{pmatrix} P_{\overline{x}\overline{x}} & P_{\overline{x}q} \\ P_{q\overline{x}} & P_{qq}\end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 H}{\partial \overline{x}_2^2} & \dfrac{\partial^2 H}{\partial \overline{x}_2 \partial q_2} \\ \dfrac{\partial^2 H}{\partial q_2 \partial \overline{x}_2} & \dfrac{\partial^2 H}{\partial q_2^2}\end{pmatrix}. \tag{A.11}$$

Note that $P_{\overline{x}q} = P_{q\overline{x}}$. Here, $P_{\overline{x}\overline{x}}$, $P_{\overline{x}q}$, and P_{qq} are given by

$$P_{\overline{x}\overline{x}} = \overline{x}_2^{-\beta}\left(\frac{vq_2}{\overline{x}_2}(1-\beta) + \frac{\gamma(\gamma-\beta)}{\overline{x}_2^2}(sI(q_2) - vq_2\underline{x}_2(q_2))\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\right), \tag{A.12}$$

$$P_{\overline{x}q} = \overline{x}_2^{-\beta}\left(v + \frac{\gamma}{\overline{x}_2}(sC\ (q_2) - v\underline{x}_2(q_2))\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\right), \tag{A.13}$$

$$P_{qq} = \overline{x}_2^{-\beta}\left(\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^{\gamma}\left(sC\ (q_2) - \left(\frac{\gamma}{\overline{x}_2} + v\right)\frac{\partial \underline{x}_2(q_2)}{\partial q_2}\right) - C\ (q_2)\right), \tag{A.14}$$

where $\partial\underline{x}_2(q_2)/\partial q_2 = (\gamma(q_2C\ (q_2) + I(q_2;F_2))/((\gamma-1)vq_2^2)$. In our numerical examples, we set the parameters to satisfy the second-order conditions in (A.10) as sufficient conditions for the local optimum.

Proof of Proposition 2

We provide the proofs of the four properties. First, we provide the proof of $\overline{x}_2 \geq \overline{x}_2$, which is equivalent to the proof of $\mathrm{d}\overline{x}_2 / \mathrm{d}\Delta F \geq 0$. Here, we show the proof of $\mathrm{d}\overline{x}_2 / \mathrm{d}\Delta F \geq 0$. By totally differentiating Equations (A.5) and (A.7), we have $Py = h$, where the matrix P is defined in Equation (A.11), and the vectors y and h are defined by

$$y = \begin{pmatrix} \mathrm{d}\overline{x}_2 \\ \mathrm{d}q_2 \end{pmatrix}, \quad h = \begin{pmatrix} -\dfrac{\beta\phi}{\overline{x}_2{}^{\beta+1}}\mathrm{d}\Delta F \\ 0 \end{pmatrix}. \tag{A.15}$$

Thus, we obtain

$$\frac{\mathrm{d}\overline{x}_2}{\mathrm{d}\Delta F} = -\frac{P_{qq}}{|P|}\frac{\beta\phi}{\overline{x}_2{}^{\beta+1}} \geq 0, \tag{A.16}$$

where we have used the second-order conditions in (A.10): $P_{qq} \leq 0$ and $|P| > 0$.

Second, we derive $q_2 \geq q_2$. Because $1 > s(\overline{x}_2/\underline{x}_2(q_2))^\gamma$ and $\beta\phi\Delta F/((\beta-1)q_2) \geq 0$, Equation (19) implies that q_2 must satisfy

$$\frac{q_2 C(q_2)}{I(q_2; F_2)} \geq \frac{\beta}{\beta - 1}. \tag{A.17}$$

By contrast, as shown in Equation (15), q_2 satisfies

$$\frac{q_2 C(q_2)}{I(q_2; F_2)} = \frac{\beta}{\beta - 1}. \tag{A.18}$$

The condition $(q_2 C(q_2)/I(q_2; F_2)) \geq 0$ ensures that there exists a unique solution q_2. We must assume that $(q_2 C(q_2)/I(q_2; F_2)) \geq 0$. See Cui and Shibata (2017) about the second-order condition for details. Thus, by using Equations (A.17) and (A.18), we obtain $q_2 \geq q_2$.

Third, we show the proof of $0 < w_1 < \Delta F$. Recall that $w_1 = (\overline{x}_1/\overline{x}_2)^\beta \Delta F$. We have already obtained $\overline{x}_1 < \overline{x}_2 \leq \overline{x}_2$. Thus, we have $\overline{x}_1/\overline{x}_2 < 1$ and $\beta > 1$, which completes the proof of $0 < w_1 < \Delta F$.

Finally, we provide the proof of $\underline{x}_2 \geq \underline{x}_2$, which is equivalent to the proof of $\mathrm{d}\underline{x}_2(q_2)/\mathrm{d}q_2 \geq 0$, because we already know that $q_2 \geq q_2$. Differentiating $\underline{x}_2(q_2)$ with q_2 gives

$$\frac{\mathrm{d}\underline{x}_2(q_2)}{\mathrm{d}q_2} = \frac{\gamma}{\gamma - 1}\frac{1}{vq_2}\left(C(q_2) - \frac{I(q_2)}{q_2}\right). \tag{A.19}$$

On the one hand, substituting $C(q_2)$ in Equation (19) into Equation (A.19) yields

$$\frac{\mathrm{d}\underline{x}_2(q_2)}{\mathrm{d}q_2} = \frac{\gamma}{\gamma - 1}\frac{1}{vq_2}\frac{1}{\beta - 1}\left(\frac{I(q_2; F_2)}{q_2} + \beta\phi\Delta F\left(1 - s\left(\frac{\overline{x}_2}{\underline{x}_2(q_2)}\right)^\gamma\right)^{-1}\right). \tag{A.20}$$

Recall that $(1 - s(\overline{x}_2 / \underline{x}_2(q_2))^{\gamma}) \quad [0,1]$, which leads to the positivity of the right-hand side of Equation (A.20). On the other hand, substituting $C(q_2) = \beta I(q_2; F_2)/((\beta-1)q_2)$ into Equation (A.19) yields $\mathrm{d}\underline{x}_2(q_2)/\mathrm{d}q_2 \geq 0$. These complete the proof.

Proof of Proposition 3

Because the optimum under asymmetric information is considered, for notational simplicity, we drop the superscript "**" and simply write q_2 and $\overline{x}_2$. In addition, we simply write $I(q_2)$ as $I(q_2; F_2)$. Differentiating $\mathrm{d}H/\mathrm{d}q_2$ in Equation (A.5) and $\mathrm{d}H/\mathrm{d}\overline{x}_2$ in Equation (A.7) with $\overline{x}_2$, q_2, and s, gives $Py = g$, where the matrix P, the vector y, and the vector g are defined by (A.11), (A.15), and

$$g = \frac{(\overline{x}_2)^{\gamma-\beta}}{(\underline{x}_2(q_2))^{\gamma}} \begin{pmatrix} \dfrac{\beta-\gamma}{1-\gamma}\dfrac{I(q_2)}{\overline{x}_2} \\ -C(q_2) + \dfrac{\gamma}{\gamma-1}\dfrac{I(q_2)}{q_2} \end{pmatrix} \mathrm{d}s, \tag{A.21}$$

respectively. The solution $\mathrm{d}q_2$ is

$$\mathrm{d}q_2 = -\frac{1}{|P|}\frac{(\overline{x}_2)^{\gamma-\beta}}{(\underline{x}_2(q_2))^{\gamma}}\left[\frac{\beta-\gamma}{1-\gamma}\frac{I(q_2)}{\overline{x}_2}P_{\overline{x}q} - \left(- C(q_2) + \frac{\gamma}{\gamma-1}\frac{I(q_2)}{q_2}\right)P_{\overline{x}\overline{x}}\right]\mathrm{d}s. \tag{A.22}$$

Using $\mathrm{d}H/\mathrm{d}q_2 = 0$ in Equation (A.6) and $\mathrm{d}H/\mathrm{d}\overline{x}_2 = 0$ in Equation (A.8), $P_{\overline{x}\overline{x}}$ and $P_{\overline{x}q}$ are rewritten as

$$P_{\overline{x}\overline{x}} = \overline{x}_2^{-\beta}\left((1-\beta)(1-\gamma)v\frac{q_2}{\overline{x}_2} - \beta\gamma\frac{I(q_2)+\phi\Delta F}{\overline{x}_2^2}\right), \tag{A.23}$$

$$P_{\overline{x}q} = \overline{x}_2^{-\beta}\left((1-\gamma)v + \gamma\frac{C(q_2)}{\overline{x}_2}\right), \tag{A.24}$$

respectively. Substituting Equations (A.23) and (A.24) into Equation (A.22), we obtain

$$\frac{\mathrm{d}q_2}{\mathrm{d}s} = -\frac{1}{|P|}\frac{(\overline{x}_2)^{\gamma-2\beta}}{(\underline{x}_2(q_2))^{\gamma}}\Big[A_1 - A_2\Big], \tag{A.25}$$

where

$$A_1 := (\beta-\gamma)v\frac{I(q_2)}{\overline{x}_2} + \frac{(\beta-\gamma)\gamma}{1-\gamma}\frac{I(q_2)}{\overline{x}_2^2}C(q_2), \tag{A.26}$$

$$\begin{aligned} A_2 := & -(1-\beta)(1-\gamma)v\frac{q_2}{\overline{x}_2}C(q_2) - (1-\beta)\gamma v\frac{I(q_2)}{\overline{x}_2} + \beta\gamma\frac{I(q_2)+\phi\Delta F}{\overline{x}_2^2}C(q_2) \\ & - \frac{\beta\gamma}{\gamma-1}\frac{(I(q_2)+\phi\Delta F)I(q_2)}{q_2\overline{x}_2^2}. \end{aligned} \tag{A.27}$$

Rearranging $A_1 - A_2$ yields

$$\begin{aligned} A_1 - A_2 &= (\beta - \gamma)\frac{I(q_2)}{\overline{x}_2^2}\left\{\frac{v\overline{x}_2}{m+n}\left(m - \frac{q_2 C\,(q_2)}{I(q_2)}\right)\right. \\ &\left. +\frac{\gamma}{1-\gamma}\frac{1}{m+n}\left[\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)C\,(q_2) - mn\frac{I(q_2)+\phi\Delta F}{q_2}\right]\right\}, \end{aligned} \tag{A.28}$$

where m and n are defined as

$$m := \frac{\beta}{\beta - 1} > 1, \quad n := \frac{\gamma}{1-\gamma} < 0,$$

where we have used $\beta > 1$ and $\gamma < 0$. Note that $m + n$, mn, $m/(m + n)$, and $n/(m + n)$ are

$$\begin{aligned} m+n &= \frac{\beta - \gamma}{(\beta - 1)(1-\gamma)} > 0, \qquad & mn &= \frac{\beta\gamma}{(\beta-1)(1-\gamma)} < 0, \\ \frac{m}{m+n} &= \frac{\beta(1-\gamma)}{\beta-\gamma} > 0, & \frac{n}{m+n} &= \frac{(\beta-1)\gamma}{\beta-\gamma} < 0, \end{aligned}$$

respectively. By removing $(\overline{x}_2/\underline{x}_2(q_2))^\gamma$ in two simultaneous equations, $\mathrm{d}H/\mathrm{d}q = 0$ in Equation (A.6) and $\mathrm{d}H/\mathrm{d}\overline{x} = 0$ in Equation (A.8), we obtain

$$\frac{v\overline{x}_2}{m+n}\left(m - \frac{q_2 C\,(q_2)}{I(q_2)}\right) = \frac{1}{m+n}\left[\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)C\,(q_2) - mn\frac{I(q_2)+\phi\Delta F}{q_2}\right]. \tag{A.29}$$

Substituting Equation (A.29) into Equation (A.25) gives

$$\frac{\mathrm{d}q_2}{\mathrm{d}s} = \frac{1}{|P|}\frac{(\overline{x}_2)^{\gamma-2\beta-2}}{(\underline{x}_2(q_2))^\gamma}\frac{\gamma-\beta}{1-\gamma}\frac{I(q_2)}{m+n}\left[\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)C\,(q_2) - mn\frac{I(q_2)+\phi\Delta F}{q_2}\right]. \tag{A.30}$$

Recall that $C\,(q_2) \geq mI(q_2)/q_2$ in Equation (A.17) under asymmetric information. Because $m + n > 0$ and $(n - m(\phi\Delta F/I(q_2))) \leq 0$, we have

$$\frac{1}{m+n}\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)C\,(q_2) \leq \frac{1}{m+n}\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)m\frac{I(q_2)}{q_2}, \tag{A.31}$$

which implies

$$\frac{1}{m+n}\left[\left(n - m\frac{\phi\Delta F}{I(q_2)}\right)C\,(q_2) - mn\frac{I(q_2)+\phi\Delta F}{q_2}\right] \leq -m\frac{\phi\Delta F}{q_2} \leq 0. \tag{A.32}$$

Substituting the inequality (A.32) into Equation (A.30), we obtain $\mathrm{d}q_2/\mathrm{d}s \geq 0$ because of $|P| \geq 0$ and $(\gamma - \beta)/(1 - \gamma) < 0$. This completes the proof.

Finally, we consider the extreme case under symmetric (full) information (i.e., the case of $\Delta F = 0$) to ensure that the above proof is correct. By substituting $\Delta F = 0$ and $C\,(q) = mI(q)/q$ into Equation (A.30), we have $\mathrm{d}q_2/\mathrm{d}s = 0$. By contrast, we confirm that, by substituting $\Delta F = 0$, the symmetric-information solution q_2 must satisfy $C\,(q_2) - mI(q_2)/q_2 = 0$.

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	Information	q	$\underline{x}$
Wong (2010)	Symmetry	Endogeneity	Endogeneity
Grenadier and Wang (2005)	Asymmetry	–	–
Cui and Shibata (2017)	Asymmetry	Endogeneity	–
Our model	Asymmetry	Endogeneity	Endogeneity

Table 1: Difference between our model and others' models

	$F = F_1$		$F = F_2$	
	Symmetry	Asymmetry	Symmetry	Asymmetry
$\overline{x}_i$	12.5412	12.5412	19.9044	26.3172
q_i	8.1321	8.1321	10.2451	11.7718
$\underline{x}_i$	1.4013	1.4013	2.2145	2.7675
w_i	–	29.3438	–	0
$V(\overline{x}_i, q_i)$	1579.16	1579.16	3157.55	4793.36
$I(q_i; F_i)$	637.78	637.78	1275.35	1831.28
	Symmetry		Asymmetry	
$O(x)$	198.6704		194.6596	

Table 2: Numerical solutions

The parameters are $r = 0.09$, $\mu = 0.025$, $\sigma = 0.3$, $p_1 = 0.5$, $F_1 = 100$, $F_2 = 200$, $s = 0.5$, and $x = 5$.

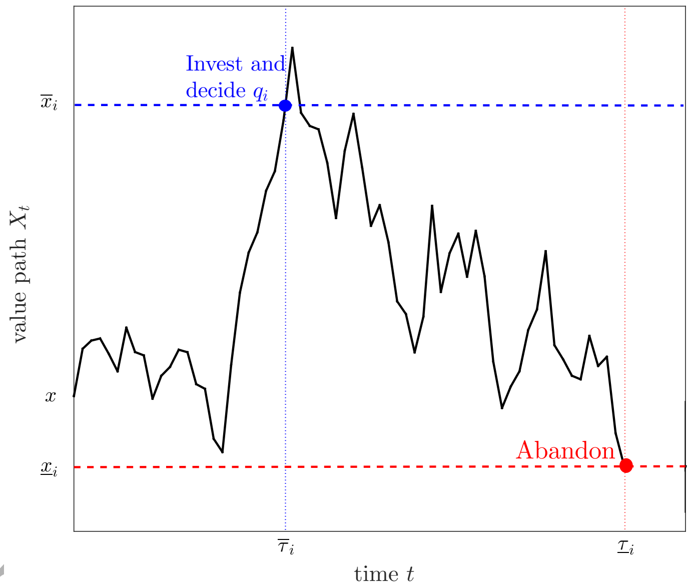

Figure 1: Scenario of our model

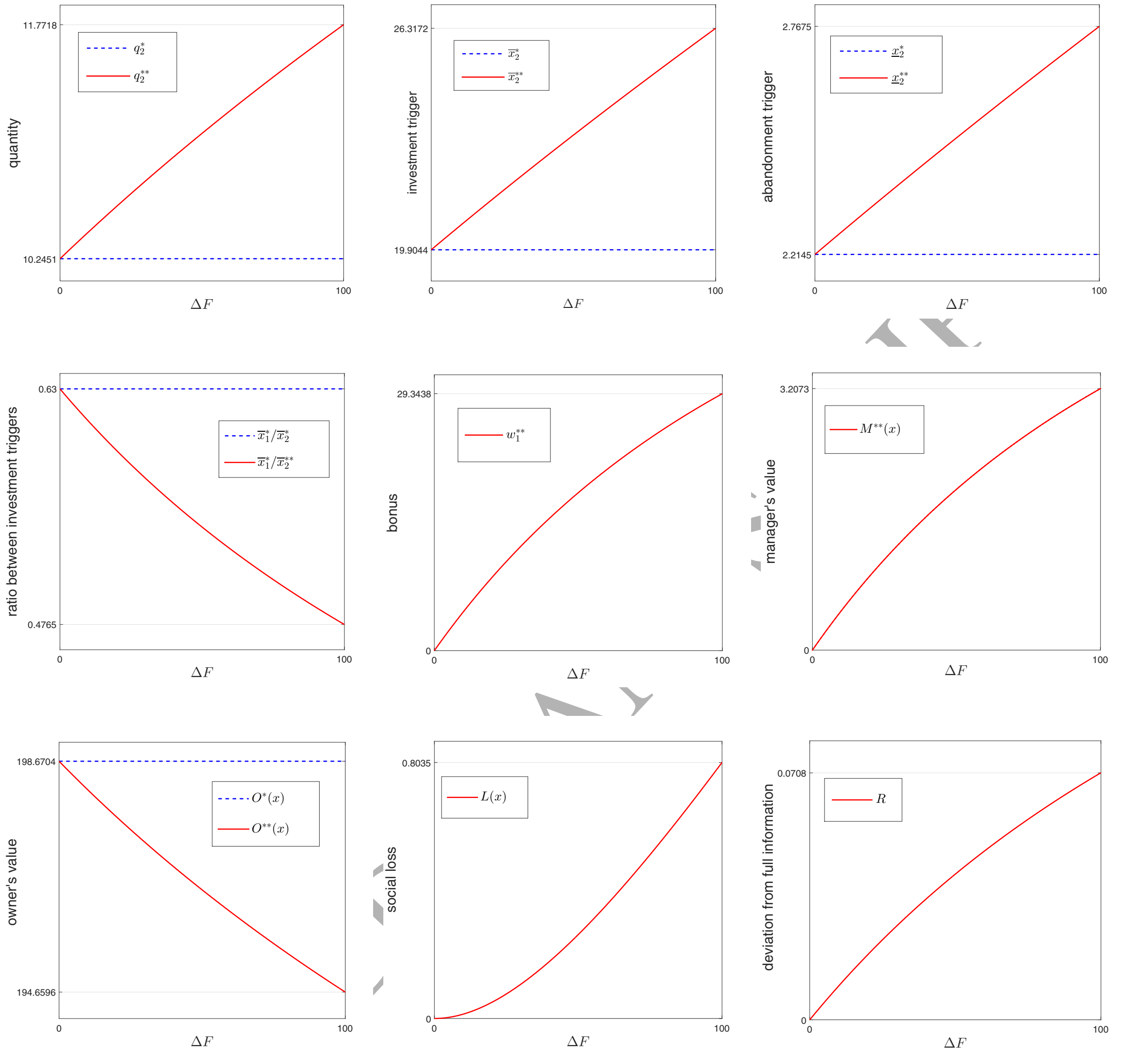

Figure 2: Effects of degree of asymmetric information (ΔF)

The parameters are $r = 0.09$, $\mu = 0.025$, $F_1 = 100$, $\sigma = 0.3$, $p_1 = 0.5$, $s = 0.5$, and $x = 5$.

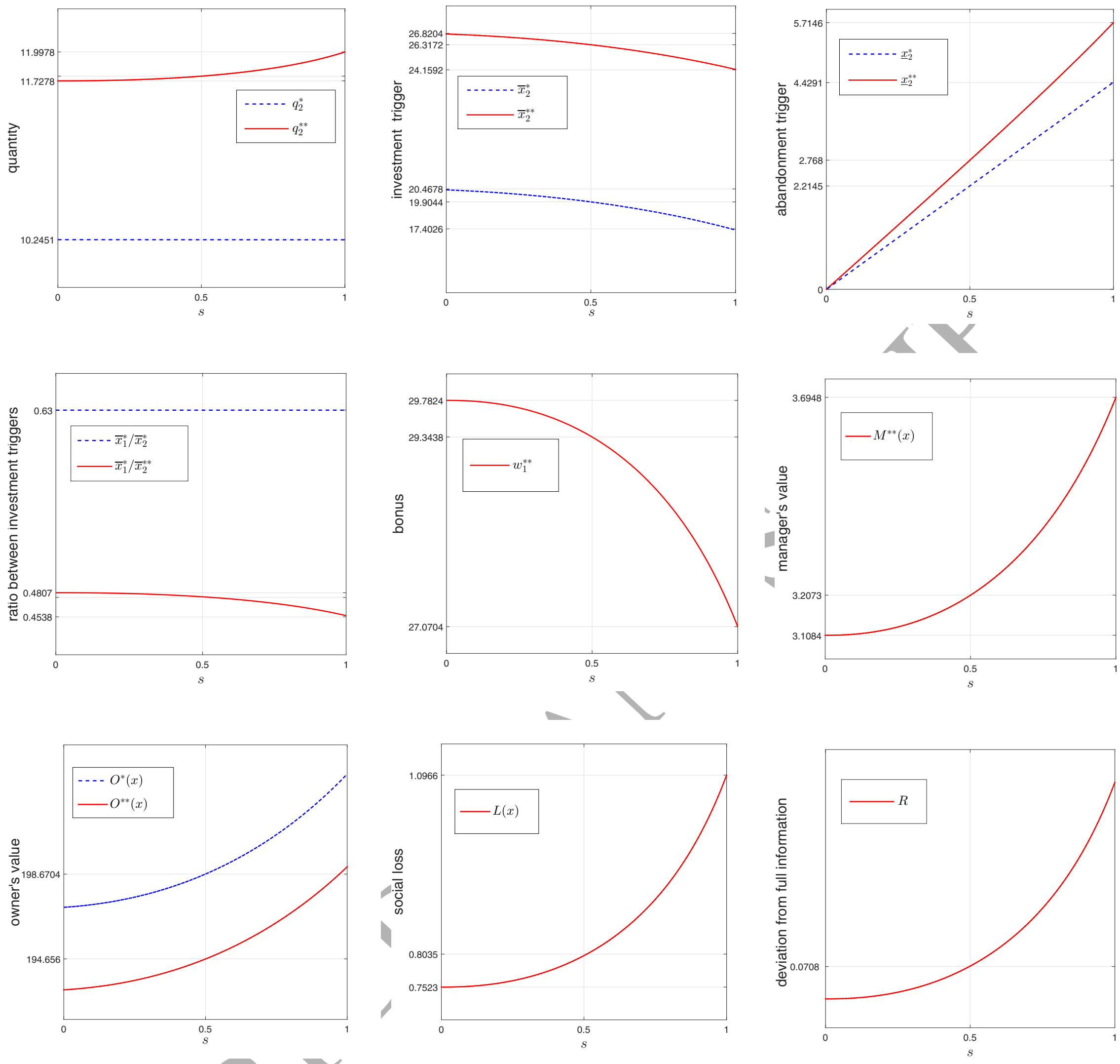

Figure 3: Effects of degree of investment reversibility (s)

The parameters are $r = 0.09$, $\mu = 0.025$, $F_1 = 100$, $F_2 = 200$, $\sigma = 0.3$, $p_1 = 0.5$, and $x = 5$.

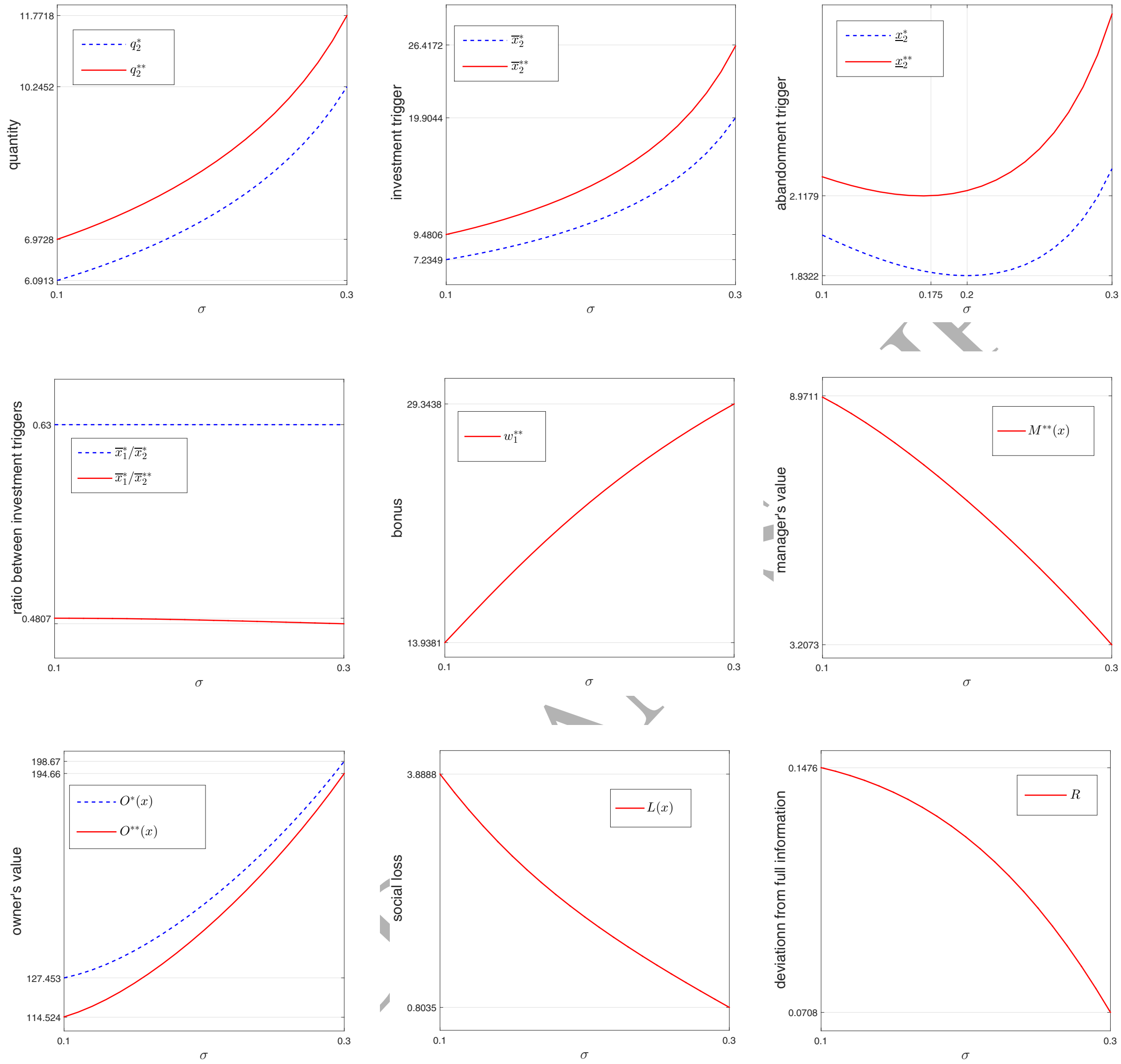

Figure 4: Effects of volatility (σ)

The parameters are $r = 0.09$, $\mu = 0.025$, $F_1 = 100$, $F_2 = 200$, $p_1 = 0.5$, $s = 0.5$, and $x = 5$.

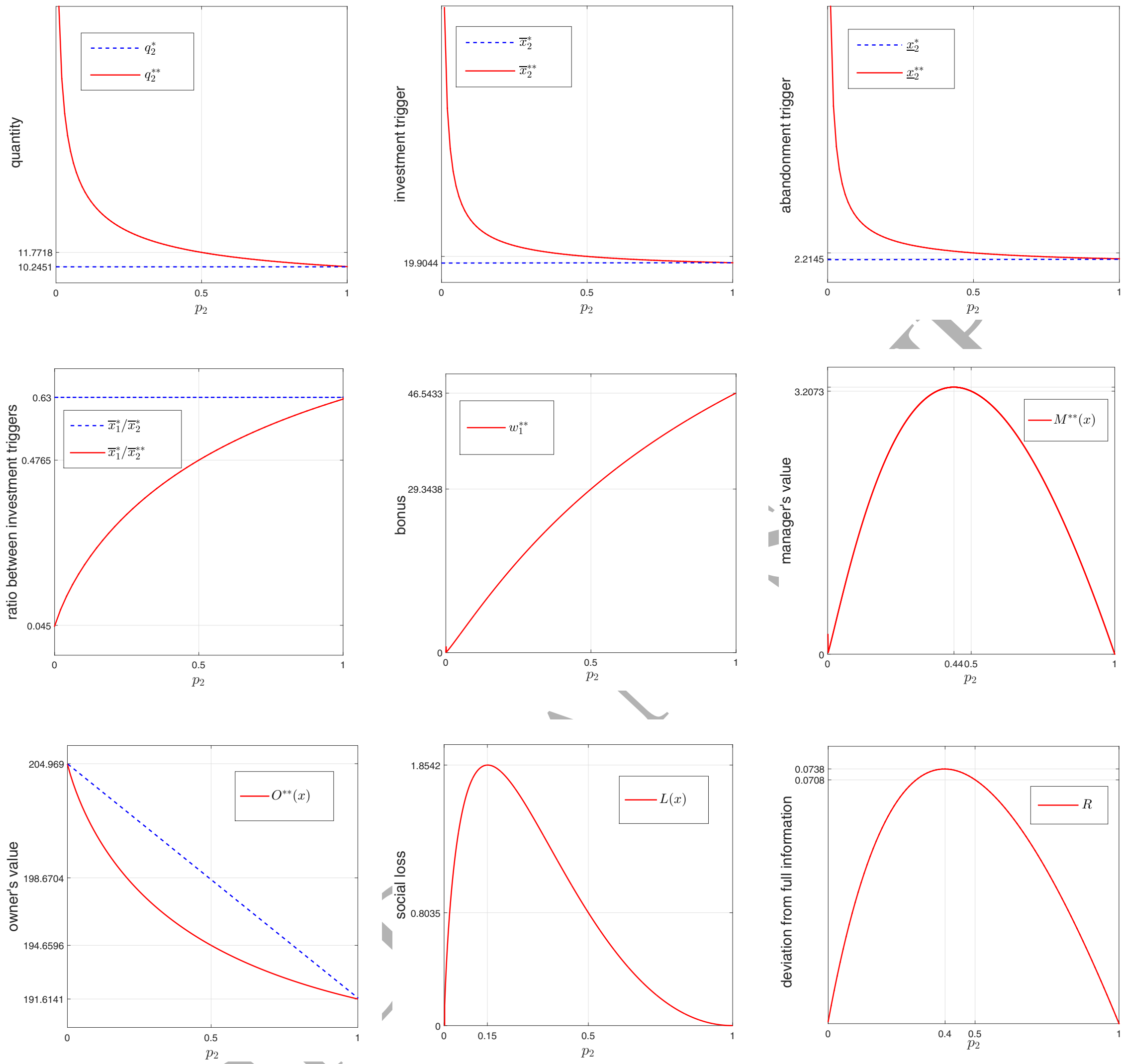

Figure 5: Effects of probability of occurrence with F_2 (p_2)

The parameters are $r = 0.09$, $\mu = 0.025$, $F_1 = 100$, $F_2 = 200$, $\sigma = 0.3$, $s = 0.5$, and $x = 5$.