Optimal dynamic policies for integrated production and marketing planning in business-to-business marketplaces

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Abstract

This work develops optimal dynamic policies for integrated production and marketing planning in a vertically decentralized single-manufacturer and single-retailer channel over a multi-period planning horizon, subject to deteriorating goods and a multivariate demand function. This work formulates the discount profit maximization problem, and provides inter-enterprise dynamic joint decisions for retail price and replenishment schedule/quantity using a calculus-based formulation combined with dynamic programming. Additionally, two alternatives for doing business, namely, retailer-managed inventory with a price-only contract and vendor-managed inventory (VMI) with a consignment contract, are applied to business-to-business traditional marketplaces (TMs) and electronic marketplaces (EMs), respectively. Numerical results demonstrate that solutions generated in EMs outperform those in TMs in terms of maximizing channel-wide total discount profits and those of manufacturer and retailer. Further, analytical results show that the proposed policy under VMI with a consignment contract in EMs significantly increases system efficiency and simultaneously achieves Pareto improvements using an extra one-part tariff for the decentralized channel.

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1. Introduction

Information technology (IT) has become increasingly important in enhancing supply chain performance in highly competitive global markets. Electronically enabled supply chains have the potential to improve supply chain performance, including increasing coordination effectiveness and transaction efficiency, by altering the quantity and velocity of information flows among supply chain partners (Chopra and Meindl, 2001; McAfee, 2002; Liu et al., 2005; Sanders, 2008). Swaminathan and Tayur (2003) demonstrated that an important link exists between an IT strategy, business processes, and supply chain performance. Indeed, effective supply chain management (SCM) requires collaboration among business processes and the integration of IT systems. Numerous studies have shown that enterprises applying business processes and IT systems outperform their competitors (Simchi-Levi et al., 2008; Grunfileh and Tarafdar, 2014). In addition to increasing operational efficiency, a highly integrated system can strengthen strategic advantages and generate related benefits (Stefanou, 2001; Mandal and Gunasekaran, 2002; Fin, 2006; Wong et al., 2014).

An important building block in effective supply chain strategies is strategic partnerships between suppliers and buyers. The benefits of inter-organizational collaboration and cooperation between upstream and downstream supply chain entities include reduced costs, reduced capital investment, reduced pooling risk, increased agility and adaptability, improved customer service, enhanced profit margins, and a focus on core competencies (Stank et al., 1999; Lee, 2000; Quinn, 2000; Vickery et al., 2003; Arshinder et al., 2008; Chou and Chang, 2008; Chen and Wei, 2012; Chen, 2013; Sarker, 2013; Wu et al., 2014). One primary coordination task is to streamline business flows of goods and information and decision-making processes among channel partners using vendor-managed inventory (VMI) for a vertically decentralized channel (Disney and Towill, 2003). Strategic partnerships alter the ways in which information is shared and inventory is managed in a supply chain, potentially eliminating the bullwhip effect (Moinzadeh, 2002; Reddy and Rajendra, 2005; Yu et al., 2008, 2010). For example, Milliken & Company works with several clothing suppliers and retailers, all of which agree to use point-of-sale data from department stores to synchronize ordering and manufacturing activities. The lead time from ordering fabric to receipt of finished goods by the department stores was reduced from 18 to 3 weeks (Schonberger, 1996). The most important requirement for an effective supply chain partnership, especially one moving toward the VMI system with consignment-based revenue-sharing contracts, is an advanced information system, including electronic data interchange...
and Internet-based exchange, which can decrease data transfer time and number of entry mistakes (Lee et al., 1999; Dejonckheere et al., 2004). This work fills the gap between production and marketing decision-making by developing model-driven optimization-based policies in a vertically decentralized single-manufacturer and single-retailer dynamic channel over a multi-period planning horizon. Moreover, this work develops policies for a VMI system in which consignment with a revenue-sharing contract is applied to solve the problem of optimally dynamic decisions in business-to-business (B2B) traditional markets (TMs) and electronic markets (EMs).

Electronic markets are an increasingly important research topic in the IT domain (Kaplan and Sawhney, 2000). Via advances in technology, Internet-based EMs, which typically have low transaction cost and are easily searched by buyers and sellers, have changed the way in which trade is conducted in a channel (Gunasekaran et al., 2002; Hausen et al., 2006). Notably, EMs reduce procurement costs by a few percent to 40% depending on the industry, on average, approximately 15% (Simchi-Levi et al., 2008). Wang and Benaroch (2004) demonstrated that supplier and buyer decisions about whether to join a B2B EM depend on the revenue structure of the EM owner, and that the optimal replenishment quantity decision with a full return policy under a single-period news-vendor supply chain in EMs can increase the channel efficiency. An EM system is an inter-organizational information system that exists away from the physical location of a TM, serves as an intermediary between buyers and sellers in a vertical market, and allows buyers and sellers to exchange information about prices and goods (Bakos, 1991). The latest TIs enhance inter-organizational interactions between production and marketing, such as those in customer relationship management, enterprise resource planning (ERP), decision support systems (DSSs), SCM, and EMs. In recent years, SCM with the growth of ERP has become an important focus of decision support applications, and the difficulty in obtaining the data required to model supply chains has decreased due to ERP (Power and Sharda, 2007). Model-driven optimization-based DSSs are typically applied to such SCM stages as transportation, manufacturing, capacity, demand, replenishment, and pricing. Vigus et al. (2001) reported that the Kellogg Company saved millions of dollars by using model-driven optimization-based systems. Smith et al. (2003) developed a model-driven DSS that maximizes profit for a retail supply chain with multiple vendors. However, information system benefits cannot be fully realized without a finely tuned alignment and reconciliation between system configurations, organizational imperatives, and core business processes (Al-Mashari et al., 2003). Further, the fundamental basis of planning and scheduling in an information system is based on fixed and static settings (Pettty et al., 2000; Hsiang, 2001). As a result, the system generates suboptimal solutions to the pricing and lot-size/scheduling problem. Supply chains are dynamic systems that evolve over time; that is, customer demand changes over time, as do supply chain relationships (Simchi-Levi et al., 2008). Numerous studies have implicitly assumed that enterprises cannot control demand, when in fact this is untrue. In a competitive business environment, enterprises have relied heavily on the dynamic pricing scheme to improve their net profits. Owing to its effectiveness in demand-side control over on-hand stock that generates considerable profits, dynamic pricing schemes are increasingly prevalent in retailing and e-commerce merchandising (Boyd and Bilegan, 2003). Furthermore, deterioration affects numerous inventory items, such as fashion goods and high-tech products, which are subject to depletion by phenomena other than demand—such as shrinkage and obsolescence. Deteriorating inventory problems have attracted recently significant research attention.

The representative research on dynamic pricing and lot-sizing planning models with deteriorating items includes Cohen (1977), Rajan et al. (1992), Abad (1996, 2001, 2003), Wei (1997), Wei and Law (2001), and Papachristos and Skouri (2003). Cohen (1977)'s model deals with simultaneously setting price and production levels for an exponentially decaying product under standard EOQ cost assumptions. The model proposed by Rajan et al. (1992) jointly determines the optimal replenishment cycle and price for inventory that is subject to continuous decay and value drop over the inventory cycle, where the price is a function of time. Abad (1996) extended the model of Rajan et al. (1992) by allowing shortages that can be partially backlogged at the end of the cycle. The subsequent models (Wei, 1997; Wei and Law, 2001; Abad, 2001, 2003; Papachristos and Skouri, 2003) are variants of Rajan et al. (1992) and Abad (1996) that deal with the problem under a single period setting and considering additional factors such as quantity discount, partially backlogged, or economic production quantity. Instead, this work develops an optimization-based scheme as a dynamic integrated production and marketing decision-making model using a calculus-based formulation combined with dynamic programming (DP) to solve the joint retail price and replenishment quantity/scheduling decision problem for deteriorating goods within a vertically decentralized single-manufacturer and single-retailer supply chain in both EMs and TMs.

This work differs from the aforementioned work, in several aspects. First, this work considers the dynamic joint decision problem over a finite planning horizon, which represents the product lifecycle and consists of multiple inventory cycles. In the aforementioned work, the pricing and replenishment policy are jointly determined within an inventory cycle. Second, this study examines the dynamic performance of decentralized channels under a retailer-managed inventory (RMI) system with a price-only (PO) contract and a VMI system with a consignment contract in EMs and TMs, respectively; the channel sells a perishable product with a multivariate demand function of price and time. A third difference between this work and past research is that the proposed scheme may adjust the selling price upward or downward periodically over the planning horizon that makes the pricing policy more responsive to the fluctuations of supply or demand over product lifecycle. The models proposed by Rajan et al. (1992) and Abad (1996) can only adjust the price downward, due to the value drop effect, within the inventory cycle. One of their major drawbacks is the inability to react to the market trend in demand over product lifecycle. In other words, they focused on short-term control over a single inventory cycle; this work dealt with mid-term to long-term control over the product lifecycle. Other models proposed by Cohen (1977), Wei (1997), Wei and Law (2001), Abad (2001, 2003), and Papachristos and Skouri (2003) assume that the endogenous price is fixed over the inventory cycle. Additionally, this research proposes the dynamic joint channel decision problem taking time-value of money into account, i.e., the net present value (NPV) approach. The reason of adopting NPV is due to its practical use in business planning and financial decision making (Sun and Queranne, 2002).

The remainder of this study is organized as follows. Section 2 describes the problem context. Sections 3 and 4 develop the mathematical models and solution procedures for decentralized channels under RMI with a PO contract and VMI with a consignment contract in B2B TMs and EMs, respectively. Section 5 compares solutions generated by the four proposed policies and applies sensitivity analysis to key parameters. Finally, Section 6 presents concluding remarks and suggestions for future research.

2. The problem context

This section characterizes the problem and contexts, including assumptions and the necessary notations. An RMI system with a PO contract is typical in vertically decentralized channel coordination, namely, the traditional trading form in which the retailer is
responsible for replenishment schedule/quantity and retail price decisions. Recently, one essential channel coordination mechanism is the VMI system with consignment contracts, in which the manufacturer is responsible for replenishment scheduling/quantity and retail price decisions. Hence, this work develops mathematical models and solution procedures for decentralized channels under RMI with a PO contract and VMI with a consignment contract in TMs and EMs, respectively. This work considers a single perishable item, whose retail price and replenishment scheduling/quantity are reviewed periodically at time \( t, t=0, 1, 2, \ldots, H \), where \( H \) is the planning horizon. Each period begins with a joint decision about scheduling a replenishment (if any) and its associated retail price. The problem is equivalent to determining the optimal sequence of issuance times for new replenishment \( z_{i-1} \), \( i=1, 2, \ldots, n \), with the retail price being reset and the lot-size specified simultaneously to maximize the profit stream over \([0, H]\). Notably, each new replenishment at \( z_{i-1} \) is set for a selling period over \([z_{i-1}, z_i]\). For sake of conciseness, we assume replenishment is instantaneous, with no in-transit inventory between upstream and downstream entities in a channel and no shortages exist.

The demand function considered in this work should satisfy the following assumptions: (i) \( D(p, t) \) is decreasing in \( p \); (ii) \( \lim_{p \to 0} D(p, t) < +\infty \) and \( \lim_{p \to \infty} D(p, t) = 0 \) for \( t \geq 0 \); and (iii) \( D(p, t) \geq 0 \) for \( p > 0 \), \( t \geq 0 \). The assumptions are assertions that demand is nonnegative, changes continuously with price and time, decreases as price increases, and is finite. There are many forms of demand functions that satisfy the assumptions (see Lau and Lau, 2003). The models and numerical analyses in this work assume the demand function has the form: \( D(p, t) = f(p)g(t) \), where \( f(p) = a - bp \), both \( a \) and \( b > 0 \), and \( g(t) = e^{\lambda t} \). Since \( e^{\lambda t} \) is nonnegative for all \( \lambda \), the demand function \( D(p, t) = (a - bp)e^{\lambda t} \) satisfies the three assumptions above. Besides, the sales trend over product lifecycle is characterized by \( \lambda \), positive (or negative) value of which represents upward (or downward) demand rate over time, and \( \lambda = 0 \) represents the time-invariant demand. Choosing this particular form is for its simplicity and for streamlining further discussions on the optimality of the discount profit maximization problem. Furthermore, this particular demand function is commonly used in the literature (see, for example, Wee, 1995; Wee and Wang, 1999; Papachristos and Skouri, 2000). Other multi-variate demand models have been proposed in pricing/inventory research such as Urban and Baker (1997) and Smith and Achabal (1998), which considered the demand rate as a function of price, time, and inventory level. Lau and Lau (2003) provided in-depth discussions on the effects of applying different demand curve functions on pricing/inventory decisions in single tiered as well as multi-tiered channels. In the profit maximization problem, revenue is generated from market sales and all relevant inventory expenses comprise holding, deterioration, replenishment, purchase, production, and transaction costs. All future revenue and costs are subject to the effects of inflation and time discounting. The net discount rate of inflation is assumed constant over time; that is, \( R = \rho - \varsigma \), where \( \varsigma \) is the inflation rate and \( \rho \) is the discount rate representing the time value of money. The value of relevant revenue or cost, i.e., the discount cash flow, is \( X_te^{-\rho t} \) for \( t \geq 0 \), where \( X_t \) is the value of \( X \) at time \( t \). The following notations are used throughout this work.

- \( \pi = \) Profit generated over period \([z_{i-1}, z_i]\)
- \( \Pi_{z_i} = \) Profit generated over period \([0, z_i]\)
- \( c = \) Per unit production cost for the manufacturer
- \( h = \) Per unit holding cost for a given unit of time
- \( w = \) Per unit purchase cost for the retailer
- \( \theta = \) Constant product deterioration rate
- \( r = \) Fraction of channel revenue retained by the retailer
- \( S_k = \) Replenishment operations cost per lot
- \( S_m = \) Production setup cost per run
- \( A_k = \) Per unit transaction cost for the retailer in the TM
- \( A_M = \) Per unit transaction cost for the manufacturer in the TM
- \( \rho_r = \) Percentage of transaction value from the retailer to the owner of the EM
- \( \rho_M = \) Percentage of transaction value from the manufacturer to the owner of the EM

3. The model under RMI with PO contracts

In a vertically decentralized channel, channel coordination typically operates such that a make-to-order manufacturer produces products using a lot-for-lot policy with a unit cost \( c \) and sells them through a coordinating retailer under RMI with a PO contract with a wholesale price; that is, the manufacturer charges the retailer the wholesale price per unit purchased, and the retailer then sets the retail price and replenishment schedule/quantity. Product ownership is transferred from the manufacturer to the retailer once the retailer receives the products. Two alternatives exist for doing business in a supply chain. One alternative is the TM mode, in which both the retailer and manufacturer in the channel conduct business using the traditional trading form. The other alternative is the EM mode, in which companies in the channel conduct commerce transactions electronically over the Internet. This work first applies the TM and then the EM to the discount profit maximization problem in a decentralized channel under RMI with a PO contract.

3.1. The base model: RMI with a PO contract in the TM (policy I)

The base model considers RMI with a PO contract in the TM for a vertically decentralized two-echelon supply chain with one manufacturer and one retailer. The retailer has the right to determine the replenishment schedule/quantity and retail price for the goods, and focuses only on profit maximization over the multi-period planning horizon, without considering the counterpart’s reaction and channel-wide profit. The retailer owns the goods and holds an inventory of goods, which is subject to a constant deterioration rate, and replenishes inventory from the manufacturer, which incurs a replenishment operations cost to meet deterministic price-dependent and time-varying demand. The work first derives a generic model for determining optimal joint decisions: retail price and lot-size over an arbitrary selling period \([z_{i-1}, z_i]\), and proves solution uniqueness. The model is then extended to a multi-period formulation, for which the original solution of replenishments, \( z_{i-1} \), and associated retail price and lot-size for \( i=1, 2, \ldots, n \), are determined using DP.

In considering the demand requirement and effect of deterioration loss over an arbitrary selling period \([z_{i-1}, z_i]\), inventory at time \( t \) can be represented by the differential equation:

\[
\frac{d}{dt} \log (p, t) = -\log (a - bp) e^{\lambda t}, \quad t \in [z_{i-1}, z_i].
\]

by multiplying \( e^{-t-z_{i-1}\theta} \) on both sides of the equation, integrating by part, and applying the assumption that the inventory level at the end of the cycle is zero, the inventory level at time \( t \) can be simplified as follows:

\[
l(p, t) = \int_t^{z_i} (a - bp) e^{\lambda u - \theta u - t} du.
\]

Derivation of Eq. (1) is given in the Appendix A. Via Eq. (1), the replenished quantity is the inventory level at the start of
replenishment

\[ Q(p) = l(p, z_{-1}) = \int_{z_{-1}}^{z_n} (a - bp) e^{\delta t + \theta(t - z_{-1})} dt. \quad (2) \]

In a decentralized channel, the retailer’s discount profit over the selling period \([z_{-1}, z_n]\) under RMI with a PO contract in the TM is the retailer’s discount revenue minus relevant costs, including purchase cost, holding cost, deterioration cost, retailer’s transaction cost, and replenishment operations cost, as follows:

\[
\Pi_{\text{RMI,TM}}(p) = \int_{z_{-1}}^{z_n} (a - bp)e^{\lambda t} dt - \int_{z_{-1}}^{z_n} w e^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - \int_{z_{-1}}^{z_n} A e^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - S e^{-R(t - z_{-1})} \]

\[
= \int_{z_{-1}}^{z_n} \left( (a - bp)e^{\lambda t} - \left( (a - bp)e^{\delta t + \theta(t - z_{-1})} - S e^{-R(t - z_{-1})} \right) \right) dt.
\]

The sequence of events for RMI with the PO model in the TM is as follows: both companies negotiate a wholesale price \(w\), based on which the retailer determines the coordinated retail price over given horizons \([z_{-1}, z_n]\) and \([z_{-1}, 0]\) for the supply chain. Hence, the optimal retail price that maximizes the retailer’s discount profit over period \([z_{-1}, z_n]\) can be acquired by taking the first-order derivative of Eq. (3) with respect to \(p\), and setting the result to equal zero as follows:

\[
\Pi'_{\text{RMI,TM}}(p) = \frac{(a(\theta + R) + (e^{\lambda t}(\theta + R) - e^{\lambda (t + \theta)}))h + (Aa + w(\theta + R)(\lambda - R))}{\lambda - R}.
\]

**Proposition 1.** The retailer’s discount profit function \(\Pi_{\text{RMI,TM}}(p)\) under RMI with a PO contract under the TM is concave in \(p\).

The proofs of Proposition 1 and the other propositions are detailed in the Appendix A. The corresponding discount profit generated from the selling period \([z_{-1}, z_n]\) for the manufacturer under RMI with a PO contract and in the TM comprises the discount wholesale revenue minus relevant costs, including variable production cost, manufacturer’s transaction cost, and production setup cost, as follows:

\[
\Pi_{\text{RMI,EM}}(p) = \int_{z_{-1}}^{z_n} (a - bp)e^{\lambda t} dt - \int_{z_{-1}}^{z_n} w e^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - \int_{z_{-1}}^{z_n} \rho_pe^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - \int_{z_{-1}}^{z_n} \rho_pe^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - S e^{-R(t - z_{-1})} \]

\[
= \int_{z_{-1}}^{z_n} \left( (a - bp)e^{\lambda t} - \left( (a - bp)e^{\delta t + \theta(t - z_{-1})} - S e^{-R(t - z_{-1})} \right) \right) dt.
\]

Substituting \(p\) for \(\rho\) into Eqs. (2), (3), and (5) yields the optimal quantity, \(Q_{\text{RMI,EM}}\), the maximal retailer’s discount profit, \(\Pi_{\text{RMI,EM}}\), and the associated maximal channel-wide discount profit, \(\Pi_{\text{RMI,EM,c}}\), over an arbitrary period \([z_{-1}, z_n]\). In the decentralized channel coordination under RMI with a PO contract in the TM, the retailer determines the sequence of replenishment and associated retail price for products. To extend a single-period planning horizon to a multi-period planning horizon, this work applies DP to determine the optimal replenishment schedule, and associated retail price, replenishment quantity, and discount profits of the retailer, manufacturer and channel-wide according to RMI with a PO contract in the TM as follows:

\[
\Pi^{**}_{\text{RMI,TM,RZ}} = \max \{ \Pi^{**}_{\text{RMI,TM,RZ},t + 1} + \Pi^{**}_{\text{RMI,TM,RZ},t} : 0 \leq z_{t-1} < z_t \leq H \},
\]

with boundary condition \(\Pi^{**}_{\text{RMI,TM,RZ},0} = 0\). This recursive procedure works in a forward fashion to determine maximal retailer’s total discount profit over the time horizon. In the last stage of the procedure, \(\Pi^{**}_{\text{RMI,TM,RZ}}\) is derived, which is the maximal retailer’s total discount profit over the planning horizon. The optimal replenishment schedule and retail price over the time horizon, denoted by \((z^{**}_{t-1}, \Pi^{**}_{\text{RMI,TM,RZ},t}, \Pi^{**}_{\text{RMI,TM,RZ},t} \Pi^{**}_{\text{RMI,TM,RZ},t-1})\), can be acquired by tracking backward from time \(H\) to time \(0\).

**3.2. RMI with a PO contract in the EM (policy II)**

In this work, a firm operating in an EM is an independent third party, which enhances the ability of marketing to assign resources optimally. The manufacturer and retailer must pay the owner of the EM a transaction fee per unit when they join the EM. The transaction fee per unit in the EM may be a transaction-based commission fee or referral fee charged by the owner of the EM; this fee is typically a percentage of transaction value (i.e., the fraction of discount revenue the retailer and manufacturer earn), denoted by \(\rho_H (0 \leq \rho_H \leq 1)\) and \(\rho_M (0 \leq \rho_M \leq 1)\) for the retailer and manufacturer, respectively. According to the above definition, the retailer’s discount profit over the selling period \([z_{-1}, z_n]\) is the retailer’s discount revenue minus relevant costs associated with purchase cost, holding cost, deterioration cost, retailer’s transaction fee, and replenishment operations cost

\[
\Pi_{\text{RMI,EM}}(p) = \int_{z_{-1}}^{z_n} (a - bp)e^{\lambda t} dt - \int_{z_{-1}}^{z_n} w e^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - \int_{z_{-1}}^{z_n} \rho_pe^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - \int_{z_{-1}}^{z_n} \rho_pe^{-R(t - z_{-1})} (a - bp)e^{\delta t + \theta(t - z_{-1})} dt - S e^{-R(t - z_{-1})} \]

\[
= \int_{z_{-1}}^{z_n} \left( (a - bp)e^{\lambda t} - \left( (a - bp)e^{\delta t + \theta(t - z_{-1})} - S e^{-R(t - z_{-1})} \right) \right) dt.
\]

The optimal retail price under RMI with a PO contract in the EM that maximizes the retailer’s discount profit over period \([z_{-1}, z_n]\) can be obtained by taking the first-order derivative of Eq. (7) with respect to \(p\) and setting the result to equal zero

\[
p_{\text{RMI,EM}} = \frac{(e^{\lambda t}(\theta + R) - e^{\lambda (t + \theta)})(h + w(\theta + R)(\lambda - R)) + (e^{\lambda t}(\theta + R) - e^{\lambda (t + \theta)}))h + (Aa + w(\theta + R)(\lambda - R))}{\lambda - R}.
\]

**Proposition 2.** The retailer’s discount profit function, \(\Pi_{\text{RMI,EM}}(p)\), under RMI with a PO contract in the EM is concave in \(p\).
cost
\[ \pi_{RMI,EM} = \int_{Z_{i-1}}^{Z_i} \left( e^{-z_i} - \left( (1 - \rho_M) e^{-R_i} - e^{-R_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}} \right) \]
\[ = \int_{Z_{i-1}}^{Z_i} \left( (1 - \rho_M) (1 - r) e^{-R_i} - c + \lambda_R e^{-\lambda_R z_i} - \lambda_R e^{-\lambda_R z_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}}. \]

(9)

Following a similar procedure developed for RMI with a PO contract in the TM, this work can obtain the optimal replenishment sequence and associated retail price, denoted by \( Z_{i-1,RMI,EM;RMI,EM}^{*}(P) \), for a multi-period planning horizon under RMI with a PO contract in the EM via DP as follows:

\[ \Pi_{RMI,EM,R_i}^{*} = \max \left\{ \Pi_{RMI,EM,R_i}^{*} + \pi_{RMI,EM,R_i}^{*} : 0 \leq z_{i-1} < z_i \leq H \right\}. \]

(10)

4. The model under VMI with consignment contracts

A VMI system is sometimes called a vendor-managed replenishment (VMR) system, in which the retailer and manufacturer partnership can be seen as a continuum. One end of this continuum comprises information sharing, which helps the manufacturer plan effectively, while the other end comprises a replenishment program, in which the manufacturer manages all operations involved in replenishment of goods. This section formulates the discount profit-maximization problem under VMI with a consignment contract for a decentralized channel in two transaction markets, the TM and EM, respectively.

4.1. VMI with a consignment contract in the TM (policy III)

In the decentralized channel coordination under VMI and a consignment contract in the TM, the manufacturer owns the goods title and determines the sequence of replenishment and retail price over the multi-period planning horizon. Let \( r(0 \leq r \leq 1) \) be the fraction of channel revenue the retailer keeps; thus, \( (1 - r) \) is the fraction the manufacturer earns. According to the above definitions, the manufacturer’s discount profit during an arbitrary selling period \([z_{i-1}, z_i]\) is the channel discount revenue that the manufacturer earns minus associated costs, including variable production cost, manufacturer’s transaction cost, holding cost, deterioration cost, production setup cost, and replenishment operations cost

\[ \pi_{VMICS,TM}(P) = \int_{Z_{i-1}}^{Z_i} \left( 1 - r \right) e^{-R_i} \left( c + \lambda_R e^{-\lambda_R z_i} - \lambda_R e^{-\lambda_R z_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}}. \]

(11)

The order of events under VMI with a consignment model in the TM is as follows. Both firms negotiate a revenue-sharing allocation \( r \) first, based on which the manufacturer determines the coordinated retail price over given \( z_{i-1} \) and \( z_i \) for the supply chain. Hence, the optimal retail price that maximizes the manufacturer’s discount profit over period \([z_{i-1}, z_i]\) can be derived by taking the first-order derivative of Eq. (11) with respect to \( p \), and setting this result to equal zero as follows:

\[ P_{VMICS,TM}^{*} \]

\[ = \int_{Z_{i-1}}^{Z_i} \left( e^{-z_i} - \left( (1 - \rho_M) e^{-R_i} - e^{-R_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}} \right) \]

(12)

Proposition 3. The manufacturer’s discount profit function, \( \pi_{VMICS,TM}(P) \), under VMI with a consignment contract in the TM is concave in \( p \).

The corresponding retailer’s discount profit generated under VMI with a consignment contract in the TM during the selling period \([z_{i-1}, z_i]\) is the channel discount revenue, which the retailer keeps, minus the retailer’s discount transaction cost

\[ \pi_{VMICS,EM}^{*} \]

\[ = \int_{Z_{i-1}}^{Z_i} \left( rpe^{-R_i} - A_M e^{\lambda_R z_i} - \lambda_R e^{-\lambda_R z_i} \right) (a - bp) e^{zt} dt. \]

(13)

The optimal replenishment sequence and associated retail price, denoted by \( Z_{i-1,VMICS,EM;VMICS,EM}^{*}(P) \), for a multi-period planning horizon under VMI with a consignment contract in the TM can be obtained through DP using a method similar to that used under RMI with a PO contract in the TM as follows:

\[ \Pi_{VMICS,EM,Z_i}^{*} = \max \left\{ \Pi_{VMICS,EM,Z_i}^{*} + \pi_{VMICS,EM}^{*} : 0 \leq z_{i-1} < z_i \leq H \right\}. \]

(14)

4.2. VMI with a consignment contract in the EM (policy IV)

In a similar manner, the manufacturer decides on both the replenishment schedule/quantity and retail price over the multi-period planning horizon for decentralized channel coordination under VMI with a consignment contract in the EM. The manufacturer’s discount profit during an arbitrary selling period \([z_{i-1}, z_i]\) is the channel discount revenue the manufacturer earns minus associated costs, including the manufacturer’s transaction fee, variable production cost, holding cost, deterioration cost, production setup cost, and replenishment operations cost

\[ \pi_{VMICS,EM}(P) = \int_{Z_{i-1}}^{Z_i} \left( 1 - r \right) e^{-R_i} \left( c + \lambda_R e^{-\lambda_R z_i} - \lambda_R e^{-\lambda_R z_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}}. \]

(15)

The optimal retail price that maximizes the manufacturer’s discount profit over period \([z_{i-1}, z_i]\) under VMI with a consignment contract in the TM can be obtained by taking the first-order derivative of Eq. (15) with respect to \( p \), and setting the result to equal zero as follows:

\[ P_{VMICS,EM}^{*} \]

\[ = \int_{Z_{i-1}}^{Z_i} \left( e^{-z_i} - \left( (1 - \rho_M) e^{-R_i} - e^{-R_i} \right) (a - bp) e^{zt} dt - S_M e^{-R_{i-1}} \right) \]

(16)
The manufacturer’s discount profit function $\Pi_{\text{VMICS,EM}}(p)$ under VMI with a consignment contract in the EM is concave in $p$.

The retailer’s corresponding discount profit under VMI with a consignment contract in the EM over the selling period $[z_{i-1}, z_i]$ is the retailer’s discount revenue minus the retailer’s discount transaction fee

$$ \pi_{\text{VMICS,EM,R}} = \int_{z_{i-1}}^{z_i} \left(1 - \rho_k \right) p_r e^{-\rho_k (a - bp)e^{ct} dt}. \quad (17) $$

The optimal replenishment schedule and associated retail price for a multi-period planning horizon under VMI with a consignment contract in the EM is denoted by $(z_{i-1}, \text{VMICS,EM} \cdot \pi_{\text{VMICS,EM,M}^*_i})$, and can be obtained by

$$ \Pi_{\text{VMICS,EM,M}, z_i}^* = \max \{ \Pi_{\text{VMICS,EM,M}, z_{i-1}} + \pi_{\text{VMICS,EM,M}}^* : 0 \leq z_{i-1} < z_i \leq H \}. \quad (18) $$

5. Numerical analysis

This work conducted several experiments to obtain qualitative insights into the structures of the proposed policies and their sensitivity to key parameters. The four policies in this work, incorporating the proposed solution procedures, were implemented on a personal computer with a 2.13-GHz ×2.13-GHz Pentium CPU and 1.5 GB RAM with Windows 7 running Mathematica. The computing time required for DP was, on average, < 15 s. This work focused particularly on investigating the benefit generated from VMI with a consignment contract in the EM (policy IV).

5.1. An illustrative example

The basic settings in this work are as follows: number of periods $H = 15$; cost parameters $S_M = 100$, $S_p = 50$, $A_H = 0.5$, $A_R = 0.1$, $\rho_M = 0.02$, $\rho_R = 0.01$, $c = 1$, $w = 5$, and $h = 0.05$; rate of deterioration $\theta = 0.05$; net discount inflation rate $R = 0.05$; fraction of channel revenue $r = 0.4$; and demand parameters $a = 500$, $b = 25$, and $\lambda = -0.1$. This scenario simulates the diminishing effect of the demand function toward the end of a product lifecycle. Table 1 lists numerical results generated by policies I–IV, including number of replenishments, $n$, and the associated replenishment schedule, $(z_{i-1}^*, z_i^*)$, for $i = 1, 2, \ldots, n$, optimal retail price, $p^*$, lot size, $Q^*$, retailer’s discount profit, $\pi_{\text{VMICS,EM}, M}^*$, manufacturer’s discount profit, $\pi_{\text{VMICS,EM}}^*$, and channel-wide discount profit, $\pi_{\text{VMICS}}^*$.

Under VMI with a consignment contract, the EM (policy IV) generates lower retail prices (11,046, 11,12, and 11,179) than the TM (policy III) (11,505, 11,592, and 11,68), and accordingly yields a higher total replenishment quantity (1886.6 vs. 1786.6). In terms of total discount profits, the EM outperforms the TM by 7.62% (12,335.4 vs. 11,585.8) for the channel-wide, 2.92% (5835.7 vs. 5672.9) for the retailer, and 9.92% (6497.9 vs. 5912.9) for the manufacturer. Under RMI with a PO contract, numerical results are similar.

Comparing solutions generated by the four policies, VMI with a consignment contract in the EM (policy IV) tends to generate lower retail prices, larger total demand, and more channel-wide total discount profit (12,335.4 vs. 10,862.3, 11,452.3, and 11,585.8 for policy IV vs. policies I, II, and III) for the decentralized channel. Furthermore, VMI with a consignment contract in the EM (policy IV) is unacceptable to the retailer when compared with the base model (RMI with a PO contract in the TM (policy I)), because of a lack of Pareto improvements in two-echelon channel coordination (6731.7 vs. 5835.7 for the retailer’s total discount profit with policy I vs. that with policy IV). To achieve a win-win situation for both parties in the channel, an extra one-part tariff (i.e., slotting allowance from the manufacturer to the retailer) is required to coordinate the cooperative decentralized channel effectively. Table 2 summarizes numerical details. The case with the fraction of channel revenue $r = 0.4$ is equivalent to policy IV with the retailer’s total discount profit of 5835.7, manufacturer’s

### Table 1

Numerical results generated by the policies in the base case.

<table>
<thead>
<tr>
<th>Policies</th>
<th>$r$</th>
<th>$[z_{i-1}^<em>, z_i^</em>]$</th>
<th>$p^*$</th>
<th>$Q^*$</th>
<th>$\pi_{\text{VMICS,EM}, M}^*$</th>
<th>$\pi_{\text{VMICS,EM}}^*$</th>
<th>$\pi_{\text{VMICS}}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMI with a PO contract in the TM (policy I)</td>
<td>1</td>
<td>[0, 4]</td>
<td>13.125</td>
<td>623.1</td>
<td>3045.0</td>
<td>2081.0</td>
<td>5585.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>[4, 8]</td>
<td>13.125</td>
<td>477.7</td>
<td>1909.8</td>
<td>1115.1</td>
<td>3024.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>[8, 15]</td>
<td>13.566</td>
<td>426.9</td>
<td>1317.4</td>
<td>934.5</td>
<td>2251.9</td>
</tr>
<tr>
<td></td>
<td>$\Sigma$</td>
<td></td>
<td>1467.7</td>
<td>6731.7</td>
<td>4130.6</td>
<td>10862.3</td>
<td></td>
</tr>
<tr>
<td>RMI with a PO contract in the EM (policy II)</td>
<td>1</td>
<td>[0, 4]</td>
<td>13.096</td>
<td>625.8</td>
<td>3499.0</td>
<td>2340.6</td>
<td>5839.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>[4, 7]</td>
<td>12.951</td>
<td>329.1</td>
<td>1589.7</td>
<td>969.0</td>
<td>2558.6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>[7, 11]</td>
<td>13.096</td>
<td>310.8</td>
<td>1206.7</td>
<td>783.6</td>
<td>1990.3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>[11, 15]</td>
<td>13.096</td>
<td>208.3</td>
<td>652.7</td>
<td>411.0</td>
<td>1063.8</td>
</tr>
<tr>
<td></td>
<td>$\Sigma$</td>
<td></td>
<td>1474.0</td>
<td>6948.1</td>
<td>4504.2</td>
<td>11452.3</td>
<td></td>
</tr>
<tr>
<td>VMI with a consignment contract in the TM (policy III)</td>
<td>1</td>
<td>[0, 3]</td>
<td>11.505</td>
<td>591.6</td>
<td>2301.9</td>
<td>1464.9</td>
<td>4766.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>[3, 6]</td>
<td>11.505</td>
<td>438.3</td>
<td>1467.8</td>
<td>958.2</td>
<td>3006.0</td>
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<td></td>
<td>3</td>
<td>[6, 10]</td>
<td>11.592</td>
<td>418.2</td>
<td>1816.0</td>
<td>1185.7</td>
<td>2346.7</td>
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<tr>
<td></td>
<td>4</td>
<td>[10, 15]</td>
<td>11.680</td>
<td>338.5</td>
<td>742.2</td>
<td>724.1</td>
<td>1466.3</td>
</tr>
<tr>
<td></td>
<td>$\Sigma$</td>
<td></td>
<td>1786.6</td>
<td>5672.9</td>
<td>5912.9</td>
<td>11585.8</td>
<td></td>
</tr>
<tr>
<td>VMI with a consignment contract in the EM (policy IV)</td>
<td>1</td>
<td>[0, 3]</td>
<td>11.046</td>
<td>623.6</td>
<td>2365.5</td>
<td>2697.4</td>
<td>5062.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>[3, 6]</td>
<td>11.046</td>
<td>450.2</td>
<td>1508.0</td>
<td>1686.5</td>
<td>3194.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>[6, 10]</td>
<td>11.112</td>
<td>442.1</td>
<td>1195.7</td>
<td>1309.0</td>
<td>2504.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>[10, 15]</td>
<td>11.179</td>
<td>358.9</td>
<td>766.2</td>
<td>806.8</td>
<td>1573.0</td>
</tr>
<tr>
<td></td>
<td>$\Sigma$</td>
<td></td>
<td>1886.6</td>
<td>5835.7</td>
<td>6499.7</td>
<td>12335.4</td>
<td></td>
</tr>
</tbody>
</table>
5.2. Sensitivity analysis

Under the above base settings, the sensitivity of vertically decentralized system efficiency under VMI with a consignment contract in the EM (policy IV) vs. the base model (i.e., RMI) is analyzed with respect to key parameters—the magnitude of potential demand, $a$, coefficient of price sensitivity, $b$, net discount rate of inflation, $R$, and rate of deterioration, $\theta$. These values were changed by up to $\pm 50\%$ from base settings. Policy IV outperforms policy I significantly in channel-wide total discount profit as $a$ decreases and/or as $b$ increases (Fig. 1). Other parameters (i.e., $R$ and $\theta$) seem insignificant to channel-wide total discount profit improvements with policies I and IV.

6. Concluding remarks

This work summarizes emerging research on managing a vertically decentralized single-retailer and single-manufacturer multi-period channel that produces and sells deteriorating goods in a marketplace. This work formulated four decision models for decentralized channel coordination under RMI with a PO contract and VMI with a consignment contract in B2B EMs and TMs, respectively. Numerical results show that solutions generated in the EM outperform those in the TM in maximizing channel-wide total discount profits and those of manufacturer and retailer. Further, the proposed policy under VMI with a consignment contract in the EM significantly increases system efficiency (i.e., channel-wide total discount profit), and simultaneously achieves Pareto improvements using an extra one-part tariff: slotting allowance for the vertically decentralized channel. A natural extension of this work is to adjust the model to consider additional demand and deterioration functions, such as uncertain demand and fuzzy-modeled deterioration. Furthermore, future studies can focus on multiple retailers and/or suppliers.

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Appendix A

Derivation of Eq. (1). By considering the demand requirement and effect of deterioration loss during period $[z_{t-1}, z_t]$ we have

$$l(p, t) = \int_t^{z_t} (l(p, u)\theta + D(p, u))du$$

$$= \int_t^{z_t} l(p, u)\theta + (a - b)e^{z_{t-1}u}du, \text{ for } z_{t-1} \leq t \leq z_t.$$ 

Differentiating $l(p, t)$ with respect to $t$ yields

$$\frac{\partial}{\partial t}l(p, t) = -l(p, t)\theta - (a - b)e^{z_{t-1}u}.$$ 

Multiplying $e^{(z_{t-1} - z_t)\theta}$ on both sides of the equation yields

$$\frac{\partial}{\partial t}e^{(z_{t-1} - z_t)\theta} + l(p, t)\theta e^{(z_{t-1} - z_t)\theta} = -(a - b)e^{z_{t-1}u + (z_{t-1} - z_t)\theta}.$$ 

Combining the two terms on the left hand side of the equation, we have

$$\frac{\partial}{\partial t}e^{(z_{t-1} - z_t)\theta} = -(a - b)e^{z_{t-1}u + (z_{t-1} - z_t)\theta}.$$ 

Taking another integration yields

$$l(p, z_t)e^{(z_{t-1} - z_t)\theta} - l(p, t_0)e^{(z_{t-1} - z_t)\theta} = -\int_t^{z_t} (a - b)e^{z_{t-1}u + (z_{t-1} - z_t)\theta}du.$$ 

Noting that $l(p, z_0) = 0$, we finally have

$$l(p, t) = e^{-(z_{t-1} - z_0)\theta}\int_t^{z_t} (a - b)\theta e^{z_{t-1}u + (z_{t-1} - z_0)\theta}du$$

$$= \int_t^{z_t} (a - b)e^{z_{t-1}u + \theta e^{z_{t-1}u - z_0}}du.$$ 

Proof of Proposition 1. The second-order derivative of the retailer’s discount profit, Eq. (3), with respect to $p$ is

$$\frac{\partial^2 l(p, z_t)e^{(z_{t-1} - z_t)\theta}}{\partial p^2} = 2b(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda).$$ 

Since $b > 0$ and $(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda) < 0$ for $z_{t-1} < z_t$, regardless of the value of $\lambda$ and $R$, the second-order derivative of the retailer’s discount profit function under RMI with a PO contract in the TM with respect to $p$ is strictly negative. □

Proof of Proposition 2. The second-order derivative of the retailer’s discount profit, Eq. (7), with respect to $p$ is

$$\frac{\partial^2 l(p, z_t)e^{(z_{t-1} - z_t)\theta}}{\partial p^2} = 2b(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda).$$ 

Since $b > 0$, $0 \leq p \leq 1$, and $(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda) > 0$ for $z_{t-1} < z_t$, regardless of the value of $\lambda$ and $R$, the second-order derivative of the retailer’s discount profit function under RMI with a PO contract in the TM with respect to $p$ is negative. □

Proof of Proposition 3. The second-order derivative of the manufacturer’s discount profit, Eq. (11), with respect to $p$ is

$$\frac{\partial^2 l(p, z_t)e^{(z_{t-1} - z_t)\theta}}{\partial p^2} = 2b(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda).$$ 

Since $b > 0$, $0 \leq r < 1$, and $(e^{(z_{t-1} - z_t)\theta} - e^{(z_{t-1} - z_t)\theta}/R - \lambda) > 0$ for $z_{t-1} < z_t$, regardless of the value of $\lambda$ and $R$, the second-order derivative of
the manufacturer's discount profit function under VMI with a consignment contract in the TM with respect to \( p \) is negative.\( \square \)

**Proof of Proposition 4.** The second-order derivative of the manufacturer's discount profit, Eq. (15), with respect to \( p \) is
\[
\frac{\partial^2 \pi_{VMI,EM,EM}(p)}{\partial p^2} = 2b\left(e^{-(\lambda - R)} - e^{-1} \right) (1 - \lambda) - e^{-1} \left(1 - \frac{1}{\rho M P M} - \frac{1}{R - \lambda}\right).
\]
Since \( b > 0 \), \( 0 \leq \rho M P M \leq 1 \) and \( (e^{-(\lambda - R)} - e^{-1}) (1 - \lambda) / (R - \lambda) < 0 \) for \( \lambda < R \), regardless of the value of \( \lambda \) and \( R \), the second-order derivative of the manufacturer's discount profit function under VMI with a consignment contract in the EM with respect to \( p \) is negative.\( \square \)

**References**


